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Introduction to hub polynomial of graphs

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Abstract
In this paper we introduce hub polynomial of a connected graph G.The hub polynomial of a connected graph G of order n is the polynomial $H_G(x) = \sum_{i=h(G)}^{n} h_{G,i} x^i$ where $h_{G,i}$ denotes the number of hub sets of G of cardinality i and $h(G)$ is the hub number of G.We obtain hub polynomial of some special classes of graphs.We study hub roots of some graph G.Also we obtain hub polynomial of join of two graphs.We define H-unique graphs and obtain some family of H-unique graphs.

Keywords
Hub set, Hub Polynomial, Hub roots, H-unique Graphs.

AMS Subject Classification
05C40, 05C99

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Contents

1. Introduction

By a graph $G = (V,E)$ we mean a finite ordered graph with no loops and no multiple edges.For graph theoretic terminology we refer [3] . We use the following concepts in this article.

A wheel graph W_n is a graph with n vertices obtained from a cycle C_{n-1} by adding a new vertex and edges joining it to all vertces of the cycle.A double star graph $S_{m,n}$ is a tree obtained from the graph K_2 with two vertices u and v by attaching m pendedant edges in u and n pendant edges in v.

The Dutch windmill graph D_3^m is the graph obtained by selecting one vertex in each of m triangles and identifying them.

A lollipop graph$L_{(m,n)}$ is the graph obtained by joining the complete graph K_m to a path graph P_n with a bridge.

A fan graph is denoted by $F_{1,n}$ and is defined as $P_n + K_1$, where P_n is the path graph with n vertices.

The join of two graphs G_1 and G_2 is the graph with vertex set $V(G_1) \cup V(G_2)$ and edge set
$E(G_1) \cup E(G_2) \cup \{uv/u \in V(G_1), v \in V(G_2)\}$.

We use following results to prove our main results.

Theorem 1.1. *[5] If C_n is the cycle with n vertices then, $h(C_n) = n-3$.*

Theorem 1.2. *[5] If P_n is the path with n vertices then, $h(P_n) = n-2$.*

Theorem 1.3. *(De Gua's Theorem) If the polynomial lacks 2m consecutive terms,that is,the coefficients of these terms vanish ,then this polynomial has no less than 2m complex roots.If 2m+1 consecutive terms are missing ,then if they are between terms of different signs,the polynomial has no less than 2m complex roots ,whereas if the missing terms are between terms of the same sign the polynomial has no less than 2m+2 complex roots.*

2. Hub Polynomial of Graphs

Let $G = (V,E)$ be a connected graph.The concept of hub set is introduced by M walsh[5]. A subset H of V is called a hub set of G if for any two distinct vertices $u, v \in V - H$, there exists a u-v path P in G such that all the internal vertices of P are in H. The minimum cardinality of a hub set of G is called the hub number of G and is denoted by $h(G)$. In this paper we introduce the Hub Polynomial of a Graph G of order n as

$$H_G(x) = \sum_{i=h(G)}^{n} h_{G,i} x^i$$

where $h_{G,i}$ denotes the number of hub sets of cardinality i .

We consider the hub polynomial of the complete graph K_n as a special case since $h(K_n) = 0$.We define it as $H_{K_n}(x) = (1+x)^n - 1$ since it has nC_i hub sets of cardinality i for each $i = 1,2,3....n$.

A root of the hub polynomial is called hub root of G.

From the very definition of hub polynomial we obtain the following results.

Theorem 2.1. *Let G be a connected graph of order n,Then*

1. $h_{G,n} = 1$
2. $h_{G,i} = 0$ *if and only if* $i \leq h(G) - 1$ *or* $i \geq n+1$.
3. $H_G(x)$ *has no constant term.*
4. *If* G_1 *is any sub graph of G,then* $deg(H_G(x) \geq deg(H_{G_1}(x))$.
5. *zero is a root of* $H_G(x)$ *of multiplicity* $h(G)$ *for all non complete graph G.*
6. *Sum of coefficients of* $H_G(x)$ = *Total number of hub sets of G.*
7. *Coefficients of* x^{n-2} *in* $H_G(x)$ *is* nC_2.

