Malaya Journal of Matematik, Vol. 8, No. 4, 1580-1582, 2020

https://doi.org/10.26637/MJM0804/0042

Connected edge Detour global domination number of a graph

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Abstract

In this paper, we introduce the concept of connected edge detour global domination number of a graph is introduced. A subset D of the vertex set $V(G)$ of a connected graph G is called a connected edge detour global dominating set if D is an edge detour global dominating set and the induced subgraph $<D>$ is connected. The connected edge detour global domination number $\gamma_{cedg}(G)$ of G is the minimum cardinality taken over all connected edge detour global dominating sets in G. A connected edge detour global dominating set of cardinality $\gamma_{cedg}(G)$ is called a γ_{cedg}-set of G. We determine $\gamma_{cedg}(G)$ for some standard and special graphs and its properties are studied.

Keywords

Edge detour global domination number, connected edge detour global domination number.

AMS Subject Classification

05C12.

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Article History: Received **17** July **2020**; Accepted **21** September **2020**

Contents

1. Introduction

By a graph $G=(V,E)$, we consider a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by n, m respectively. Edge Detour Global Dominating graphs were introduced and studied by Punitha Tharani and Ferdina [12]. For underlying definition and results, see references [1-14].

Theorem 1.1. *For any connected graph of order $n \geq 2$. Then, $2 \leq dn(G) \leq \gamma_{dg}(G) \leq n$.*

Theorem 1.2. *Let G be a graph of order n. Then $\gamma_{dg}(G)=n$ iff G contains only end and full vertices.*

Theorem 1.3. *For the path graph $P_n, \gamma_{edg}(P_n) = \left\lceil \frac{n-4}{3} \right\rceil + 2, n \geq 5$*

Theorem 1.4. *For the complete graph $K_n, \gamma_{edg}(K_n) = n, n \geq 2$*

2. Connected Edge Detour Global Domination Number of a Graph

Definition 2.1. *A subset D of V of a connected graph $G=(V,E)$ is called a connected edge detour global dominating set of G if D is an edge detour global dominating set and the induced subgraph $<D>$ is connected. The Connected edge detour global domination number $\gamma_{cedg}(G)$ of G is the minimum cardinality taken over all connected edge detour global dominating sets in G. A connected edge detour global dominating set of cardinality $\gamma_{cedg}(G)$ is called a γ_{cedg}-set of G.*

Example 2.2. *Consider the graph G given in Figure 1.*

Here, $D_1=\{v_1,v_4,v_6\}$, $D_2=\{v_1,v_4,v_5\}$, $D_3=\{v_1,v_3,v_5\}$ are γ_{edg}-sets of G and so $\gamma_{edg}(G)=3$. Now $D_5=\{v_1,v_2,v_3,v_4\}$, $D_6=\{v_1,v_2,v_3,v_6\}$, $D_7=\{v_1,v_2,v_5,v_6\}$ are γ_{cedg}-set of G. Then $\gamma_{cedg}(G)=|D_5|=|D_6|=|D_7|=4$.

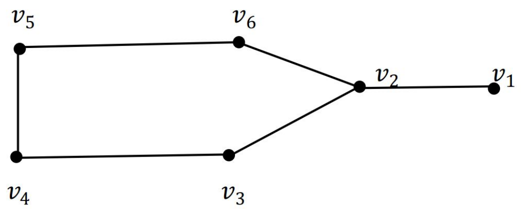

Figure 1

Remark 2.3. *Every γ_{cedg}-set is an edge detour global dominating set but the converse is not true. From the above figure, $D = \{v_1, v_4, v_5\}$ forms a γ_{edg}-set but not γ_{cedg}-set of* G.

