Malaya Journal of Matematik, Vol. 8, No. 4, 1587-1591, 2020
https://doi.org/10.26637/MJM0804/0044

On anti-fuzzy EPWI-ideals of lattice pseudo W-algebras

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Abstract
Characterizations of Anti-fuzzy EPWI-ideals of a Lattice pseudo W-algebras are established. The notions of anti-fuzzy CCPWI-ideal and anti-fuzzy EPWI-ideal are introduced. Several examples and some of their properties are investigated. Moreover, the connection between anti-fuzzy EPWI-ideal and EPWI-ideal are obtained.

Keywords
CCPWI-ideal, EPWI-ideal, fuzzy CCPWI-ideal, fuzzy EPWI-ideal, anti-fuzzy CCPWI- ideal, anti-fuzzy EPWI-ideal and lattice pseudo W-algebras.

AMS Subject Classification
06B10, 03E72, 03G10.

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Article History: Received **21** July **2020**; Accepted **19** September **2020**

Contents

1. Introduction

The notion of fuzzy ideal has been introduced in many algebraic structures such as lattice, rings and algebras. The concept of fuzzy sets formulated by Zadeh [10], which can to study the basic logic framework of fuzzy set theory. The concept of an anti-fuzzy subgroups of groups was introduced by Biswas [1] in 1990. The concepts of W-algebras (Wajsberg algebras) was presented by Mordchaj Wajsberg [9]. Ceterchi Rodica [2] introduced the concept of lattice structure pseudo W-algebras. In their paper [8] introduced on fuzzy EPWI-ideals and obtained their properties with illustrations.

In this paper, we introduce notions of anti-fuzzy CCPWI-ideal and anti-fuzzy EPWI-ideal. We investigate these ideals and the properties of anti-fuzzy EPWI-ideal of lattice pseudo W-algebras are provided.

2. Preliminaries

This part refers to the required earlier information, including basic definitions and results of Anti-fuzzy $EPWI$-ideals of lattice pseudo W-algebras.

Definition 2.1 ([7]). *Let $\mathscr{M}$ be a lattice pseudo W-algebra. Then $K \neq \phi \subset \mathscr{M}$ is called a completely closed PWI-ideal ($CCPWI$-ideal) of $\mathscr{M}$, if it satisfied the following axioms for all $p,q \in K$.*

(i) $0 \in K$

(ii) $(p \rightarrow q)^{\sim}, (p \rightsquigarrow q)^{-} \in K, q \in K$ *implies* $p \in K$.

Definition 2.2 ([10]). *Let $\mathscr{M}$ be a set, then a function $\delta : \mathscr{M} \rightarrow [0,1]$ is called a fuzzy subset on $\mathscr{M}$ for each $x \in \mathscr{M}$, the value of $\delta(x)$ describes a degree of membership of x in δ.*

Definition 2.3 ([10]). *Let δ be a fuzzy set in $\mathscr{M}$, then for $\beta \in [0,1]$, the set $\delta_\beta = \{x \in \mathscr{M} / \delta(x) \geq \beta\}$ is called a level subset of δ.*

Definition 2.4 ([10]). *Let δ and λ be a non-empty two fuzzy subsets of $\mathscr{M}$. Then δ is called a fuzzy subset of λ if $\delta(p) \leq \lambda(p)$ for all $p \in \mathscr{M}$.*

Definition 2.5 ([10]). *Let δ and λ be two fuzzy subsets in $\mathscr{M}$, then*

(i) $(\delta \cap \lambda)(p) = \min\{\delta(p), \lambda(p)\}$ *for all* $p \in \mathscr{M}$

(ii) $(\delta \cup \lambda)(p) = \max\{\delta(p), \lambda(p)\}$ *for all* $p \in \mathscr{M}$,

where, $(\delta \cap \lambda)$ *and* $(\delta \cup \lambda)$ *are fuzzy sets in* $\mathscr{M}$. *In general, if* $\{\lambda\_i, i \in \psi\}$ *is a family of fuzzy sets in* $\mathscr{M}$, *then* $\cap_{i\in\psi}\lambda_i(p) = \inf\{\lambda_i(p)/i \in \psi\}$ *for all* $p \in \mathscr{M}$ *and* $\bigcup_{i\in\psi}\lambda_i(p) = \sup\{\lambda_i(p)/ i \in \psi\}$ *for all* $p \in \mathscr{M}$. *Where* ψ *is the index set. Then for more preliminaries the references [2,3],[6-8].*