Theorem 2.2. *If* G_1 *and* G_2 *are two isomorphic graphs then,* $H_{G_1}(x) = H_{G_2}(x)$

Theorem 2.3. *The hub polynomial of the path* P_n *is* $H_{P_n}(x) = \binom{n}{2}x^{n-2} + \binom{n}{1}x^{n-1} + x^n$

Proof. Let $P_n = (v_1, v_2, ...v_n)$ be a path .Then we have $h(P_n) = n-2$.Also every sub set of vertex set of P_n consisting of n-2 elements and all its super sets form a hub set for the path P_n.Hence $h_{P_n,n-2} = \binom{n}{n-2}$,$h_{P_n,n-1} = \binom{n}{n-1}$ and $h_{P_n,n} = 1$ □

Theorem 2.4. *The hub polynomial of the cycle* C_n *is* $H_{C_n}(x) = \binom{n}{3}x^{n-3} + \binom{n}{2}x^{n-2} + \binom{n}{1}x^{n-1} + x^n$

Proof. Let $C_n = (v_1, v_2, ...v_n, v_1)$ be a cycle .Then we have $h(C_n) = n-3$.Also every sub set of vertex set of C_n consisting of n-3 elements and all its super sets form a hub set for the cycle C_n.Hence $h_{C_n,n-3} = \binom{n}{n-3}$, $h_{C_n,n-2} = \binom{n}{n-2}$, $h_{C_n,n-1} = \binom{n}{n-1}$ and $h_{C_n,n} = 1$ □

Theorem 2.5. *The hub polynomial of the star graph* $K_{1,n}$, $n \geq 3$ *is* $H_{K_{1,n}}(x) = x(1+x)^n + nx^{n-1} + x^n$

Proof. Let u be the central vertex of $K_{1,n}$ and $v_1, v_2....v_n$ are the pendent vertices.Then $\{u\}$ is the unique hub set of cardinality one. Therefore $h(K_{1,n}) = 1$.

For $1 \leq i \leq n-2$, every hub set must include the central vertex u.Hence a hub set of cardinality i can be obtained by choosing $i-1$ vertices from $v_1, v_2, ...v_n$.This can be done in $\binom{n}{1}$ ways.For $i = n-1$,there are $\binom{n}{2}$ hub sets which contain u namely $\{u, v_1, v_2, ...v_{n-2}\}$, $\{u, v_1, v_2, ...v_{n-3}, v_n\}$.......$\{u, v_3, v_4, ...v_n\}$ and $\binom{n}{1}$ hub set which donot contain the central vertex u ,namely $\{v_1, v_2, ...v_{n-1}\}$....$\{v_2, v_3, ...v_n\}$. Hence there are $(\binom{n}{2} + \binom{n}{1})$ hub sets of cardinality $n-1$. For $i = n$,there are $\binom{n}{1}$ hub sets which contain the central vertex u and one hub set $\{v_1, v_2, ...v_n\}$ which do not contain u. Hence $H_{K_{1,n}}(x) = \sum_{i=1}^{n-2}\binom{n}{i-1}x^i + (\binom{n}{2} + \binom{n}{1})x^{n-1} + (\binom{n}{1} + 1)x^n + x^{n+1}$ $= x(1+x)^n + nx^{n-1} + x^n$ □

Theorem 2.6. *The hub polynomial of* $K_{2,n}$ *is* $H_{K_{2,n}}(x) = x(x+2)(1+x)^n + nx^{n-1} + x^n$.

Proof. Let $\{v_1, v_2\}$,$\{u_1, u_2...u_n\}$ are the bipartition of vertex set of $K_{2,n}$.Now $h(K_{2,n}) = 1$ and $\{v_1\}$ and $\{v_2\}$ are the only hub sets of cardinality one.

