Malaya Journal of Matematik, Vol. 8, No. 4, 1635-1642, 2020
https://doi.org/10.26637/MJM0804/0052

Intuitionistic fuzzy classes of implicative filters in RLW-algebras

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Abstract
The goal of this paper is propose the definitions of intuitionistic fuzzy positive, associative and fantastic implicative filters of a RLW - algebras (Residuated Lattice Wajsberg algebras), and to investigate those properties with illustrations. In addition, we obtain some equivalent conditions of intuitionistic fuzzy positive, associative and fantastic implicative filters.

Keywords
Wajsberg algebra; Lattice Wajsberg algebra; RLW- algebra; Fuzzy implicative filter; Intuitionistic fuzzy implicative filter; Intuitionistic fuzzy positive implicative filter; Intuitionistic fuzzy associative implicative filter; Intuitionistic fuzzy fantastic implicative filter.

AMS Subject Classification
03B52, 03G10, 03E72.

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Article History: Received **17** July **2020**; Accepted **22** September **2020**

Contents

1. Introduction

In 1965, Lotfi. A. Zadeh [16] described the concept of fuzzy logic as the generalization of ordinary subsets. Krassimir .T .Atanassov [1,2] proposed the idea of fuzzy sets, known as intuitionistic fuzzy sets in 1983. Morgan Ward and R.P. Dilworth presented the idea of residuated lattices [14]. Mordchaj Wajsberg introduced the concept of Wajsberg algebras in [15]. Ibrahim and Basheer Ahamed [3,4] identified the concepts of fuzzy implicative filter and an anti-fuzzy implicative filter of lattice Wajsberg algebras and obtained some properties. Ibrahim and Jeya lekshmi [12] introduced the notion of intuitionistic fuzzy pseudo- boolean implicative filters of lattice pseudo -Wajsberg algebras. Ibrahim and Shajitha Begum [13] proposed the concepts of fuzzy positive implicative and Associative WI- ideals of lattice Wajsberg algebras and discussed some of their properties. The authors [6,7,8,9,10,11] proposed the notions of implicative filter, fuzzy implicative filter, intuitionistic fuzzy implicative filter, positive, associative and fantastic implicative filter, fuzzy positive, associative and fantastic implicative filters of RLW-algebra and discussed some of their properties.

In this research article, we present the notions of intuitionistic fuzzy positive, associative and fantastic implicative filters of RLW- algebra. Likewise, we examine a portion of their related properties. Finally, we discuss some equivalent conditions with illustrations.

2. Preliminaries

In this part we review some essential definitions and their properties which are useful to build up our primary outcomes.

Definition 2.1 ([5]). *Let* $(\mathscr{G}, \rightarrow, *, 1)$ *be an algebra with a* *binary operation " $\rightarrow$ " and a quasi complement "*". Then it is called Wajsberg algebra, if the following conditions are satisfied for all $r,s,t \in \mathscr{G}$,*

(i) $1 \rightarrow r = r$

(ii) $(r \rightarrow s) \rightarrow ((s \rightarrow t) \rightarrow (r \rightarrow t)) = 1$

(iii) $(r \rightarrow s) \rightarrow s = (s \rightarrow r) \rightarrow r$

(iv) $(r^* \rightarrow s^*) \rightarrow (s \rightarrow s) = 1.$

Proposition 2.2 ([5]). *Let $(\mathscr{G}, \rightarrow, *, 1)$ be a Wajsberg algebra. Then the following axioms are satisfied for all $r,s,t \in \mathscr{G}$,*

(i) $r \rightarrow r = 1$

(ii) If $(r \rightarrow s) = (s \rightarrow r) = 1$ *then* $r = s$

(iii) $r \rightarrow 1 = 1$

(iv) $(r \rightarrow (s \rightarrow r)) = 1$

(v) If $(r \rightarrow s) = (s \rightarrow t) = 1$ *then* $s \rightarrow t = 1$

(vi) $(r \rightarrow s) \rightarrow ((t \rightarrow r) \rightarrow (t \rightarrow s)) = 1$

(vii) $r \rightarrow (s \rightarrow t) = s \rightarrow (r \rightarrow t)$

(viii) $r \rightarrow 0 = r \rightarrow 1^* = r^*$

(ix) $(r^*)^* = r$

(x) $(r^* \rightarrow s^*) = s \rightarrow r.$

Definition 2.3 ([5]). *Let $(\mathscr{G}, \rightarrow, *, 1)$ be a Wajsberg algebra. Then it is called a lattice Wajsberg algebra, if the following conditions are satisfied for all $r,s \in \mathscr{G}$,*

(i) The partial ordering " $\leq$ " on a lattice Wajsberg algebra $\mathscr{G}$ such that $u \leq v$ if and only if $r \rightarrow r = 1$

(ii) $(r \vee r) = (r \rightarrow s) \rightarrow s$

(iii) $(r \wedge s) = (r^* \rightarrow s^*) \rightarrow r^*)^*.$

Thus, $(\mathscr{G}, \vee, \wedge, *, \rightarrow, 0, 1)$ is a lattice Wajsberg algebra with lower bound 0 and upper bound 1 .