Definition 2.6 ([8])**.** *Let* $\mathscr{M}$ *be a lattice pseudo* W*-algebra. Let* K *be a non-empty subset of* $\mathscr{M}$ *which is not necessary an ideal of* $\mathscr{M}$, *a subset* $\mathscr{N}$ *of* $\mathscr{M}$ *is called an extended PWI-ideal* ($EPWI-$ *ideal) of* $\mathscr{M}$, *if it satisfied*

(i) K *is a subset of* $\mathscr{N}$

(ii) $0 \in \mathscr{N}$

(iii) *For all* $p \in \mathscr{M}, q \in K$ *then* $(p \to q)^{\sim}, (p \rightsquigarrow q)^{-} \in K$ *implies* $p \in \mathscr{N}$.

Definition 2.7 ([8])**.** *Let* $\mathscr{M}$ *be a lattice pseudo* W*-algebra. Then the fuzzy subset* λ *of* $\mathscr{M}$ *is called a fuzzy completely closed PWI-ideal (fuzzy CCPWI-ideal) of* $\mathscr{M}$, *if it satisfied the following axioms for all* $p, q \in \mathscr{M}$

(i) $\lambda(0) \geq \lambda(p)$

(ii) $\lambda(p) \geq \min\{\lambda((p \to q)^{\sim}), \lambda((p \rightsquigarrow q)^{-}), \lambda(q)\}$.

Definition 2.8 ([8])**.** *Let* δ *and* λ *be fuzzy subsets of* $\mathscr{M}$, *then* λ *is called fuzzy extended PWI-ideal (fuzzy EPWI-ideal) of* $\mathscr{M}$, *if it satisfied the following axioms for all* $p, q \in \mathscr{M}$

(i) δ *is a fuzzy subset of* λ

(ii) $\lambda(0) \geq \lambda(p)$

(iii) $\lambda(p) \geq \min\{\delta((p \to q)^{\sim}), \delta((p \rightsquigarrow q)^{-}), \delta(q)\}$.

3. Anti-fuzzy $EPWI$-ideal of lattice pseudo W-algebras

In this section, we introduce the notions of anti-fuzzy $CCPWI$-ideal and anti- fuzzy $EPWI$-ideal. Further, we investigate these ideals and the properties of anti-fuzzy $EPWI$-ideal of lattice pseudo W-algebras are provided.

Definition 3.1. *Let* $\mathscr{M}$ *be a lattice pseudo* W*-algebra. Then the fuzzy subset* δ *of* $\mathscr{M}$ *is called an anti-fuzzy CCPWI-ideal of* $\mathscr{M}$, *if it satisfies the following axioms for all* $p, q \in \mathscr{M}$

(i) $\delta(0) \leq \delta(p)$

(ii) $\delta(p) \leq \max\{\delta((p \to q)^{\sim}), \delta((p \sim q)^{-}), \delta(q)\}$.

Example 3.2. *Consider a set* $\mathscr{M} = \{0, e, f, g, 1\}$. *Define a partial ordering "* $\leq$ *"on* $\mathscr{M}$, *such that* $0 \leq e \leq f \leq g \leq 1$ *with the following tables (1),(2),(3) and (4).*

Table: 1	
x	x^{-}
0	1
e	f
f	e
g	e
1	0

Table: 2					
$\to$	0	e	f	g	1
0	1	1	1	1	1
e	f	1	1	1	1
f	e	e	1	1	1
g	e	e	f	1	1
1	0	e	f	g	1

Table: 3	
x	$x^{\sim}$
0	1
e	g
f	e
g	f
1	0

Table: 4					
$\rightsquigarrow$	0	e	f	g	1
0	1	1	1	1	1
e	g	1	1	1	1
f	e	e	1	1	1
g	f	e	f	1	1
1	0	e	f	g	1

Then $\{\mathscr{M}, \mathrm{V}, \wedge, -, \sim, \longrightarrow, \rightsquigarrow, 0, 1\}$ *is a lattice pseudo* W*-algebra and consider the fuzzy subsets* δ_1 *and* δ_2 *on* $\mathscr{M}$ *as,*

$$\delta_1(x) = \begin{cases} 0.2 \text{ if } x = 0, e \\ 0.8 \text{ if } x = f, g, 1. \end{cases}$$

