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Double domination on bipolar fuzzy graphs with strong edge and its properties

R. Muthuraj [1*] and A. Kanimozhi[2]

Abstract
In this article, the definition of double dominance is introduced in the bipolar fuzzy graph. It provides descriptions of the size, order, degree etc of a bipolar fuzzy graph. With sufficient examples, the double dominance number of a bipolar fuzzy graph has been clarified. It addresses the properties of double dominance on the bipolar fuzzy graph. Some simple theorems have also been proposed relating to the claimed supremacy.

Keywords
Bipolar fuzzy graph, dominating set, double dominating set, doubledomination number on bipolar fuzzy graph.

AMS Subject Classification
03E72, 03E55.

[1] *PG and Research Department of Mathematics, H.H. The Rajah's College, Pudukkottai–622 001, Tamil Nadu, India.*
[2] *Research Scholar, PG and Research Department of Mathematics, H.H. The Rajah's College, Pudukkottai–622 001, Tamil Nadu, India.*
[2] *Department of Mathematics, NPR College of Engineering and Technology, Natham- 624401, Tamil Nadu, India.*
[1,2] *Affiliated to Bharathidasan University, Tiruchirappalli- 620024, Tamil Nadu, India.*
*Corresponding author: rmr1973@yahoo.co.in; [2]kanimozhi2k22@yahoo.com.
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Contents

1. Introduction

From the definition of fuzzy relation introduced by L.A. Zadeh [13] in the year 1965, Kaufmann. A, first presented the idea of fuzzy graph. Another comprehensive definition was introduced by Rosenfeld [10] in 1975, including fuzzy vertex and fuzzy edges and other fuzzy analogues of graph theoretical concepts such as paths, cycles, connectedness, etc. In 1998, A. Somasundaram, S. Somasundaram [11] studied the definition of dominance in fuzzy graphs. In the year 2000, Harey and Haynes [2] introduced the idea of double dominance in Graphs. In the year 2011, Muhammad Akram [5,6] first presented the idea of a bipolar fuzzy graph (BFG) and also presented the idea ofa regular BFG in 2012. In the year 2015, Nagoor Gani, Muhammed Akram and Anupriya [9] defined the concept of double dominance on intuitionistic fuzzy graph. In this article, the idea of double dominance is extended to BFG and discussed its properties.

2. Preliminaries

The basic definitions of a BFG are redefined and explained with suitable example. Throughout this paper,

(i) The edge between the vertices r and t as rt.

(ii) $\mathrm{G}=(\mathrm{A},\mathrm{B})$ be a BFG, mean that G be a BFG with underlying graph $\mathrm{G}^*=(\mathrm{M},\mathrm{N})$.

Definition 2.1 ([6]). *A fuzzy set α on a set U is a map $\alpha : U \to [0,1]$. A map $\beta : U \times U \to [0,1]$ is called a fuzzy relation on X if $\beta(r,t) \leq \min(\alpha(r),\alpha(t))$ for all $r,t \in U$. A fuzzy relation β is symmetric if $\beta(r,t) = \beta(r,t)$ for $r,t \in U$.*

Definition 2.2 ([5]). *Let U be a non-void set. A bipolar fuzzy set H in U is an object having the form*

$$H = \left\{ \left(\mathrm{r}, \alpha_{\mathrm{H}}^{\mathrm{P}}(\mathrm{r}), \alpha_{\mathrm{H}}^{\mathrm{N}}(\mathrm{r})\right) / \mathrm{r} \in \mathrm{U} \right\},$$

where, $\alpha_{\mathrm{H}}^{\mathrm{P}} : \mathrm{U} \to [0,1]$ and $\alpha_{\mathrm{H}}^{\mathrm{N}} : \mathrm{U} \to [-1,0]$ are mappings. The positive membership degree $\alpha_H^P(r)$ to denote the degree of satisfaction of an element with the property corresponding to a bipolar fuzzy set H*, and the negative membership degree*