Proposition 2.4 ([5]). *Let $(\mathscr{G}, \rightarrow, *, 1)$ be a Wajsberg algebra. Then the following axioms are satisfied for all $u,s,w \in \mathscr{G}$,*

(i) If $r \leq s$ then $r \rightarrow t \geq s \rightarrow t$ and $t \rightarrow r \leq t \rightarrow s$

(ii) $r \leq s \rightarrow r$ if and only if $s \leq r \rightarrow t$

(iii) $(r \vee s)^* == (\ r^* \wedge z^*)$

(iv) $(r \wedge s)^* = (r^* \vee s^*)$

(v) $(r \vee s) \rightarrow t = (r \rightarrow t) \wedge (s \rightarrow t)$

(vi) $r \rightarrow (s \wedge t) = (r \rightarrow s) \wedge (r \rightarrow t)$

(vii) $(u \rightarrow s) \vee (s \rightarrow r) = 1$

(viii) $r \rightarrow (s \vee t) = (r \rightarrow s) \vee (r \rightarrow t)$

(ix) $(r \wedge s) \rightarrow t = (r \rightarrow t) \vee (s \rightarrow t)$

(x) $(r \wedge s) \vee t = (r \vee t) \wedge (s \vee t)$

(xi) $(r \wedge s) \rightarrow t = (r \rightarrow s) \rightarrow (r \rightarrow t).$

Definition 2.5 ([14]). *Let $(\mathscr{G}, \vee, \wedge, \odot, \rightarrow, 0, 1)$ be an algebra of type $(2,2,2,2,0,0)$. Then, it is called a residuated lattice, if the following axioms are satisfied for all $r,s,t \in \mathscr{G}$,*

(i) $(\mathscr{G}, \vee, \wedge, 0, 1)$ is a bounded lattice

(ii) $(\mathscr{G}, \odot, 1)$ is a commutative monoid

(iii) $r \odot s \leq t$ if and only if $r \leq s \rightarrow t$.

Definition 2.6 ([5]). *Let $(\mathscr{G}, \vee, \wedge, \rightarrow, *, 0, 1)$ be a lattice Wajsberg algebra. Then, it is called a RLW - algebra, if a binary operation "$\odot$"on $\mathscr{G}$, satisfied the condition $r \odot s = (r \rightarrow s^*)^*$ for all $t,s, \in \mathscr{G}$.*

Note 2.7. *From the Definition 2.6, we have $(\mathscr{G}, \vee, \wedge, \odot, \rightarrow, *, 0, 1)$ is called a RLW - algebra.*

Definition 2.8 ([5]). *Let $\mathscr{G}$ be a lattice Wajsberg algebra. Then a subset $\mathscr{P}$ of $\mathscr{G}$ is called an implicative filter of $\mathscr{G}$, if the following axioms are satisfied for all $r,s \in \mathscr{G}$,*

(i) $1 \in \mathscr{P}$

(ii) $r \in \mathscr{P}$ *a n d* $r \rightarrow s \in \mathscr{P}$ *imply* $r \in \mathscr{P}$.

Definition 2.9 ([9]). *Let $\mathscr{G}$ be a RLW - algebra. Then the non-empty subset $\mathscr{P}$ of $\mathscr{G}$ is called a positive implicative filter of g, if the following axioms are satisfied for all $r,s,t \in \mathscr{G}$*

(i) $1 \in \mathscr{P}$

(ii) $(s \odot r) \rightarrow t \in \mathscr{P}$ *and* $r \rightarrow s \in \mathscr{P}$ *imply* $r \rightarrow t \in \mathscr{P}$

(iii) $(r \rightarrow s) \rightarrow s \in \mathscr{P}$ *and* $r \in \mathscr{P}$ *imply* $s \in \mathscr{P}$.

Definition 2.10 ([9]). *Let $\mathscr{G}$ be a RLW - algebra. Then the non-empty subset $\mathscr{P}$ of $\mathscr{G}$ is called a associative implicative filter of $\mathscr{G}$, if the following axioms are satisfied for all $r,s,t \in \mathscr{G}$,*

(i) $1 \in \mathscr{P}$

(ii) $r \odot (s \odot t) \in \mathscr{P}$ *and* $r \odot s \in \mathscr{P}$ *imply* $t \in \mathscr{P}$

(iii) $r \rightarrow (s \rightarrow t) \in \mathscr{P}$ *and* $r \rightarrow s \in \mathscr{P}$ *imply* $t \in \mathscr{P}$.

Definition 2.11 ([11]). *Let g be a RLW$-$ algebra. Then a subset $\mathscr{P}$ of $\mathscr{G}$ is called a fantastic implicative filter of $\mathscr{G}$, if the following axioms are satisfied for all $r,s,t \in \mathscr{G}$,*

(i) $1 \in \mathscr{P}$

(ii) If $r,s \in \mathscr{G}$ *then* $r \odot v \in \mathscr{P}$

(iii) $t \rightarrow (s \rightarrow r) \in \mathscr{P}$ *and* $t \in \mathscr{P}$ *imply* $((r \rightarrow s) \rightarrow s) \rightarrow r \in \mathscr{P}$.

Definition 2.12 ([15]). *Let $\mathscr{G}$ be a set. A function $\eta : \mathscr{G} \to [0,1]$ is called a fuzzy subset on $\mathscr{G}$, for each $r \in \mathscr{G}$. The value of $\eta(r)$ describes a degree of membership of r in η.*

Definition 2.13 ([15]). *Let η be a fuzzy subset of X, then the complement of η is denoted by $\overline{\eta(r)}$ and defined as $\overline{\eta(r)} = 1 - \eta(r)$ for all $u \in X$.*

Definition 2.14 ([10]). *Let $\mathscr{G}$ be a RLW - algebra. Then the fuzzy subset η of $\mathscr{G}$ is called a fuzzy positive implicative filter of $\mathscr{G}$, if it satisfies the following axioms for all $u, v, w \in \mathscr{G}$,*

(i) $\eta(1) \geq \eta(r)$

(ii) $\eta(r \to t) \geq \min\{\eta((s \odot r) \to r), \eta(r \to s)\}$

(iii) $\eta(s) \geq \min\{\eta((r \to s) \to s), \eta(r)\}$.