Then $\delta_1(x)$ *is an anti-fuzzy CCPWI*$-$ *ideal of* $\mathscr{M}$, *but*

$$\delta_2(x) = \begin{cases} 0.3 \text{ if } x = 0, 1 \\ 0.7 \text{ if } x = e, f, g. \end{cases}$$

Then $\delta_2(x)$ *is not an anti-fuzzy CCPWI-ideal of* $\mathscr{M}$ *since,* $\delta_2(e) \leq \max\{\delta_2((e \to 1)^{\sim}), \delta_2((e \rightsquigarrow 1)^{-}), \delta_2(1)\}$

$$\delta_2(e) \leq \max\{\delta_2(0), \delta_2(0), \delta_2(1)\} \Rightarrow 0.7 \nleqslant 0.3.$$

Thus $\delta_2(e) \nleqslant \max\{\delta_2((e \to 1)^{\sim}), \delta_2((e \rightsquigarrow 1)^{-}), \delta_2(1)\}$.

Definition 3.3. *Let* δ *and* φ *be fuzzy subsets of* $\mathscr{M}$, *then* φ *is called an anti-fuzzy EPWI-ideal of* $\mathscr{M}$, *if it satisfies the following axioms for all* $p, q \in \mathscr{M}$.

(i) φ *is a fuzzy subset of* δ

(ii) $\varphi(0) \leq \varphi(p)$

(iii) $\varphi(p) \leq \max\{\delta((p \to q)^{\sim}), \delta((p \rightsquigarrow q)^{-}), \delta(q)\}$.

Example 3.4. *Consider a set* $\mathscr{M} = \{0, m, n, 1\}$. *Define a partial ordering "* $\leq$ *" on* $\mathscr{M}$, *such that* $0 \leq m \leq 1; 0 \leq n \leq 1$ *with the following tables (5),(6),(7) and (8).*

Table: 5	
x	x^{-}
0	1
m	n
n	m
1	0

Table: 6				
$\to$	0	m	n	1
0	1	1	1	1
m	n	1	1	1
n	m	n	1	1
1	0	m	n	1

Then $\{\mathscr{M}, \vee, \wedge, -, \sim, \to, \rightsquigarrow, 0, 1\}$ *is a lattice pseudo* W*-algebra. Consider the fuzzy subsets* δ *and* φ *on* $\mathscr{M}$ *as,*

$$\delta(x) = \begin{cases} 0.8 \text{ if } x = 0 \\ 0.6 \text{ if } x = m, n, 1 \end{cases}, \quad \varphi(x) = \begin{cases} 0.2 \text{ if } x = 0 \\ 0.5 \text{ if } x = m, n, 1 \end{cases}$$

Table: 7	
x	$x^{\sim}$
0	1
m	n
n	m
1	0

Table: 8				
$\rightsquigarrow$	0	m	n	1
0	1	1	1	1
m	n	1	1	1
n	m	m	1	1
1	0	m	n	1

Then, φ is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ.

Example 3.5. *Consider a set $\mathscr{M}=\{0,c,d,1\}$. Define a partial ordering " $\leq$ " on $\mathscr{M}$, such that $0\leq c\leq 1; 0\leq d\leq 1$ with the following tables (9),(10),(11) and (12). Then,*

Table: 9	
x	x^{-}
0	1
c	d
d	c
1	0

Table: 10				
$\rightarrow$	0	c	d	1
0	1	1	1	1
c	d	1	1	1
d	c	c	1	1
1	0	c	d	1

Table: 11	
x	$x^{\sim}$
0	1
c	d
d	c
1	0

Table: 12				
$\rightsquigarrow$	0	c	d	1
0	1	1	1	1
c	d	1	1	1
d	c	c	1	1
1	0	c	d	1

$\{\mathscr{M},\vee,\wedge,-,\sim,\rightarrow,\rightsquigarrow,0,1\}$ is a lattice pseudo W-algebra. Consider the fuzzy subsets δ and φ on $\mathscr{M}$ as,

$$\delta(x)=\begin{cases}0.5 \text{ if } x=0\\ 0.7 \text{ if } x=c,d\\ 0.9 \text{ if } x=1\end{cases},\quad \varphi(x)=\begin{cases}0.2 \text{ if } x=0,1\\ 0.6 \text{ if } x=c,d\end{cases}$$

Then, φ is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ. But, φ is not an anti-fuzzy CCPWI- ideal. since, $\varphi(d)\leq \max\{\varphi((d\rightarrow 1)^{\sim}),\varphi((d\rightsquigarrow 1)^{-}),\varphi(1)\}\Rightarrow 0.6\nleq 0.2$.
Next, we show that the union of two anti-fuzzy $EPWI$-ideals is also be an anti-fuzzy $EPWI-$ ideal from the example given below.