For $2 \leq i \leq n-2$ there are

1. $\binom{n}{i-1}$ hub sets which contains v_1 but not v_2
2. $\binom{n}{i-1}$ hub sets which contains v_2 but not v_1
3. $\binom{n}{i-2}$ hub sets which contain boh v_1 and v_2

For $i = n, n-1$,there are

1. $\binom{n}{i-1}$ hub sets which contains v_1 but not v_2
2. $\binom{n}{i-1}$ hub sets which contains v_2 but not v_1
3. $\binom{n}{i-2}$ hub sets which contain boh v_1 and v_2
4. $\binom{n}{i}$ hub sets which contain neither v_1 nor v_2

For $i = n+1$ there are $\binom{n}{1}$ hub sets containing both v_1 and v_2 and one hub set containing v_1 but not v_2 and one hub set containing v_2 but not v_1. Hence

$H_{K_{2,n}}(x) = 2x + \sum_{i=2}^{n-2}(\binom{n}{i-1} + \binom{n}{i-1} + \binom{n}{i-2})x^i + (\binom{n}{n-2} + \binom{n}{n-2} + \binom{n}{n-3} + \binom{n}{n-1})x^{n-1} + (\binom{n}{n-1} + \binom{n}{n-1} + \binom{n}{n-2} + \binom{n}{n})x^n + (\binom{n}{n} + \binom{n}{n} + \binom{n}{1})x^{n+1} + x^{n+2}$

$= x(x+2)(1+x)^n + nx^{n-1} + x^n$. □

Theorem 2.7. *The hub polynomial of the double star* $S_{m,n}$ *is* $H_{S_{m,n}}(x) = x^2(1+x)^{m+n} + (2m+2n+1)x^{m+n} + 2x^{m+n+1}$

Proof. Let $U = \{u_1, u_2, ...u_m\}$,$V = \{v_1, v_2....v_n\}$ and $\{u, v\}$ be the vertices of $S_{m,n}$ such that u and v are adjacent,every vertices in U are adjacent to u and every vertices in V are adjacent to v.Clearly the hub number of $S_{m,n}$ is two and $\{v_1, v_2\}$ is the only hub set of cardinality 2.For $3 \leq i \leq m+n-1$,every hub set must contain both the vertices u and v.Hence there are $\binom{m+n}{i-2}$ such hub sets

.For $i = m+n$

1. There are $\binom{m+n}{2}$ hub sets which contain both u and v.
2. There are $\binom{m+n}{1}$ hub sets which contains u but not v.
3. There are $\binom{m+n}{1}$ hub sets which contains v but not u.

4. $\{u_1, u_2, ...u_m, v_1, v_2...v_n\}$ is also hub set of cardinality $m+n$

For $i = m+n+1$ there are $m+n+2$ hub sets.Hence
$H_{S_{m,n}}(x) = x^2 + \binom{m+n}{1}x^3 + \binom{m+n}{2}x^4 + ... + \binom{m+n}{m+n-3}x^{m+n-1} + (\binom{m+n}{2} + 2m + 2n + 1)x^{m+n} + (m+n+2)x^{m+n+1} + x^{m+n+2}$
$= x^2(1+x)^{m+n} + (2m+2n+1)x^{m+n} + 2x^{m+n+1}$. □

Theorem 2.8. *The hub polynomial of the lollipop graph $L_{(n,1)}$ is*

$H_{L_{(n,1)}}(x) = 2x + \sum_{r=2, r\neq n-1}^{n+1}(2\binom{n}{r-1} - \binom{n-1}{r-2})x^r + (2\binom{n}{n-2} - \binom{n-1}{n-3} + 1)x^{n-1}$

Proof. Let $v_1, v_2, ...v_n$ be the vertices of the complete graph and let v_1 is adjacent to v.Then $\{v\}$ and $\{v_1\}$ are the only hubstes of cardinality 1.Therefore $h_{L_{(n,1)},1} = 2$.