Definition 2.15 ([10]). *Let $\mathscr{G}$ be a RLW− algebra. Then the fuzzy subset η of $\mathscr{G}$ is called a for all $u, s, t \in \mathscr{G}$,*

(i) $\eta(1) \geq \eta(r)$

(ii) $\eta(t) \geq \min\{\eta(r \odot (s \odot t)), \eta(r \odot s)\}$

(iii) $\eta(t) \geq \min\{\eta(r \to (s \to t)), \eta(r \to s)\}$.

Definition 2.16 ([11]). *Let g be a RLW - algebra. Then a fuzzy subset η of G is called a fuzzy fantastic implicative filter of g, if it satisfies the following axioms for all $r, s, t \in \mathscr{G}$,*

(i) $\eta(1) \geq \eta(r)$

(ii) $\eta(r \odot s) \geq \min\{\eta(r), \eta(s)\}$

(iii) $\eta(((r \to s) \to s) \to r) \geq \min\{\eta(t \to (s \to r)), \eta(t)\}$.

Definition 2.17 ([1]). *An intuitionistic fuzzy set H in a non-empty set X is an object having the for*

$$H = \{(r, \eta_H(r), \theta_H(r)) / r \in X\} = (\eta_H, \theta_H),$$

where the functions $\eta_H : X \to [0,1]$ and $\theta_H : X \to [0,1]$ denote the degree of membership and the degree of nonmembership respectively, and $0 \leq \eta_H(r) + \theta_H(r) \leq 1$ for any $r \in X$.

Definition 2.18 ([8]). *Let $(\mathscr{G}, \vee, \wedge, \odot, \to, *, 0, 1)$ be a RLW - algebra. Then, an intuitionistic fuzzy set $H = (\eta_H, \theta_H)$ of $\mathscr{G}$ is said to be an intuitionistic fuzzy implicative filter of $\mathscr{G}$ if the following axioms are satisfied for all $r, s \in \mathscr{G}$,*

(i) $\eta_H(1) \geq \eta_H(r)$ *and* $\theta_H(1) \leq \theta_H(r)$

(ii) $\eta_H(s) \geq \min\{\eta_H(r \to s), \eta_H(r)\}$

(iii) $\theta_H(s) \leq \max\{\theta_H(r \to s), \theta_H(r)\}$

(iv) $\eta_H(r \odot s) \geq \min\{\eta_H(r), \eta_H(s)\}$

(v) $\theta_H(r \odot s) \leq \max\{\theta_H(r), \theta_H(s)\}$.

3. Main Results

In this section, all propositions are discussed under the condition of intuitionistic fuzzy positive, associative and fantastic implicative filters of *RLW* - algebra and obtain some useful results with illustrations.

3.1 Intuitionistic Fuzzy Positive Implicative Filter

Definition 3.1. *Let $(\mathscr{G}, \vee, \wedge, \odot, \to, *, 0, 1)$ be a RLW- algebra. Then, an intuitionistic fuzzy set $H = (\eta_H, \theta_H)$ of $\mathscr{G}$ is called an intuitionistic fuzzy positive implicative filter of $\mathscr{G}$, if it satisfies the following axioms for all $r, s, t \in \mathscr{G}$,*

(i) $\eta_H(1) \geq \eta_H(r)$ *and* $\theta_H(1) \leq \theta_H(r)$

(ii) $\eta_H(r \to t) \geq \min\{\eta_H((s \odot r) \to t), \eta_H(r \to s)\}$

(iii) $\theta_H(r \to t) \leq \max\{\theta_H((s \odot r) \to t), \theta_H(r \to s)\}$

(iv) $\eta_H(s) \geq \min\{\eta_H((r \to s) \to s), \eta_H(u)\}$

(v) $\theta_H(s) \leq \max\{\theta_H((r \to s) \to s), \theta_H(r)\}$.

Example 3.2. *Let $\mathscr{G} = \{0, p, q, \ell, m, n, o, 1\}$ be a set with Figure 1 as a partial ordering. Define a binary operation $\to$ and a quasi-complement on $\mathscr{G}$ as in Tables 2 and 3.*

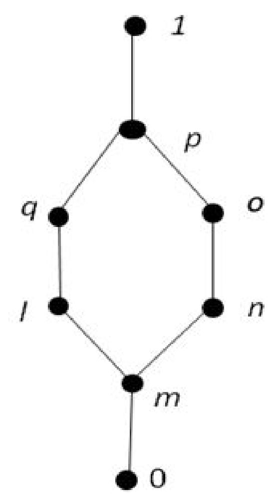

Figure 1. Lattice diagram

Table 1. Complement

r	r^*
0	1
p	m
q	n
ℓ	o
m	p
n	q
o	ℓ
1	0

Table 2. Implication

$\rightarrow$	0	p	q	ℓ	m	n	o	1
0	1	1	1	1	1	1	1	1
p	1	1	p	p	o	o	o	1
q	m	1	1	p	o	o	o	1
ℓ	o	1	1	1	o	o	o	1
m	p	1	1	1	1	1	1	1
n	1	1	p	p	p	1	1	1
o	ℓ	1	p	p	p	p	1	1
1	0	p	q	ℓ	m	n	o	1