Example 3.6. *In Example 3.4, consider the fuzzy subsets δ_1,δ_2 and φ_1,φ_2 on $\mathscr{M}$ as,*

$$\delta_1(x)=\begin{cases}0.3 \text{ if } x=0\\ 0.6 \text{ if } x=m\\ 0.9 \text{ if } x=n,1\end{cases},\quad \varphi_1(x)=\begin{cases}0.2 \text{ if } x=0\\ 0.5 \text{ if } x=m\\ 0.7 \text{ if } x=n,1\end{cases}$$

$$\delta_2(x)=\begin{cases}0.3 \text{ if } x=0\\ 0.7 \text{ if } x=n\\ 0.8 \text{ if } x=m,1\end{cases},\quad \varphi_2(x)=\begin{cases}0.1 \text{ if } x=0\\ 0.5 \text{ if } x=n\\ 0.7 \text{ if } x=m,1\end{cases}$$

Then, φ_1 and φ_2 are an anti-fuzzy EPWI-ideal of $\mathscr{M}$ related to δ_1 and δ_2 respectively.

$$\delta_1\cup\delta_2(x)=\begin{cases}0.3 \text{ if } x=0\\ 0.6 \text{ if } x=m\\ 0.7 \text{ if } x=n\\ 0.9 \text{ if } x=1\end{cases},\varphi_1\cup\varphi_2(x)=\begin{cases}0.2 \text{ if } x=0\\ 0.5 \text{ if } x=m,n\\ 0.7 \text{ if } x=1\end{cases}$$

And also $\varphi_1\cup\varphi_2$ is an anti-fuzzy EPWI-ideal of A related to $\delta_1\cup\delta_2$. Since,

$$\begin{aligned}\beta_1\cup\beta_2(1)\leq&\max\{\delta_1\cup\delta_2((1\rightarrow n)^{\sim}),\delta_1\cup\delta_2\\&((1\rightsquigarrow n)^{-}),\delta_1\cup\delta_2(n)\}\\\leq&\max\{\delta_1\cup\delta_2(m),\delta_1\cup\delta_2(m),\\&\delta_1\cup\delta_2(n)\}=0.7=0.7.\end{aligned}$$

Proposition 3.7. *Let $\{\varphi_i/(i\in\psi)\}$ be a family of anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to a fuzzy subset δ of $\mathscr{M}$. Then $\cup_{i\in\psi}\beta_i$ is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ.*

Proof. Let $\{\varphi_i/(i\in\Psi)\}$ be a family of anti-fuzzy $EPWI$-ideal of $\mathscr{M}$,

(i) Let $p\in\mathscr{M}$, If φ_i is a fuzzy subset of δ for all $i\in\psi$. Then, we have $\varphi_i(p)\leq\delta(p)$ for all $i\in\psi$ Thus, sup $\{\varphi_i(p)/i\in\Psi\}\leq\mu(p)$ and $\cup_{i\in\psi}\varphi_i(p)\leq\mu(p)$. Hence, we get $\cup_{i\in\psi}\varphi_i$ is a fuzzy subset of δ.

(ii) Let $p\in\mathscr{M}$, then $\cup_{i\in\Psi}\varphi_i(0)=\sup\{\varphi_i(0)/i\in\psi\}\leq\varphi(p)$, since $\varphi_i(0)\leq\varphi(p)$ for all $i\in\psi$.