$\alpha_H^N(r)$ *to denote the degree of satisfaction of an element r with some implicit counter-property corresponding to a bipolar fuzzy set H. If* $\alpha_H^P(r) \neq 0$ *and* $\alpha_H^N(r) = 0$ *then* H *is known to have only positive satisfaction degree. If* $\alpha_H^P(r) = 0$ *and* $\alpha_H^N(r) \neq 0$, *then the condition that x does not fulfil* H *'s property, but rather satisfies* H′s *counter property. It is possible for a factor r to be such that* $\alpha_H^P(r) \neq 0$ *and* $\alpha_H^N(r) \neq 0$. *When the property's membership feature overlaps that of its counter property over some portion of U. We will use the symbol,* $\mathrm{H} = \left(\alpha_H^P, , \alpha_H^N\right)$ *for the sake of simplicity, and for the bipolar fuzzy set* $\mathrm{H} = \left\{\left(\mathrm{r}, \alpha_{\mathrm{H}}^{\mathrm{P}}(\mathrm{r}), \alpha_{\mathrm{H}}^{\mathrm{N}}(\mathrm{r})\right) / \mathrm{r} \in \mathrm{U}\right\}$.

Definition 2.3 ([6])**.** *Let U be a non-void set. Then, the mapping* $H = \left(\alpha_H^P, \alpha_H^N\right) : X \times X \to [-1,1] \times [-1,1]$ *a bipolar fuzzy relation on X such that* $\alpha_H^P(r,t) \in [0,1]$ *and* $\alpha_H^N(r,t) \in [-1,0]$.

Definition 2.4 ([6])**.** *A BFG, is denoted as a pair* $G = (A,B)$, *where* $A = \left(\alpha_A^P, \alpha_A^N\right)$ *and* $B = \left(\alpha_{\mathrm{B}}^{\mathrm{P}}, \alpha_{\mathrm{B}}^{\mathrm{N}}\right)$ *are bipolar fuzzy sets and* $\alpha_{\mathrm{A}}^{\mathrm{P}} : \mathrm{M} \to [0,1], \alpha_{\mathrm{A}}^{\mathrm{N}} : \mathrm{M} \to [-1,0]$, *and* $\alpha_B^P : M \times M \to [0,1]$, $\alpha_B^N : M \times M \to [-1,0]$ *are bipolar fuzzy mappings such that* $\alpha_B^P(\mathrm{rt}) \leq \min\left\{\alpha_A^P(r), \alpha_A^P(t)\right\}$ *and* $\alpha_B^N(rt) \geq \max\left\{\alpha_A^N(r), \alpha_A^N(t)\right\}$ *for all rt* $\in$ *N. A is called the bipolar fuzzy vertex set of* M *and* B *the bipolar fuzzy edge set of* N *respectively. Note that* B *is a symmetric bipolar fuzzy relation on A. That is,* $\mathrm{G} = (\mathrm{A}, \mathrm{B})$ *is a BFG of the underlying crisp graph* $\mathrm{G}^* = (\mathrm{M}, \mathrm{N})$, *where* M *is a vertex set and the edge set* $\mathrm{N} \subseteq \mathrm{M} \times \mathrm{M}$ *such that* , $\alpha_{\mathrm{B}}^{\mathrm{P}}(\mathrm{rt}) \leq \min\left\{\alpha_A^P(r), \alpha_A^P(t)\right\}$ *and* $\alpha_B^N(rt) \geq \max\left\{\alpha_A^N(r), \alpha_A^N(t)\right\}$ *for all rt* $\in N$.

Definition 2.5 ([3])**.** A BFG $G = (\mathrm{A}, \mathrm{B})$ *of a graph* $\mathrm{G}^* = (\mathrm{M}, \mathrm{N})$ *is called strong if* $\alpha_{\mathrm{B}}^{\mathrm{P}}(\mathrm{rt}) = \min\left\{\alpha_A^P(r), \alpha_A^P(t)\right\}$ *and* $\alpha_B^N(rt) = \max\left\{\alpha_A^N(r), \alpha_A^N(t)\right\}$ *for all rt* $\in N$.

Definition 2.6. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the cardinality of* M *or the order of* G *is defined by*

$$p = |M| = \sum_{r \in M} \frac{1 + \alpha_A^P(r) + \alpha_A^N(r)}{2}.$$

Definition 2.7. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the cardinality of* N *or the size of* G *is defined as*

$$q = |N| = \sum_{\mathrm{rt} \in N} \frac{1 + \alpha_B^P(rt) + \alpha_B^N(rt)}{2}.$$

Definition 2.8. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the degree of the vertex is denoted as* deg(r) *and* it *is defined as*

$$deg(\mathrm{r}) = \sum_{\mathrm{r} \in \mathrm{M}} \frac{1 + \alpha_{\mathrm{B}}^{\mathrm{P}}(\mathrm{rt}) + \alpha_{\mathrm{B}}^{\mathrm{N}}(\mathrm{rt})}{2}.$$

Definition 2.9. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the maximum degree of a BFG is denoted by* $\Delta(\mathrm{G}) = \max\{\deg(\mathrm{r}) / \mathrm{r} \in \mathrm{M}\}$.