Define $\vee,\wedge$ and $\odot$ operations on $(\mathscr{G},\vee,\wedge,0,1)$ as follow $(r\vee s)=(r\rightarrow s)\rightarrow s$, $(r\wedge s)=((r^*\rightarrow s^*)\rightarrow s^*)^*$. $r\odot s=(r\rightarrow s^*)^*$ for all $r,s\in\mathscr{G}$. Then, $(\mathscr{G},\vee,\wedge,\odot,\rightarrow,*0,1)$ is a $RLW-$ algebra. Consider an intuitionistic fuzzy subset $H=(\eta_H,\theta_H)$ on $\mathscr{G}$ is defined by

$$\eta_H(r)=\begin{cases}1 & \text{if } r=1 \text{ for all } r\in\mathscr{G}\\ 0.2 & \text{otherwise for all } r\in\mathscr{G},\end{cases}$$

$$\theta_H(r)=\begin{cases}0 & \text{if } r=1 \text{ for all } r\in\mathscr{G}\\ 0.3 & \text{otherwise for all } r\in\mathscr{G}.\end{cases}$$

Then, H is an intuitionistic fuzzy positive implicative filter of RLW - algebra $\mathscr{G}$.

From the previous example, let us consider an intuitionistic fuzzy subset $H=(\eta_H,\theta_H)$ on $\mathscr{G}$ as,

$$\eta_H(r)=\begin{cases}0.7 \text{ if } & r\in\{0,p,\ell\} \text{ for all } r\in\mathscr{G}\\ 0.5 \text{ otherwise for all } r\in\mathscr{G},\end{cases}$$

$$\theta_H(u)=\begin{cases}0.4 \text{ if } r\in\{0,p,\ell\} \text{ for all } r\in\mathscr{G}\\ 0.6 \text{ otherwise for all } r\in\mathscr{G}.\end{cases}$$

Then, H is not an intuitionistic fuzzy positive implicative filter of RLW-algebra $\mathscr{G}$. Since,

$$\eta_H(1\rightarrow q)<\min\{\eta_H((p\odot 1)\rightarrow q),\eta_H(1\rightarrow p)\}$$

and

$$\theta_H(1\rightarrow q)>\max\{\theta_H((p\odot 1)\rightarrow q),\theta_H(1\rightarrow p)\}.$$

Proposition 3.3. *Every intuitionistic fuzzy positive implicative filter of RLW - algebra $\mathscr{G}$ is an intuitionistic fuzzy implicative filter.*

Proof. Let $H=(\eta_H,\theta_H)$ be an intuitionistic fuzzy positive implicative filter of $RLW-$ algebra $\mathscr{G}$.
From (ii) of Definition 3.1

$$\eta_H(r\rightarrow t)\geq\min\{\eta_H((s\odot r)\rightarrow t),\eta_H(r\rightarrow s)\}.$$

We know that, $(s\odot r)\rightarrow t=s\rightarrow(r\rightarrow t)$,

$$\eta_H(r\rightarrow t)\geq\min\{\eta_H(s\rightarrow(r\rightarrow t)),\eta_H(r\rightarrow s)\}.$$

Taking $r=1$,, from (i) of Definition 2.1 we get,

$$\eta_H(1\rightarrow t)\geq\min\{\eta_H(s\rightarrow(1\rightarrow t)),\eta_H(1\rightarrow s)\},$$

$\eta_H(w)\geq\min\{\eta_H(s\rightarrow t),\eta_H(s)\}$. Similarly, from (iii) of Definition 3.1

$$\theta_H(r\rightarrow t)\leq\max\{\theta_H((s\odot r)\rightarrow t),\theta_H(r\rightarrow s)\},$$

$$\theta_H(r\rightarrow t)\leq\max\{\theta_H(s\rightarrow(r\rightarrow t)),\theta_H(r\rightarrow s)\}.$$

Taking $r=1$, From (i) of Definition 2.1, we get

$$\theta_H(1\rightarrow t)\leq\max\{\theta_H(s\rightarrow(1\rightarrow t)),\theta_H(1\rightarrow s)\},$$

$\theta_H(t)\leq\max\{\theta_H(s\rightarrow t),\theta_H(s)$. Hence, $H=(\eta_H,\theta_H)$ be an intuitionistic fuzzy implicative filter of $\mathscr{G}$. ☐

Proposition 3.4. *Let $H=(\eta_H,\theta_H)$ be an intuitionistic fuzzy implicative filter of $RLW-$ algebra $\mathscr{G}$, then $H=(\eta_H,\theta_H)$ is order preserving.*

Proof. Let $H=(\eta_H,\theta_H)$ be an intuitionistic fuzzy implicative filter of RLW-algebra $\mathscr{G}$. Then,

$$\begin{aligned}\eta_H(s)&\geq\min\{\eta_H(r\rightarrow s),\eta_H(r)\}, \text{[From (ii) of Definition 2.14]}\\ &=\min\{\eta_H(1),\eta_H(r)\} \quad \text{[From (i) of Definition 2.3]}\\ &=\eta_H(r).\end{aligned}$$

Thus,

$$\begin{aligned}&\eta_H(s)\geq\eta_H(r)\\ &\theta_H(s)\leq\max\{\theta_H(r\rightarrow s),\theta_H(r)\} \text{[From (ii) of Definition2.14]}\\ &\quad=\max\{\theta_H(1),\theta_H(r)\} \text{[From (i) of Definition2.3]}\\ &\quad=\theta_H(r).\end{aligned}$$