(iii) Let $p,q\in\mathscr{M}$. Then, we have

$$\begin{aligned}\cup_{i\in\psi}\varphi_i(p)&=\sup\{\varphi_i(p)/i\in\psi\}\\&\leq\sup\left\{\delta((p\rightarrow q)^{\sim}),\delta\left((p\rightsquigarrow q)^{-}\right),\delta(q)\right\}.\end{aligned}$$

Since,

$$\begin{aligned}\varphi_i(p)&=\sup\{\varphi_i(p)/i\in\psi\}\\&\leq\sup\left\{\delta((p\rightarrow q)^{\sim}),\delta\left((p\rightsquigarrow q)^{-}\right),\delta(q)\right\}\text{ for all } i\in\psi\\U_{i\in\psi}\varphi_i&\leq\sup\left\{\delta((p\rightarrow q)^{\sim}),\delta\left((p\rightsquigarrow q)^{-}\right),\delta(q)\right\}.\end{aligned}$$

Therefore, $\cup_{i\in\psi}\varphi_i$ is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ. ☐

Proposition 3.8. *Let $\mathscr{M}$ be a lattice pseudo W-algebra, δ and φ be fuzzy subsets of $\mathscr{M}$ and also φ is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ if and only if the level subset δ_α is an $EPWI$-ideal of $\mathscr{M}$ related to φ_α for all $\alpha\in[0,\varphi(0)]$, $\varphi(0)=\inf_{x\in\mathscr{M}}\varphi(p)$.*

Proof. Let φ be an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ and $\alpha\in[0,\varphi(0)]$.
To prove: δ_α is an $EPWI$-ideal of $\mathscr{M}$ related to φ_α

(i) It is clear that $\varphi_\alpha\subseteq\delta_\alpha$ and $\varphi(0)\leq\alpha$ implies $0\in\varphi_\alpha$.

(ii) Let $p,q\in\mathscr{M}$, such that $(p\rightarrow q)^{\sim},(p\rightsquigarrow q)^{-}\in\varphi_\alpha$ and $q\in\varphi_\alpha$ implies $\delta((p\rightarrow q)^{\sim})\leq\alpha,\delta((p\rightsquigarrow q)^{-})\leq\alpha$ and $\delta(q)\leq\alpha$.

So $\max\{\delta((p\rightarrow q)^{\sim}),\delta((p\rightsquigarrow q)^{-}),\ \delta(q)\}\leq\alpha$.
But

$$\varphi(p)\leq\max\left\{\delta((p\rightarrow q)^{\sim}),\delta\left((p\rightsquigarrow q)^{-}\right),\delta(q)\right\}.$$

Since φ is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ. Therefore, $\varphi(p)\leq\alpha$ implies $x\in\delta_\alpha$. Hence, δ_α is an $EPWI$-ideal

of $\mathscr{M}$ related to φ_α.
Conversely,
Let δ_α be an $EPWI$-ideal of $\mathscr{M}$ related to φ_α for all $\alpha \in [0, \varphi(0)], \varphi(0) = {}_{x\in\mathscr{M}} \inf \varphi(x)$ To prove, φ is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ.

(i) Let $p \in \mathscr{M}$ and $\alpha = \varphi(p) \Rightarrow p \in \varphi_\alpha$ implies $p \in \delta_\alpha$, since $\varphi_\alpha \subseteq \delta_\alpha$ is an $EPWI$-ideal of $\mathscr{M}$, so that $\varphi(p) \leq \alpha \Rightarrow \varphi(p) \leq \delta(p) \Rightarrow \varphi \subseteq \delta$.

(ii) It is clear that $\varphi(0) \leq \varphi(p)$ for all $p \in \mathscr{M}$.

(iii) Let $p, q \in \mathscr{M}$, such that

$$\max\left\{\delta\left((p \to q)^\sim\right), \delta\left((p \rightsquigarrow q)^-\right), \delta(q)\right\} = \alpha.$$

implies $\delta((p \to q)^\sim) \leq \alpha, \delta((p \rightsquigarrow q)^-) \leq \alpha$ and $\delta(q) \leq \alpha \Rightarrow (p \to q)^\sim, (p \rightsquigarrow q)^- \in \varphi_\alpha$ and $q \in \varphi_\alpha$ implies $p \in \delta_\alpha$ since δ_α be an EPWI-ideal of $\mathscr{M}$ related to φ_α. So that, $\varphi(p) \leq \alpha$ implies that

$$\varphi(p) \leq \max\left\{\delta\left((p \to q)^\sim\right), \delta\left((p \rightsquigarrow q)^-\right), \delta(q)\right\}.$$

Therefore, φ is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ. □

Note 3.9. *Let φ be a fuzzy subset of lattice pseudo W-algebra of $\mathscr{M}$, the set $\{p \in \mathscr{M} / \varphi(p) = \varphi(0)\}$ is denoted by $\mathscr{M}_\varphi$.*

Proposition 3.10. *Let $\mathscr{M}$ be a lattice pseudo W-algebra. If φ is an anti-fuzzy $EPWI$-ideal related to δ such that $\varphi(0) = \delta(0)$, then the set $\mathscr{M}_\delta$ is an $EPWI$-ideal of $\mathscr{M}$ related to $\mathscr{M}_\varphi$.*

Proof. Let φ be an anti fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ.