Theorem 2.4. *Let* G *be a connected graph of order n. Then $2 \leq \gamma_{edg}(G) \leq \gamma_{cedg}(G) \leq n$.*

Proof. Let D be an edge detour global dominating set. Every set D needs at least two vertices so that $\gamma_{edg}(G) \geq 2$. Again, every connected edge detour global dominating set is an edge detour global dominating set, $\gamma_{\mathrm{ced}g}(G) \geq \gamma_{edg}(G)$ since the set of all vertices of G is always a connected edge detour global dominating set. Therefore $n \geq \gamma_{\mathrm{ced}g}(G)$. Hence $2 \leq \gamma_{edg}(G) \leq \gamma_{cedg}(G) \leq n$. ☐

Remark 2.5. *For a connected graph G with $n \geq 2$,*

(i) $\gamma_{dg}(G) \leq \gamma_{cedg}(G)$.

(ii) $\gamma_{edg}(G) \leq \gamma_{cedg}(G)$.

(iii) Strict inequality is also true in the above relation.

(iv) From the above Example 2.2 $n = 6, \gamma_{edg}(G) = 3$, $\gamma_{\mathrm{ced}g}(G) = 4$, the bound (Theorem 2.4) is sharp.

Observation 2.6. *(i) Path P_n of order $n (n \geq 2), \gamma_{cedg}(P_n) = |V(P_n)|$.*

(ii) Cycle C_n of order $n(n \geq 3), \gamma_{cedg}(C_n) = |V(C_n)| - 2$.

(iii) Complete graph K_n of order $n(n \geq 2), \gamma_{cedg}(K_n) = |V(K_n)|$.

(iv) complete bipartite graph $K_{m,n}$.

$$\gamma_{\mathrm{ced}g}(K_{m,n}) = \left\{ \begin{array}{cc} 2 & \text{if } m = n = 1 \\ |V(K_{m,n})| - m + 1 & \text{if } n \geq 2, m = 1 \\ 3 & \text{if } m, n \geq 2 \end{array} \right\}$$

(v) Star graph $K_{1,n}, \gamma_{cedg}(K_{1,n}) = |V(K_{1,n})|$.

(vi) Bistar graph $B_{n,n}, \gamma_{cedg}(B_{n,n}) = 2n + 2$.

(vii) Wheel graph $W_n (n \geq 5), \gamma_{ced\,g}(W_n) = 3$.

Theorem 2.7. *Every γ_{cedg}-set of a connected graph G contains all the pendant vertices of G.*

Proof. Let D be a connected edge detour global dominating set of G. Then every set D contains all the pendant vertices, since the pendant edges lie only in the detour joining the corresponding pendant vertex with some other vertex. ☐

Theorem 2.8. *Every $\gamma_{\mathrm{ced}g}$-set of a connected graph G contains all the vertices of* G *has degree $n - 1$.*

Proof. Let w be a vertex of a connected graph G has degree $n - 1$. Then the vertex w belongs to every dominating set in the complement $\bar{G}$ of G. since w is dominate itself in $\bar{G}$. Then all the full vertices of G belong to the global dominating set of G. Hence, every γ_{cedg}-set contains all the full vertices. ☐

Theorem 2.9. *Let G be a connected graph of order $n \geq 2$ and D be a γ_{cedg}-set of G. Then for any cut vertex x of G, every component of $G - x$ contains an element of D.*

Proof. Let x be a cut vertex of a connected graph G and D be a connected edge detour global dominating set. Let H be one of the components of $G - x$. Suppose no vertex of D belongs to H. Then any pendant vertex of G does not belong to H (by Theorem 2.7). Therefore, H has at least one edge, say $u_t u_{t+1}$. Since D is a $\gamma_{\mathrm{ced}g}$-set, there exists vertices $u, w \in D$ such that $u_t u_{t+1}$ lies on some $u - w$ detour. $P; u = u_1, u_2, \ldots u_t, u_{t+1}, \ldots, u_n = w$ in G or both the ends u_t and u_{t+1} of the edge $u_t u_{t+1}$ are in D. Suppose that $u_t u_{t+1}$ lies on the detour P. Let P_a be the subpath of P, say $u - u_t$ and P_b be the subpath of P, say $u_t - w$. Since x is a cut vertex of G, then x belongs to both P_a and P_b so that P is not a detour, which is a contradiction to the fact. Suppose that u_t and u_{t+1} are in D, then H contains vertices of D, which is again a contradiction. ☐