Thus, $\theta_H(s)\leq\theta_H(r)$. Hence, $H=(\eta_H,\theta_H)$ is order preserving. ☐

3.2 Intuitionistic Fuzzy Associative Implicative Filter

Definition 3.5. *Let $(\mathscr{G},\vee,\wedge,\odot,\rightarrow,*,0,1)$ be a $RLW-$ algebra. Then, an intuitionistic fuzzy set $H=(\eta_H,\theta_H)$ of $\mathscr{G}$ is called an intuitionistic fuzzy associative implicative filter of $\mathscr{G}$, if it satisfies the following axioms for all $r,v,w\in\mathscr{G}$,*

(i) $\eta_H(1)\geq\eta_H(u)$ and $\theta_H\leq\theta_H(r)$.

(ii) $\eta_H(t)\geq\min\{\eta_H(r\rightarrow(s\rightarrow t)),\eta_H(r\rightarrow s)\}$.

(iii) $\theta_H(t)\leq\max\{\theta_H(r\rightarrow(s\rightarrow t),\theta_H(r\rightarrow s)\}$.

(iv) $\eta_H(t)\geq\min\{\eta_H(r\odot(s\odot t)),\eta_H(r\odot s)\}$.

(v) $\theta_H(t)\leq\max\{\theta_H(r\odot(s\odot t)),\theta_H(r\odot s)\}$.

Example 3.6. *Let $\mathscr{G}=\{0,f,g,n,o,1\}$ be a set with Figure 2 as a partial ordering. Define a binary operation "$\rightarrow$" and a quasi-complement "$*$" on $\mathscr{G}$ as in Tables 3 and 4.*

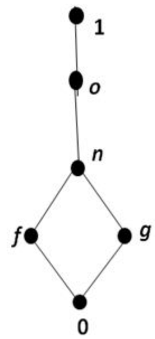

Figure 2. Lattice diagram

Table 3. Complement

r	r^*
0	1
f	g
g	f
n	0
o	0
1	0

Table 4. Implication

$\to$	0	f	g	n	o	1
0	1	1	1	1	1	1
f	g	1	g	1	1	1
g	f	n	1	1	1	1
n	0	f	g	1	1	1
o	0	f	g	n	1	1
1	0	f	g	n	o	1

Define $\vee, \wedge$ *and* $\odot$ *operations on* $(\mathscr{G}, \vee, \wedge, 0, 1)$ *as follow* $(r \vee s) = (r \to s) \to s$, $(r \wedge s) = ((r^* \to s^*) \to s^*)^*$. $r \odot s = (r \to s^*)^*$ *for all* $r, s \in \mathscr{G}$. *Then,* $(\mathscr{G}, \vee, \wedge, \odot, \to, *0, 1)$ *is a RLW - algebra. Consider an intuitionistic fuzzy subset* $H = (\eta_H, \theta_H)$ *on* $\mathscr{G}$ *is defined by*

$$\eta_H(r) = \begin{cases} 1 & \text{if } r = 1 \text{ for all } r \in \mathscr{G} \\ 0.3 & \text{otherwise for all } r \in \mathscr{G} \end{cases}$$

$$\theta_H(r) = \begin{cases} 0 & \text{if } r = 1 \text{ for all } r \in \mathscr{G} \\ 0.4 & \text{otherwise for all } r \in \mathscr{G}. \end{cases}$$

Then, H is an intuitionistic fuzzy associative implicative filter of RLW-algebra $\mathscr{G}$.

From the Example 3.6 let us consider an intuitionistic fuzzy subset $H = (\eta_H, \theta_H)$ on $\mathscr{G}$ as,

$$\eta_H(r) = \begin{cases} 0.8 \text{ if} & r \in \{0, g, n\} \text{ for all } r \in \mathscr{G} \\ 0.6 & \text{otherwise for all } r \in \mathscr{G}, \end{cases}$$

$$\theta_H(r) = \begin{cases} 0.2 \text{ if } r \in \{0, g, n\} \text{ for all } r \in \mathscr{G} \\ 0.5 \text{ otherwise for all } r \in \mathscr{G}. \end{cases}$$

Then, H is not an intuitionistic fuzzy associative implicative filter of *RLW* - algebra A. Since,

$$\eta_H(1) < \min\{\eta_H((f \odot (g \odot 1)), \eta_H(f \odot g)\}$$

and

$$\theta_H(1) > \max\{\theta_H((f \odot (g \odot 1)), \theta_H(f \odot g)\}.$$

Proposition 3.7. *Every intuitionistic fuzzy associative implicative filter of RLW - algebra* $\mathscr{G}$ *with respect to 1 is an intuitionistic fuzzy implicative filter.*

Proof. Let $H = (\eta_H, \theta_H)$ be an intuitionistic fuzzy associative implicative filter of *RLW*-algebra $\mathscr{G}$ with respect to 1 . Then,

$$\eta_H(t) \geq \min\{\eta_H(1 \to (s \to t)), \eta_H(1 \to s)\}$$
[From (ii) of Definition 3.5]
$$= \{\eta_H(s \to t), \eta_H(s)\}$$ [From (i) of Definition 2.1]

Therefore, $\eta_H(t) \geq \{\eta_H(s \to t), \eta_H(s)\}$. Now,

$$\theta_H(t) \leq \max\{\theta_H(1 \to (s \to t)), \theta_H(1 \to s)\}$$
[From (iii) of Definition 3.5]
$$= \max\{\theta_H(s \to t), \theta_H(s)\}$$ [From (iii) of Definition 2.1]