(i) Since $\varphi(0) = \delta(0)$ and $\varphi(p) \leq \delta(p)$ for all $p \in \mathscr{M}$ implies $\mathscr{M}_\varphi$ is a subset of $\mathscr{M}_\delta$.

(ii) Let $(p \to q)^\sim, (p \rightsquigarrow q)^- \in \mathscr{M}_\varphi$ and $q \in \mathscr{M}_\varphi$.

Then, we have $\delta((p \to q)^\sim) = \delta((p \rightsquigarrow q)^-) = \delta(q) = \delta(0)$. Thus, $\max\{\delta((p \to q)^\sim), \delta((p \rightsquigarrow q)^-), \delta(q)\} = \delta(0)$. But, $\varphi(p) \leq \max\{\delta((p \to q)^\sim), \delta((p \rightsquigarrow q)^-), \delta(q)\} = \delta(0)$.
Then, $\varphi(p) = \delta(0)$ implies $\varphi(p) = \varphi(0) \Rightarrow p \in \mathscr{M}_\delta$. Hence, $\mathscr{M}_\delta$ is an $EPWI$-ideal of $\mathscr{M}$ related to $\mathscr{M}_\varphi$. □

Note 3.11. *Let φ be a fuzzy subset of a lattice pseudo W-algebra of $\mathscr{M}$, then φ' is defined to be $\varphi'(p) = \varphi(p) + 1 - \varphi(0)$ for all $p \in \mathscr{M}$.*

Proposition 3.12. *Let δ and φ be fuzzy subsets of lattice pseudo W-algebra of $\mathscr{M}$ such that $\varphi(0) = \delta(0)$. Then φ is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ if and only if φ' is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ'.*

Proof. Let φ be an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ.

(i) Let $p \in \mathscr{M}$ then $\varphi(p) \leq \delta(p)$ implies $\varphi(p) + 1 - \varphi(0) \leq \varphi(p) + 1 - \varphi(0)$. Then, we have $\varphi'(p) \leq \delta'(p)$. Therefore φ' is a fuzzy subset of δ'.

(ii) $\varphi'(0) = \varphi(0) + 1 - \varphi(0) \Rightarrow \varphi'(0) = 1 \Rightarrow \varphi'(0) \leq \varphi'(p)$ for all $p \in \mathscr{M}$.

(iii) Let $p, q \in \mathscr{M}$ then

$$\begin{aligned}
\varphi'(p) =& \varphi(p) + 1 - \varphi(0) \\
&\leq \max\left\{\delta\left((p \to q)^\sim\right), \delta\left((p \rightsquigarrow q)^-\right), \delta(q)\right\} \\
&+ 1 - \delta(0) \\
&\leq \max\left\{\delta\left((p \to q)^\sim\right) + 1 - \delta(0), \delta\left((p \rightsquigarrow q)^-\right)\right. \\
&\left. + 1 - \delta(0), \delta(q) + 1 - \delta(0)\right\} \\
&\leq \max\left\{\delta'\left((p \to q)^\sim\right), \delta'\left((p \rightsquigarrow q)^-\right), \delta'(q)\right\} \\
\varphi'(p) &\leq \max\left\{\delta'\left((p \to q)^\sim\right), \delta'\left((p \rightsquigarrow q)^-\right), \delta'(q)\right\}.
\end{aligned}$$

Hence φ' is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ'.
Conversely,

(i) Let φ' is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ'. Then we have φ' is a fuzzy subset of δ' and $\varphi'(p) \leq \delta'(p)$ for all $p \in \mathscr{M}$. Thus, $\varphi(p) + 1 - \varphi(0) \leq \varphi(p) + 1 - \varphi(0)$ for all $p \in \mathscr{M}$ and $\varphi(p) \leq \mu(p)$ for all $p \in \mathscr{M}$ since, $\varphi(0) = \delta(0)$. Therefore, we have φ is a fuzzy subset of δ.