Theorem 2.10. *Every γ_{cedg}-set of a connected graph G contains all the cut vertices of G.*

Proof. Let x be a cut vertex of a connected graph G of order $n \geq 2$ and D be a connected edge detour global dominating set of G. Then $G - x$ has more than one component, say $G_1, G_2, \ldots, G_i (i \geq 2)$. Then γ_{cedg}-set D contains at least one vertex from each component $G_k (1 \leq k \leq i)$ of $G - x$ (by Theorem 2.9). Since induced subgraph $< D >$ is connected it follows that $x \in D$. ☐

Corollary 2.11. *Every γ_{cedg}-set of a connected graph G contains pendant vertices, full vertices and cut vertices of G.*

Proof. The proof follows from Theorem 2.7, 2.8 and 2.10. ☐

Corollary 2.12. *For any tree T of n vertices, $\gamma_{cedg}(T) = |V(T)|, n \geq 2$.*

Proof. The proof follows from Corollary 2.11. ☐

Corollary 2.13. *Let G be any connected graph with l pendant vertices, m full vertices and n cut vertices. Then* $\max\{2, l + m + n\} \leq \gamma_{\mathrm{ced}g}(G) \leq n$.

Proof. The proof follows from Theorem 2.4 and Corollary 2.11. ☐

Theorem 2.14. *For $3 \leq j \leq n (\forall j, n \in \mathbb{Z})$, there exists a connected graph* G *of order* n *with $\gamma_{\mathrm{ced}g}(G) = j$.*

Proof. **Case 1:** If $j = n$, Let $G = P_n$. Then by Observation 2.6 (i), $\gamma_{cedg}(G) = j$.
Case 2. If $3 = j < n$, Let $G = W_n$. Then by Observation 2.6 (vii), $\gamma_{cedg}(G) = j$.
Case 3. $3 < j < n$, Let G be a connected graph obtained from W_{n-j+3}. Let $V(G) = \{v, v_1, v_2, v_3, \ldots, v_{n-j+2}, w_1, w_2, \ldots, w_{j-3}\}$. The graph G is shown in Figure 2.
Let $V(W_{n-j+3}) = \{v, v_1, v_2, v_3, \ldots, v_{n-j+2}\}$ and $w_1, w_2, \ldots,$

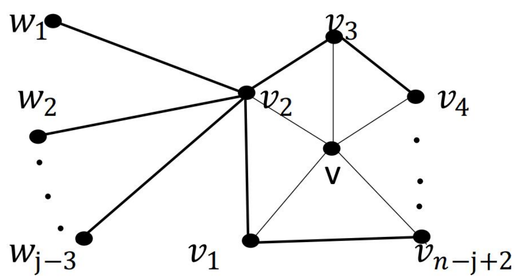

Figure 2

w_{j-3} be the new vertices which are joining to v_2. Now we have to prove that $\gamma_{cedg}(G) = j$. Then the set $D = \{w_1, w_2, \ldots, w_{j-3}\}$ together with a cut vertex v_2 is a subset of every γ_{cedg}-set G. It is clear that D is a global dominating set but not an edge detour set of G. Let $D' = D \cup \{v, v_{j-3}\}$. Then every edge of G lies on a detour joining a pair of vertices of D'. Clearly, the set D' is γ_{edg}-set and $< D' >$ is connected. Therefore, D' is a connected edge detour global dominating set of minimum cardinality,

$$\begin{aligned}
|D'| &= |D \cup \{v, v_{j-3}\}| \\
&= |D| + |\{v, v_{j-3}\}| \\
&= |\{w_1, w_2, \ldots, w_{j-3}\}| + |v_2| + |\{v, v_{j-3}\}| \\
&= j - 3 + 1 + 2 = j.
\end{aligned}$$

Hence $\gamma_{cedg}(G) = j$. □

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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