Therefore, $\theta_H(t) \leq \max\{\theta_H t, \theta_H(s)\}$.Hence, $H = (\eta_H, \theta_H)$ be an intuitionistic fuzzy implicative filter of $\mathscr{G}$. □

Proposition 3.8. *Let* $H = (\eta_H, \theta_H)$ *be an intuitionistic fuzzy implicative filter of RLW – algebra* $\mathscr{G}$, *then* $H = (\eta_H, \theta_H)$ *be an intuitionistic fuzzy associative implicative filter of* $\mathscr{G}$ *if and only if satisfies the following conditions:*

(i) $\eta_H((r \to s) \to t) \geq \eta_H(r \to (s \to t))$
(ii) $\theta_H((r \to s) \to t) \leq \theta_H((r \to (s \to t))$, *for all* $r, s, t \in \mathscr{G}$. (3.1)

Proof. Let $H = (\eta_H, \theta_H)$ be an intuitionistic fuzzy implicative filter of *RLW*-algebra $\mathscr{G}$ and satisfies $\eta_H((r \to s) \to t) \geq \eta_H(r \to (s \to t))$ and $\gamma_M((r \to s) \to t) \leq \gamma_M((r \to (s \to t))$ for all $r, s, t \in \mathscr{G}$. Then,

$$\eta_H(t) \geq \min\{\eta_H((r \to s) \to z), \eta_H(r \to s)\}$$
[From (ii) of Definition 3.5]
$$\geq \min\{\eta_H(r \to (s \to t)), \eta_H(r \to s)\}$$
[From (vii) of Proposition 2.2]
$$\theta_H(t) \leq \max\{\theta_H((r \to s) \to z)\theta_H(r \to s)\}$$
[From (ii) of Definition 3.5]
$$\leq \max\{\theta_H(r \to (s \to t)), \theta_H(r \to s)\} \text{ for all } r, s, t \in \mathscr{G}$$
[From (vii) of Proposition 2.2].

Therefore, $H = (\eta_H, \theta_H)$ is an intuitionistic fuzzy associative implicative filter of $\mathscr{G}$.
Conversely, assume that $H = (\eta_H, \theta_H)$ be an intuitionistic fuzzy associative implicative filter of *RLW* - algebra and for all $r, s, t \in \mathscr{G}$. Then, we have

$$r \to ((s \to t) \to t) = (s \to w) \to (r \to t)$$
$$= (s \to t) \to ((r \to s) \to (r \to t))$$
[From (vii) of Proposition 2.2]
$$= (r \to s) \to ((s \to t) \to (r \to t))$$
[From (vii) of Proposition 2.2]
$$= 1 \in F$$ [From (vi) of Proposition 2.2]

Now,

$\eta_H((r\to s)\to t)$
$\geq \min\{\eta_H(r\to((s\to t)\to((r\to s)\to t))),\eta_H(r\to t)\}$
$=\min\{\eta_H(1),\eta_H(r\to(s\to t))\}$, [From (i) of Proposition 2.2]
$=\eta_H(r\to(s\to t))$.

Hence, $\eta_H((r\to s)\to t)\geq \eta_H(r\to(s\to t))$. Similarly,

$\theta_H((r\to s)\to t)$
$\leq \max\{\theta_H(r\to((s\to t)\to t)),\theta_H(r\to(s\to t))\}$
$=\min\{\theta_H(1),\theta_H(r\to t)\}$, [From (i) of Proposition 2.2]
$=\theta_H(r\to(s\to t))$.

Hence, $\theta_H((r\to s)\to t)\leq \theta_H(r\to(s\to t))$ □

Proposition 3.9. *An intuitionistic fuzzy set $H=(\eta_H,\theta_H)$ is an implicative filter of RLW − algebra $\mathscr{G}$. Then, $H=(\eta_H,\theta_H)$ is an intuitionistic fuzzy associative implicative filter of RLW algebra $\mathscr{G}$ if and only if satisfies the following axioms:*

$$(i)\ \eta_H(s)\geq \eta_H(r\to(r\to s))$$
$$(ii)\ \theta_H(s)\leq \theta_H(r\to(r\to s))\ for\ all r,s\in\mathscr{G}. \quad (3.2)$$

Proof. Let $H=(\eta_H,\theta_H)$ be an intuitionistic fuzzy implicative filter of *RLW* - algebra $\mathscr{G}$.
To prove: $H=(\eta_H,\theta_H)$ is an intuitionistic fuzzy associative implicative filter of $\mathscr{G}$.
It is enough to show that (3.1) and (3.2) are equivalent. Let $r=s$ in equation (3.2), then

$$\eta_H((s\to s)\to t))\geq \eta_H(s\to(s\to t)).$$

From (i) of Proposition 2.2, $\eta_H(t)\geq \eta_H(s\to(s\to t))$ and $\theta_H((s\to s)\to t))\leq \theta_H(s\to(s\to t))$. Also From (i) of Proposition 2.2, $\theta_H(t)\leq \theta_H(s\to(s\to t))$. Hence, (3.2) and (3.2) are equivalent.
Conversely, assume that if (3.2.2) hold