(ii) Let $p \in \mathscr{M}$ then $\varphi'(0) \leq \delta'(p)$ implies $\varphi(0) \leq \delta(p)$.

(iii) Let $p, q \in \mathscr{M}$ then

$$\begin{aligned}
\varphi'(p) &\leq \max\left\{\delta'\left((p \to q)^\sim\right), \delta'\left((p \rightsquigarrow q)^-\right), \delta'(q)\right\} \\
\varphi'(p) &\leq \max\left\{\delta\left((p \to q)^\sim\right) + 1 - \delta(0), \delta\left((p \rightsquigarrow q)^-\right)\right. \\
&\quad \left. + 1 - \delta(0), \delta(q) + 1 - \delta(0)\right\} \\
\varphi'(p) &\leq \max\left\{\delta\left((p \to q)^\sim\right), \delta\left((p \rightsquigarrow q)^-\right), \delta(q)\right\} \\
&\quad + 1 - \delta(0) \\
\varphi(p) &\leq \max\left\{\delta\left((p \to q)^\sim\right), \delta\left((p \rightsquigarrow q)^-\right), \delta(q)\right\}.
\end{aligned}$$

Hence φ is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ. □

Note 3.13. *Let φ be a fuzzy subset of lattice pseudo W-algebra of $\mathscr{M}$, then the set $\{p \in \mathscr{M} / \varphi(v) \leq \varphi(p)\}$ is denoted by $(\varphi(v))^*$.*

Proposition 3.14. *Let $\mathscr{M}$ be a lattice pseudo W-algebra and $v \in \mathscr{M}$. If φ is an anti-fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ such that $\varphi(v) = \delta(v)$, then $(\delta(v))^*$ is an $EPWI$-ideal of $\mathscr{M}$ related to $(\varphi(v))^*$.*

Proof. Let φ is an anti fuzzy $EPWI$-ideal of $\mathscr{M}$ related to δ. To prove that $(\delta(v))^*$ is an $EPWI$-ideal of $\mathscr{M}$ related to $(\varphi(v))^*$.

(i) If $p \in (\varphi(v))^*$ implies $\varphi(v) \leq \varphi(p) \Rightarrow \delta(v) \leq \delta(p)$, since $\varphi(v) = \delta(v)$ and $\varphi(p) \leq \delta(p)$.

(ii) since $\varphi(0) \leq \varphi(v)$ implies $0 \in (\varphi(v))^*$.

(iii) If $p \in \mathscr{M}$ and $q \in (\varphi(v))^*$ such $(p \to q)^\sim, (p \rightsquigarrow q)^- \in (\varphi(v))^*$ and $q \in (\varphi(v))^*$ implies, then, we have

$$\delta\left((p \to q)^\sim\right) \leq \varphi(v), \delta\left((p \rightsquigarrow q)^-\right) \leq \varphi(v)$$

and $\delta(q) \leq \varphi(v)$. Thus,

$$\varphi(v) \geq \delta\left((p \to q)^\sim\right), \varphi(v) \geq \delta\left((p \rightsquigarrow q)^-\right)$$

and $\varphi(v) \geq \delta(q)$. Therefore,

$$\varphi(v) \geq \max\left\{\delta\left((p \to q)^\sim\right), \delta\left((p \rightsquigarrow q)^-\right), \delta(q)\right\}.$$

But,

$$\varphi(p) \leq \max\left\{\delta\left((p \to q)^\sim\right), \delta\left((p \rightsquigarrow q)^-\right), \delta(q)\right\}.$$

Then, we have

$$\varphi(v) \leq \varphi(p) \Rightarrow \delta(w) \leq \delta(p) \Rightarrow p \in (\delta(v))^*.$$

Hence, $(\mu(v))^*$ is an $EPWI$-ideal of $\mathscr{M}$ related to $(\varphi(v))^*$.

□

4. Conclusion

In this paper, we have discussed some representations of anti-fuzzy $CCPWI$-ideal and anti-fuzzy $EPWI$-ideal of lattice pseudo W-algebras. We show that the connection between anti-fuzzy $EPWI$-ideal and $EPWI$-ideal. This idea can further be generalized to intuitionistic fuzzy $EPWI$-ideal for new results in our future work.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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