Definition 2.10. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the minimum degree of a* BFG *is denoted by* $\delta(\mathrm{G}) = \min\{\deg(\mathrm{r}) / \mathrm{r} \in \mathrm{M}\}$.

Definition 2.11. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the degree of an edge rt* $\in$ N *is denoted as deg* (rt) *and it is defined as,*

$$deg(\mathrm{rt}) = \sum_{\mathrm{rt} \in \mathrm{N}} \frac{1 + \alpha_{\mathrm{B}}^{\mathrm{P}}(\mathrm{rt}) + \alpha_{\mathrm{B}}^{\mathrm{N}}(\mathrm{rt})}{2}.$$

Definition 2.12. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the neighbors (neighborhood) of* r *or an open neighbor of* $\mathrm{r} \in \mathrm{M}$ *of* G *is denoted by* N(r) *and is defined as* $N(r) = \left\{r \in M / \alpha_{\mathrm{B}}^{\mathrm{P}}(\mathrm{rt}) = \min\left\{\alpha_{\mathrm{A}}^{\mathrm{P}}(\mathrm{r}), \alpha_{\mathrm{A}}^{\mathrm{P}}(\mathrm{t})\right\}\right.$ *and* $\alpha_{\mathrm{B}}^{\mathrm{N}}(\mathrm{rt}) = \max\left\{\alpha_{\mathrm{A}}^{\mathrm{N}}(\mathrm{r}), \alpha_{\mathrm{A}}^{\mathrm{N}}(\mathrm{t})\right\}$ *for all* $\left.\mathrm{rt} \in \mathrm{N}\right\}$ *The closed neighbors of* $\mathrm{r} \in \mathrm{M}$ *of* G *is written by* N[r] *and is stated as* $\mathrm{N}[\mathrm{r}] = \mathrm{N}(\mathrm{r}) \cup \{\mathrm{r}\}$.

Definition 2.13. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the neighbourhood degree of* $\mathrm{r} \in \mathrm{M}$ *is denoted as* $\deg_{\mathrm{N}}(\mathrm{r})$ *and is defined as*

$$\deg_N(r) = \sum_{r \in N(u)} \frac{1 + \alpha_A^P(r) + \alpha_A^N(r)}{2}.$$

Definition 2.14. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the maximum neighbourhood degree of a BFG is denoted by* $\Delta_{\mathrm{N}}(G) = \max\{\deg_N(r) / r \in M\}$.

Definition 2.15. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the minimum neighbourhood degree of a BFG is denoted by* $\delta_{\mathrm{N}}(G) = \min\{\deg_{\mathrm{N}}(\mathrm{r}) / \mathrm{r} \in \mathrm{M}\}$.

Definition 2.16. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *an edge of* G *is said to be an effective or strong edge if* $\alpha_B^P(rt) = \min\{\alpha_A^P(r), \alpha_A^P(t)\}$ *and* $\alpha_B^N(rt) = \max\left\{\alpha_A^N(r), \alpha_A^N(t)\right\}$ *for all rt* $\in N$.

Definition 2.17. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the effective degree of a vertex* $\mathrm{r} \in \mathrm{M}$ *in* G *is defined as*

$$\deg_E(r) = \sum_{r \in M} \frac{1 + \alpha_B^P(rt) + \alpha_B^N(rt)}{2},$$

where rt is an effective edge.

Definition 2.18. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the maximum effective degree of a BFG is denoted by* $\Delta_{\mathrm{E}}(\mathrm{G}) = \max\{\deg_{\mathrm{E}}(\mathrm{r}) / \mathrm{r} \in \mathrm{M}\}$.

Definition 2.19. *For any* BFG, $\mathrm{G} = (\mathrm{A}, \mathrm{B})$, *the minimum effective degree of a* BFG *is denoted by* $\delta_{\mathrm{E}}(\mathrm{G}) = \min\{\deg_{\mathrm{E}}(\mathrm{r}) / \mathrm{r} \in \mathrm{M}\}$.

Example 2.20.