For $1 \leq r \leq n-2$, every hub set consists of r elements cotains either v_1 or v or both.Hence $h_{L_{(n,1)},r} = 2\binom{n}{r-1} - \binom{n-1}{r-2}$.
For $r = n-1$, there are $2\binom{n}{n-2} - \binom{n-1}{n-3}$ hub sets ,which contain either v or v_1 and one hub set $\{v_2, v_3, ...v_n\}$ which does not contain v and v_1 .

For $r = n$ there are $n+1$ hubsets.For $r = n+1$,there is only one hub set.

Hence
$H_{L_{(n,1)}}(x) = 2x + \sum_{r=2, r\neq n-1}^{n+1} 2\binom{n}{r-1} - \binom{n-1}{r-2})x^r + (2\binom{n}{n-2} - \binom{n-1}{n-3} + 1)x^{n-1}$ □

Theorem 2.9. *The hub polynomial of the lollipop graph $L_{(3,n)}$ is*

$H_{L_{(3,n)}}(x) = (n+1)x^n + \binom{n+3}{2}x^{n+1} + (n+3)x^{n+2} + x^{n+3}$

Proof. Let v_1, v_2, v_3 be the vertices of the complete graph K_3 and $u_1, u_2, ...u_n$ be the vertices of the path graph P_n.Let v_1 is adjacent u_1. Then $h(L(3,n)) = n$ and the hub sets of cardinality n are the subsets of $\{v_1, u_1, u_2, ..., u_n\}$ of cardinality n. Hence
$h_{L_{(3,n)},n} = n+1$. All subsets of $V(L_{(3,n)})$ of cardinality $n+1, n+2, n+3$ are hub sets.Hence the result. □

Theorem 2.10. *The hub polynomial of the Dutch wind mill graph D_3^m is*

$H_{D_3^m}(x) = x(1+x)^{2m} + x^{2m-2}(x^2 + 2mx + m)$,$m \geq 2$.

Proof. Let x be the central vertex of D_3^m.Then $\{x\}$ is the only hubset of cardinality 1.For $2 \leq i \leq 2m-3$ there are exactly $\binom{2m}{i-1}$ hubsets of cardinality i which contains the central vertex x.

For $i = 2m-2$ there are $\binom{2m}{3}$ hub sets which contains x and there are m hub sets which doesnt conains x,namely the set of vertices obtained by removing each triangles one by one .

For $i = 2m-1$, there are $\binom{2m}{2}$ hub sets which contains x and there are 2m hub sets $\{1, 2, 3....2m-1\}$, $\{1, 2, 3, ...2m-2, 2m\}$.....$\{2, 3, ...2m\}$ which does not contain x.

For $i = 2m$ there are $\binom{2m}{1}$ hubsets which contain x and one hub set $\{1, 2, 3, ...2m\}$ which does not contain x.

Hence $H_{D_3^m}(x) = x(1+x)^{2m} + x^{2m-2}(x^2 + 2mx + m)$ □

Now we find the hub polynomial of a graph construction.Let $C_k = (v_1, v_2, ...v_k, v_1)$ be the cycle with k vertices.Then we denote the graph obtained from C_k by identifying the vertex v_k of c_k with an end vertex of the path graph P_n with n vertices by $C(k,n)$.Here we find the hub polynomial of the graph $C(4,n)$.

Theorem 2.11. *The hub polynomial of the graph $C(4,n)$ is*

$H_{C(4,n)}(x) = (3n+1)x^n + \binom{n+3}{2}x^{n+1} + (n+3)x^{n+2} + x^{n+3}$

Proof. Let v_1, v_2, v_3, v_4 are the vertices of the cycle and let $u_1, u_2, ...u_n$are the vertices of the path graph P_n.Let v_4 is identified with u_1.Clearly $h(C(4,n)) = n$. Then the subsets of $V(P_n)$ of cardinality $n-1$,union with $\{v_i\}, i = 1,2,3$ form hub sets of cardinality n.Also $V(P_n)$ is a hubset. Hence
$h_{C(4,n),n} = 3n+1$.All subsets of $V(C(4,n)$ of cardinality $n+1, n+2, n+3$ are hub sets.Hence the result. □

3. Hub polynomial of join of two graphs.

Theorem 3.1. *Let G_1 and G_2 be two connected graphs of order n_1 and n_2 respectively and let $G = G_1 + G_2$.Then hub polynomial of G is*

$H_G(x) = [(1+x)^{n_1} - 1][(1+x)^{n_2} - 1] + H_{G_1}(x) + H_{G_2}(x)$
.