Let$(r\to(s\to t))\to(r\to(r\to((r\to s)\to t))$
$=1\to((r\to(s\to t))\to(r\to(r\to((r\to s)\to t)))$
$=((s\to t)\to(r\to(r\to s)\to t))$
$\quad\to((r\to t)\to(r\to(r\to(r\to((r\to s)\to t))))$
$=(r\to(s\to t))\to((s\to t)\to(r\to(r\to s)\to t))))$
$\quad\to(r\to(r\to((r\to s)\to t')))$
$=(r\to(s\to t))\to(r\to(s\to t))$
$=1$ [From (i) of Proposition 2.2]

Since,

$$(r\to(s\to t))\to(r\to(r\to((r\to s)\to t)))\in\mathscr{P}$$

Let,

$\eta_H((r\to s)\to t)\geq\{\eta_H(r\to((r\to s)\to t)))\}$
$\geq\min\{\eta_H(1),\eta_H(r\to(s\to t))\}$
$=\eta_H(r\to(s\to t))$

Similarly, $\theta_H((r\to s)\to t)\leq\theta_H(r\to(s\to t))$. Hence, $H=(\eta_H,\theta_H)$ is an intuitionistic fuzzy associative implicative filter of $\mathscr{G}$. □

3.3 Intuitionistic Fuzzy Fantastic Implicative Filter

Definition 3.10. *Let $(\mathscr{G},\vee,\wedge,\odot,\to,*,0,1)$ be a RLW-algebra. Then, an intuitionistic fuzzy set $H=(\eta_H,\theta_H)$ of $\mathscr{G}$ is called an intuitionistic fuzzy fantastic implicative filter of $\mathscr{G}$, if it satisfies the following axioms for all $r,s,t\in\mathscr{G}$,*

(i) $\eta_H(1)\geq\eta_H(r)$ *and* $\theta_H(1)\leq\theta_H(r)$

(ii) $\eta_H(((r\to 8)\to s)\to r)\geq\min\{\eta_H(t\to(s\to r)),\eta_H(t)\}$

(iii) $\theta_H(((r\to s)\to s)\to r)\leq\max\{\theta_H(t\to(s\to r)),\theta_H(t)\}$

(iv) $\eta_H(r\odot s)\geq\min\{\eta_H(r),\eta_H(s)\}$

(v) $\theta_H(r\odot s)\leq\max\{\theta_H(r),\theta_H(s)\}$.

Example 3.11. *Let $\mathscr{G}=\{0,n,o,1\}$ be a set with Figure 3 as a partial ordering. Define a binary operation" $\to$ " and a quasi-complement "*" on $\mathscr{G}$ as in Tables 5 and 6.*

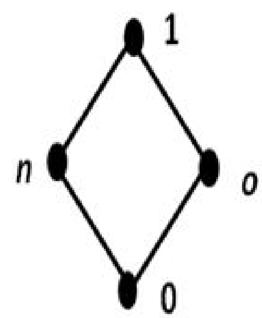

Figure 3. Lattice diagram

Table 5. Complement

r	r^*
0	1
n	o
o	n
1	0

Table 6. Implication

$\to$	0	n	o	1
0	1	1	1	1
n	n	1	1	1
o	n	n	1	1
1	0	n	o	1

Define $\vee,\wedge$ and $\odot$ operations on $(\mathscr{G},\vee,\wedge,0,1)$ as follow $(r\vee s)=(r\to s)\to s$, $(r\wedge s)=((r^*\to s^*)\to s^*)^*$ $r\odot s=(r\to s^*)^*$ for all $r,s\in\mathscr{G}$. Then, $(\mathscr{G},\vee,\wedge,\odot,\to,*0,1)$ is a

RLW-algebra. Consider an intuitionistic fuzzy subset $H = (\eta_H, \theta_H)$ *on* $\mathscr{G}$ *is defined by*

$$\eta_H(r) = \begin{cases} 1 & \text{if } r = 1 \text{ for all } r \in \mathscr{G} \\ 0.5 & \text{otherwise for all } r \in \mathscr{G} \end{cases}$$

$$\theta_H(r) = \begin{cases} 0 & \text{if } r = 1 \text{ for all } r \in \mathscr{G} \\ 0.7 & \text{otherwise for all } r \in \mathscr{G}. \end{cases}$$

Then, H is an intuitionistic fuzzy fantastic implicative filter of RLW - algebra $\mathscr{G}$.

In the same Example 3.11 let us consider an intuitionistic fuzzy subset $H = (\eta_H, \theta_H)$ on $\mathscr{G}$ as,

$$\eta_H(r) = \begin{cases} 0.6 & \text{if } r \in \{0,1\} \text{ for all } r \in \mathscr{G} \\ 0.8 & \text{if } \quad r \in \{n,o\} \text{ for all } r \in \mathscr{G} \end{cases},$$

$$\gamma_M(r) = \begin{cases} 0.3 & \text{if } r \in \{0,1\} \text{ for all } r \in \mathscr{G} \\ 0.2 & \text{if } r \in \{n,o\} \text{ for all } r \in \mathscr{G}. \end{cases}$$

Then, H is not an intuitionistic fuzzy fantastic implicative filter of *RLW* - algebra $\mathscr{G}$. Since, $\eta_H(n \odot o) < \min\{\eta_H(n), \eta_H(o)\}$ and $\theta_H(n \odot o) > \min\{\theta_H(n), \theta_H(o)\}$.

Proposition 3.12. *Every intuitionistic fuzzy fantastic implicative filter of RLW-algebra* $\mathscr{G}$ *is an intuitionistic fuzzy implicative filter.*

Proof. Let $H = (\eta_H, \theta_H)$ be an intuitionistic fuzzy fantastic implicative filter of $RLW-$ algebra $\mathscr{G}$.