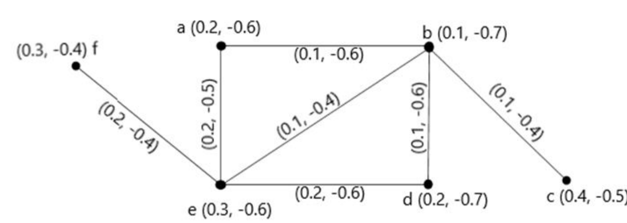

Figure 1. Bipolar Fuzzy graph G

Definition 2.21 ([4]). *Consider* $\mathrm{G} = (\mathrm{A}, \mathrm{B})$ *be a BFG. Let* $\mathrm{r}, \mathrm{t} \in \mathrm{M}$. *The vertex* r *is said to be dominates* t *in* G *if* $\alpha_{\mathrm{B}}^{\mathrm{P}}(\mathrm{rt}) = \min\left\{\alpha_{\mathrm{A}}^{\mathrm{P}}(\mathrm{r}), \alpha_{\mathrm{A}}^{\mathrm{P}}(\mathrm{t})\right\}$ *and* $\alpha_{\mathrm{B}}^{\mathrm{N}}(\mathrm{rt}) = \max\left\{\alpha_{\mathrm{A}}^{\mathrm{N}}(\mathrm{r}), \alpha_{\mathrm{A}}^{\mathrm{N}}(\mathrm{t})\right\}$ *for all* $\mathrm{rt} \in \mathrm{N}$. *A subset D of M is said to be a dominating set in G if for every* $t \in M - D$ *there exist* $\mathrm{r} \in \mathrm{D}$ *such that* r *dominates t.*

A dominating set D *of* M *is said to be a minimal dominating set if no proper subset of* D *is a dominating set of G.*

The minimum fuzzy cardinality of a minimal dominating set in G *is called the domination number of* G *and is denoted by* $\gamma(\mathrm{G})$ *or simply* γ *and the corresponding minimal dominating set is called the minimum dominating set of G.*

Example 2.22.

Figure 2

From the above example, Dominating set, $\mathrm{D} = \{\mathrm{a}, \mathrm{c}\}, \mathrm{M} - \mathrm{D} = \{\mathrm{b}, \mathrm{d}\}$ *The domination number of* G, $\gamma(\mathrm{G}) = 1.05$.

3. Double Domination on BFG

The related principles of double dominance on the BFG and their properties are discussed in this section.

Definition 3.1. *For any* $\mathrm{BFG}, \mathrm{G} = (\mathrm{A}, \mathrm{B})$, *a subset* $\mathrm{D_d}$ *of* M *is a double dominating set of* G, *if for each vertex in* $\mathrm{M} - \mathrm{D_d}$ *is dominated by atleast two vertices in* $\mathrm{D_d}$.

A double dominating set $\mathrm{D_d}$ *of* M *is said to be a minimal double dominating set if no proper subset of* $\mathrm{D_d}$ *is a double dominating set of* G.

The minimum fuzzy cardinality of a minimal double dominating set in G *is called the double domination number of* G *and is denoted by* $\gamma_{\mathrm{D_d}}(\mathrm{G})$ *and the corresponding minimal double dominating set is called the minimum double dominating set of G.*

Example 3.2. *From the BFG G in Fig 3, we have*
The Dominating set $D = \{a, b\}$.
The Domination number of $\mathrm{G}, \gamma(\mathrm{G}) = 0.9$.
The double dominating set of $\mathrm{G}, \mathrm{D_d} = \{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$.
The double domination number of $\mathrm{G}, \gamma_{\mathrm{D_d}}(\mathrm{G}) = 1.4$.

Theorem 3.3. *For any* BFG, *then* $\gamma(\mathrm{G}) \leq \gamma_{\mathrm{D_d}}(\mathrm{G})$

Proof. Let $G = (\mathrm{A}, \mathrm{B})$ be any BFG. Let $\mathrm{D} \subseteq \mathrm{M}$ be a dominating set and $\mathrm{D_d} \subseteq \mathrm{M}$ be a double dominating set of G. If $\mathrm{D} = \mathrm{D_d}$, then $\gamma(\mathrm{G}) = \gamma_{\mathrm{D_d}}(G)$. If $D \neq D_d$, then D_d has atleast one vertices more than D and hence, $\gamma(G) < \gamma_{D_d}(G)$ Hence, $\gamma(G) \leq \gamma_{D_d}(G)$. □