Proof. If H is a hub set of G_j for $j = 1,2$ of cardinality i then H is also a hub set of G of cardinality i.Also for every $H_1 \subset V(G_1)$ and $H_2 \subset V(G_2)$,$H_1 \cup H_2$ is a hub set of G of cardinality $i = i_1 + i_2$, where i_j is the cardinality of H_j for $j = 1,2$. Thus,
$H_G(x) = [(1+x)^{n_1} - 1][(1+x)^{n_2} - 1] + H_{G_1}(x) + H_{G_2}(x)$. □

Corollary 3.2. *If $n \geq 3$,the hub polynomial of the wheel graph W_{n+1} is*

$H_{W_{n+1}}(x) = x(1+x)^n + \binom{n}{3}x^{n-3} + \binom{n}{2}x^{n-2} + nx^{n-1} + x^n$.

Proof. The proof follows from Theorem 3.1 since
$W_{n+1} = C_n + K_1$ □

Corollary 3.3. *The hub polynomial of the fan graph $F_{1,n}$ is*
$H_{F_{1,n}}(x) = x(1+x)^n + \binom{n}{2}x^{n-2} + nx^{n-1} + x^n$.

Proof. The proof follows from Theorem 3.1 since
$F_{1,n} = K_1 + P_n$ □

4. Hub Roots of a Graph

A root of the hub polynomial of a graph G is called a hub root of G.We denote $Z(H_G(x))$ as the set of all hub roots of G.The number of real hub roots of the graph G is called the H-number of G and is denoted by $H(G)$

Proposition 4.1. *Let G b a simple connected graph having exactly one hub root,then $G = K_1$*

Proof. Zero is a hub root of multiplicity $h(G)$,of any graph G. Hence $G = K_1$. □

Proposition 4.2. *If G has exactly two distinct hub roots then* $H(G) = 2$.

Proof. Zero is a hub root of any graph G,and complex roots occur in conjugate pairs.Therefore G has exactly 2 real hub roots and hence $H(G) = 2$ □

Proposition 4.3. .
$Z(H_{K_n}(x)) = \{exp(2\pi ki/n) - 1, k = 0,1,2,...,n-1\}$

Proof. The hub polynomial of K_n is $(1+x)^n - 1$.Then its n roots are given by $exp(2\pi ik/n) - 1$ □

Proposition 4.4.

$$H(P_n) = \begin{cases} 1 \quad or \quad 3 & \textit{if n is odd} \\ 0 \quad or \quad 2 & \textit{if n is even} \end{cases}$$

Proof. We have $H_{P_n}(x) = \binom{n}{2}x^{n-2} + \binom{n}{1}x^{n-1} + x^n$.Here the first n-3 terms are absent.Then by De Gua's theorem ,$H_{P_n}(x)$ has atleast n-3 complex roots when n is odd and n-2 complex roots when n is even.Hence the result. □

Proposition 4.5.

$$H(C_n) = \begin{cases} 1 \quad or \quad 3 & \textit{if n is odd} \\ 0 \quad or \quad 2 \quad or \quad 4 & \textit{if n is even} \end{cases}$$

Proof. We have $H_{C_n}(x) = \binom{n}{3}x^{n-3} + \binom{n}{2}x^{n-2} + \binom{n}{1}x^{n-1} + x^n$ Here the consecutive n-4 terms are absent.Then by De Gua's theorem $H_{C_n}(x)$ has atleast n-4 complex roots when n is even and n-3 complex roots when n is odd.Hence the result. □

Proposition 4.6.

$$H(K_{1,n}) = \begin{cases} 1 \quad or \quad 3 \quad or \quad 5 & \textit{if n is even} \\ 2 \quad or \quad 4 & \textit{if n is odd} \end{cases}$$