Let $r = 1$ in (iii) of Definition 3.10, Then, we have

$$\eta_H(((r \to 1) \to 1) \to r) \geq \min\{\eta_H(t \to (1 \to r)), \eta_H(t)\}$$
$$\eta_H((1 \to 1) \to r) \geq \min\{\eta_H(t \to r), \eta_H t\}$$

From (iii) of Proposition 2.2 and from (i) of Definition 2.1, we have

$$\eta_H(1 \to r) \geq \min\{\eta_H(t \to r), \eta_H t\},$$

$$\eta_H(r) \geq \min\{\eta_H(t \to r), \eta_H(t)\}.$$

Also, from (iii) of Proposition 2.2 and from (i) of Definition 2.1, we have

$$\theta_H(((r \to 1) \to 1) \to r) \leq \max\{\theta_H(t \to (1 \to r)), \theta_H(t)\}$$
$$\theta_H((1 \to 1) \to r) \leq \max\{\theta_H(t \to r), \theta_H(t)\}$$
$$\theta_H((1 \to 1) \to r) \leq \max\{\theta_H(t \to r), \theta_H(t)\}$$
$$\theta_H(r) \leq \max\{\theta_H(t \to r), \theta_H(t)\}.$$

Hence, H is an intuitionistic fuzzy fantastic implicative filter of *RLW* - algebra $\mathscr{G}$. □

Proposition 3.13. *Let Let* $H = (\eta_H, \theta_H)$ *be an intuitionistic fuzzy fantastic implicative filter of RLW - algebra* $\mathscr{G}$, *if* $t \leq r \to v$, *then*

(i) $\eta_H(((r \to s) \to s) \to r) \geq \eta_H(t)$

(ii) $\theta_H(((r \to s) \to s) \to r) \leq \theta_H(t)$ *for all* $r,s,t \in \mathscr{G}$.

Proof. Let $H = (\eta_H, \theta_H)$ be an intuitionistic fuzzy fantastic implicative filter of *RLW* algebra G. Let $w \leq r \to s$ then

$$t \to (r \to s) = r \to (t \to s) = 1.$$

Assume that,

$$\eta_H(((r \to s) \to s) \to r) \geq \min\{\eta_H(t \to (r \to s)), \eta_H(t)\}$$
$$\eta_H(((r \to s) \to s) \to r) \geq \in \{\eta_H(1), \eta_H(t)\}$$
$$\geq \eta_H(t).$$

Similarly,

$$\theta_H(((r \to s) \to s) \to r) \leq \max\{\theta_H(t \to (s \to s)), \theta_H(t)\}$$
$$\theta_H(((r \to s) \to s) \to r) \leq \max\{\theta_H(1), \theta_H(t)\}$$
$$\leq \theta_H(t).$$

□

Proposition 3.14. *Let* $H = (\eta_H, \theta_H)$ *be an intuitionistic fuzzy fantastic implicative filter of RLW - algebra* $\mathscr{G}$ *if and only if it satisfies the following axioms*

(i) $(\eta_H(((r \to s) \to s) \to r) \geq \eta_H(s \to r)$

(ii) $(\theta_H(((r \to s) \to s) \to r) \leq \theta_H(s \to r)$ *for all* $r,s,t \in \mathscr{G}$.

Proof. Let $H = (\eta_H, \theta_H)$ be an intuitionistic fuzzy fantastic implicative filter of *RLW* algebra $\mathscr{G}$. Then, from (ii) of Definition 3.10,

$$\eta_H(((r \to s) \to s) \to r) \geq \min\{\eta_H(t \to (s \to r)), \eta_H(t)\}.$$

Let $t = 1$, from (i) of Definition 2.1, we get

$$\eta_H(((r \to s) \to s) \to r) \geq \min\{\eta_H(1 \to (s \to r)), \eta_H(1)\}$$
$$\eta_H(((s \to s) \to s) \to r) \geq \min\{\eta_H(s \to r), \eta_H(1)\}$$
$$\geq \eta_H(s \to r),$$

From (iii) of Definition 3.10,

$$\theta_H(((r \to s) \to s) \to r) \leq \max\{\theta_H(t \to (s \to r)), \theta_H(t)\}.$$

Let $t = 1$, we get

$$\theta_H(((r \to s) \to s) \to r) \leq \max\{\theta_H(1 \to (s \to r)), \theta_H(1)\}.$$

From (i) of Definition 2.1,

$$\gamma_M(((r \to s) \to s) \to r) \leq \max\{\gamma_M(s \to r), \gamma_M(1)\}$$
$$\leq \gamma_M(s \to r).$$

Conversely, assume that

$$\eta_H(((r \to s) \to s) \to r) \geq \eta_H(s \to u),$$

then

$$\eta_H(((r \to s) \to s) \to r) \geq \min\{\eta_H(t \to r), \eta_H(t)\}.$$

Similarly,

$$\eta_H(((r \to s) \to s) \to r) \leq \max\{\gamma_M(s \to (s \to r)), \gamma_M(s)\}.$$

Hence, $H = (\eta_H, \theta_H)$ is an intuitionistic fuzzy fantastic implicative filter of $RLW-$ algebra $\mathscr{G}$. □

4. Conclusion

The ideas of intuitionistic fuzzy positive, associative and fantastic implicative filters of *RLW*− algebra was defined in this paper and discussed some of their properties with examples. Furthermore, we examined an identical condition of intuitionistic fuzzy positive and associative implicative filters of *RLW* - algebra were established.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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