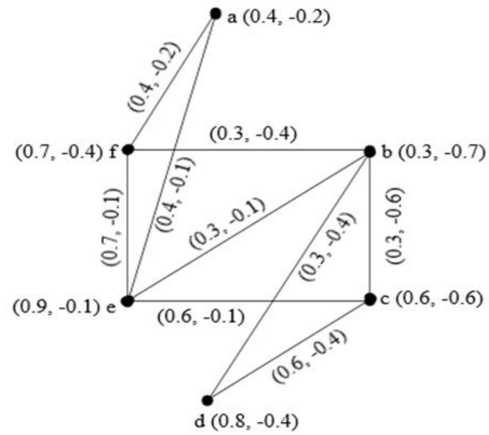

Figure 3. BFG G

Definition 3.4. *Let* $\mathrm{G} = (\mathrm{A}, \mathrm{B})$ *be a BFG. Then* G *is said to be a bipartite* BFG *if the vertex set* M *of a BFG G can be partitioned into two subsets* M_1 *and* M_2 *such that* $\alpha_{\mathrm{B}}^{\mathrm{P}}(\mathrm{rt}) = 0$ *and* $\alpha_{\mathrm{B}}^{\mathrm{N}}(\mathrm{rt}) = 0$ *for all* $\mathrm{r}, \mathrm{t} \in \mathrm{M}_1$ *or* $\mathrm{r}, \mathrm{t} \in \mathrm{M}_2$ *A bipartite BFG* $G = (A, B)$ *is said to be a complete BFG if,* $\alpha_B^P(rt) = \min\left\{\alpha_A^P(r), \alpha_A^P(t)\right\}$ *and* $\alpha_B^N(rt) = \max\left\{\alpha_A^N(r), \alpha_A^N(t)\right\}$ *for all* $r \in M_1$ *and* $t \in M_2$.

Theorem 3.5. *Let* $G = (A, B)$ *be any completely bipartite BFG with* $n > 2, n$ *is the number of vertices of G. Then double dominating set* $\mathrm{D_d}$ *of* G *exists.*

Proof. Let $G = (A, B)$ be a completely bipartite BFG. Then, $M = M_1 \cup M_2$ and $M_1 \cup M_2 = \phi$ Let $r_1, r_2 \ldots\ldots r_k \in M_1$ or $t_1, t_2 \ldots\ldots t_s \in M_2$ with either $s > 1$ or $k > 1$. Let $n > 2, n$ be the number of vertices of G.

If $s = 1$ and $k > 1$, then M_2 is the double dominating set of G. If $s > 1$ and $k = 1$, then M_1 is the double dominating set of G. If $s > 1$ and $k > 1$, then either M_1 orM $_2$ is the double dominating set of G. Hence, . Then double dominating set D_d existsfor any completely bipartite BFG with $n > 2$, n is the number of vertices of G. □

Theorem 3.6. *Let* $\mathrm{G} = (\mathrm{A}, \mathrm{B})$ *be a BFG with double dominating set. Then,* $\gamma(\mathrm{G}) + \gamma_{\mathrm{D_d}}(\mathrm{G}) \leq \mathrm{p}$.

Proof. Let $\mathrm{G} = (\mathrm{A}, \mathrm{B})$ be a BFG. Let $\mathrm{D_d}$ be the double dominating setTherefore, $\gamma(\mathrm{G}) \leq p - \gamma_{D_d}(G)$. Hence,

$$\gamma(\mathrm{G}) + \gamma_{D_d}(G) \leq \mathrm{p}.$$

□

Theorem 3.7. *For* $\mathrm{BFG}, \mathrm{G} = (\mathrm{A}, \mathrm{B})$, *then* $\gamma_{D_d}(\mathrm{G}) < \mathrm{p}$.

Proof. Let $\mathrm{G} = (\mathrm{A}, \mathrm{B})$ be a BFG. Then by Theorem 3.6, $\gamma(\mathrm{G}) + \gamma_{\mathrm{D_d}}(\mathrm{G}) \leq \mathrm{p}$. Hence, $\gamma_{\mathrm{D_d}}(\mathrm{G}) \leq \mathrm{p}$. □

4. Conclusion

The idea of double domination on bipolar fuzzy graph was presented in this article and discussed some of its properties. We can extend our research work to double total domination and other various types of bipolar fuzzy graph.

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