Proof. We have $H_{K_{1,n}}(x) = x(1+x)^n + nx^{n-1} + x^n$ .Therefore it suffices to prove that $f(x) = (1+x)^n + nx^{n-2} + x^{n-1}$ has zero or two or four real roots when n is even ,or one or three real roots when n is odd. But the number of real roots $f(x)$ is equal to the number of real roots of
$g(x) = (1+\frac{1}{x})^n + \frac{n}{x^2} + \frac{1}{x}$.Again the number of real roots of $g(x)$ is equal to the number of real roots of
$g(\frac{1}{x}) = (1+x)^n + nx^2 + x$.Consider
$g(\frac{1}{y-1}) = y^n + n(y-1)^2 + y - 1$
$= y^n + ny^2 - (2n-1)y + n - 1$.
We find the number of real roots of $h(y) = g(\frac{1}{y-1})$.Here the term containing $y^{n-1}, y^{n-2}, ...y^3$ are absent.By De Gua's rule ,$h(y)$ have atleast n-3 complex roots when n is odd and atleast n-4 complex roots when n is even.Hence the result. □

Proposition 4.7.

$$H(K_{2,n}) = \begin{cases} 1 \quad or \quad 3 \quad or \quad 5 & \textit{if n is odd} \\ 2 \quad or \quad 4 \quad or \quad 6 & \textit{if n is even} \end{cases}$$

Proof. $H_{K_{2,n}}(x) = x(x+2)(1+x)^n + nx^{n-1} + x^n$.Therefore it suffices to prove that $f(x) = (x+2)(1+x)^n + nx^{n-2} + x^{n-1}$ has zero or two or four real roots when n is odd ,and one or three or five real roots when n is even.But the number of real roots $f(x)$ is equal to the number of real roots of
$g(x) = (1+\frac{2}{x})(1+\frac{1}{x})^n + \frac{n}{x^3} + \frac{1}{x^2}$.Again the number of real roots of $g(x)$ is equal to the number of real roots of
$g(\frac{1}{x}) = (1+2x)(1+x)^n + nx^3 + x^2$.Consider,
$g(\frac{1}{y-1}) = 2y^{n+1} - y^n + ny^3 - (3n-1)y^2 + (3n-2)y - (n-1)$.We find the number of real roots of $h(y) = g(\frac{1}{y-1})$.Here the term containing $y^{n-1}, y^{n-2}, ...y^4$ are absent in $h(y)$.By De Gua's rule ,$h(y)$ have atleast n-4 complex roots when n is even and atleast n-5 complex roots when n is odd.Hence the result. □

5. H-Unique Graphs

Two graphs which are not isomorphic may have the same hub polynomial.So we introduce the concept of H-Unique graph as follows. Let H is the set of all connected graphs of order n.We define a relation on H as, $G_1 \equiv G_2$ if and only if $H_{G_1}(x) = H_{G_2}(x)$. Then $\equiv$ is an equivalence relation on H.We denote equivalence class of G by $[G]$. It is clear that there is no H-Unique connected simple graph of order 3 and P_4 is the only H-unique graph of order 4.Also there are eight H-Unique graphs of order 5.

Definition 5.1. *A graph G is called H-Unique if* $[G] = \{G\}$

Theorem 5.2. *The path* P_n *is H-Unique for all* $n \geq 4$

Proof. If G is a graph having n vertices and $G \in [P_n]$,then $h(G) = n-2$. Hence $G = P_n$ □

Theorem 5.3. *If G is a graph of orer n and* $v \in V(G)$ *be such that*

$d(v) = n-1$ *and* $G - \{v\}$ *is also connected,then G is H-Unique if and only if* $G - \{v\}$ *is H-Unique*

Proof. Since $d(v) = n-1$ we have $G = (G - \{v\}) + \{v\}$.

Hence $H_G(x) = x(1+x)^{n-1} + H_{G-\{v\}}(x)$.Therefore G is H-Unique if and only if $G - \{v\}$ is H-Unique. □

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