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Dominating function in intuitionistic fractional graph

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Abstract

In this paper, definition of intuitionistic fractional star graph, intuitionistic fractional bistar graph and intuitionistic fractional wheel graph has been introduced and we also define dominating function, minimal dominating function, intuitionistic fractional domination number ($\gamma_{i_f}(G)$) and upper intuitionistic fractional domination number ($\Gamma_{i_f}(G)$) of an intuitionistic fuzzy graph(IFG). We derived these parameters for a path, cycle, star, bistar and wheel of an intuitionistic fractional graph.

Keywords

Intuitionistic fractional graph(IFG), intuitionistic fractional domination number,intuitionistic fractional star graph, intuitionistic fractional bistar graph, intuitionistic fractional wheel graph.

AMS Subject Classification

03E72, 03F55, 05C69, 05C72, 05C38, 05C07.

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Contents

1. Introduction

Let G=(V,E) be a graph. The open neighbourhood $N(v)$ and the closed neighbourhood $N[v]$ of v are defined by $N(v) = \{u \in V : uv \in E\}$and $N[v] = N(v) \cup v$[3]. A dominating set S is a subset of the vertices in a graph such that every vertex in the graph either belongs to S or has a neighbour in S. An excellent treatment of fundamentals of domination in graphs and several advanced topics in domination are given in Haynes et al. [5, 6].

In [5], a graph $G = (V,E)$ and for a real-valued function $f : V \to R$, the weight of f is $w(f) = \sum_{v\in V} f(v)$, and for $S \subseteq V$ we define $f(S) = \sum_{v\in S} f(v)$, so $w(f) = f(V)$. For a vertex $v \in V$, we denote $f(N[v])$ by $f[v]$ for notational convenience. Let $f : V \to \{0,1\}$ be a function which assigns to each vertex of a graph an element of the set $\{0,1\}$. We say f is dominating function if for every $v \in V$, $f[v] \geq 1$. We say f is a minimal dominating function if there does not exist a dominating function $g : V \to \{0,1\}$, $f \neq g$, for which $g(v) \leq f(v)$ for every $v \in V$. That is equivalent to saying that a dominating function f is minimal if for every v such that $f(v) > 0$, there exists a vertex $u \in N[v]$ for which $f[u] = 1$. Then the domination number and upper domination number of a graph G can be defined as $\gamma(G) = min\{w(f)|f \text{ is a dominating function of } G\}$ and $\Gamma(G) = max\{w(f)|f \text{ is a minimal dominting function on } G\}$.

Fractional graph theory deals with the generalisation of integer-valued graph theoretic concepts such that they take on fractional values as their weights in the interval [0,1]. Let a graph $G = (V,E)$ and $f : V \to [0,1]$ be a function which weights are assigns to each vertex of a graph in the interval [0,1]. We say f is dominating function(DF) if $f(N[v]) = \sum_{u\in N[v]} f(u) \geq 1$ for every $v \in V$. We say a dominating function f is a minimal dominating function(MDF) if there does not exist a dominating function $f \neq g$, for which $g(v) \leq f(v)$ for every $v \in V$. Equivalently f is an MDF if for every v with $f(v) > 0$, there exist a vertex $w \in N[v]$ such that $f(N[w])$ or $\sum_{u\in N[w]} f(u) = 1$. Then the fractional domination number and upper fractional dominating number of G can be defined as $\gamma_f(G) = min\{|f| : f \text{ is an MDF of G}\}$ and $\Gamma_f(G) = max\{|f| : f \text{ is an MDF of G}\}$ in [7]. To determine the values of fractional parameter $\gamma_f(G)$ and $\Gamma_f(G)$ which has one of the standard methods for converting a graph concept from integer version to fractional version is to formulate the concept as an integer program and then to consider the linear programming relaxation. That is, a linear program (LP) is an optimization problem that can be expressed in the form Maximize $c^t x$ subject to $Ax \leq b$ and Minimize $c^t x$ subject to $Ax \geq b$" where b is an m-vector, c is an n-vector, A is an m-by-n matrix, and x varies over all n-vectors with nonnegative entries. An integer program(IP) is an optimization problem of the same form as a linear program except that the vector x is subject to the additional constraint that all its entries must be integers. In an LP or an IP, the expression $c^t x$ is called the objective function, a vector x satisfying the constraints $Ax \leq b, x \geq 0$ is called a feasible solution, and the optimum of the objective function over all feasible solutions is called the value of the program. It is natural to assign the value $-\infty$ to a maximization program with no feasible solutions and the value $+\infty$ if the objective function is unbounded on feasible solutions.Many of the fractional invariants in [10] can be defined by taking a definition of a standard graph invariant verbatim and inserting the word fuzzy in an appropriate place. Thus it is often appropriate to understand the subscript f, which we use throughout to denote a fractional analogue, to also stand for the word $fuzzy$. One can speculate what might be meant by a fuzzy or fractional graph. This could mean a pair (V,E) in which V is a finite set and E is a fuzzy set of 2-element subsets of V. Alternatively, one might allow V to be a fuzzy set as well. A detailed study of fractional graph theory and fractionalisation of various graph parameters are given in Scheinerman and Ullman [10].

In fractional or fuzzy graph $G = (V,E)$, where V be the vertex set which determines only the degree of membership and define fuzzy vertex set of a graph G. In this paper, we add an additional component which determines the degree of non-membership also and defining intuitionistic fuzzy vertex set of a graph G. In fractional or fuzzy graph into intuitionistic fractional (or) fuzzy graph(IFG) $G = (V,E)$, where the vertex set V be the fuzzy set into intuitionistic fuzzy set by way of taking the non-membership values are also into consideration.The same idea is extended to edge set also and we define a function $f : V \to [0,1]$, we obtain the fractional dominating function, minimal dominating function of an intuitionistic fuzzy graph. Articles [4, 5, 8, 10] motivated us to analyze the fractional dominating function parameters of intuitionistic fuzzy graphs. In Section 2, we review some basic concepts and definitions. Section 3, we introduce the definition of dominating function, minimal dominating function of the intuitionistic fractional graphs, intuitionistic fractional star graph, intuitionistic fractional bistar graph and intuitionistic fractional wheel graph. In Section 4, a linear programming algorithm for finding an intuitionistic fractional parameters γ_{if}, intuitionistic fractional domination number and Γ_{if}, upper intuitionistic fractional domination number of an IFG G have been formulated. We found γ_{if} and Γ_{if} for a path, a cycle, a star, a bistar and a wheel of an IFG.

2. Preliminaries

In this section, some basic definitions and observation which are used in constructing the algorithm relating to IFGs are given.

Definition 2.1. *[8] An Intuitionistic Fuzzy Graph (IFG) is of the form $G = (V,E)$, where*
(i) $V = \{v_1, v_2, \ldots v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\nu_1 : V \to [0,1]$ denote the degrees of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V$ $(i = 1,2,\ldots,n)$.
(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\nu_2 : V \times V \to [0,1]$ are such that

$$\mu_2(v_i,v_j) \leq min[\mu_1(v_i), \mu_1(v_j)]$$
$$\nu_2(v_i,v_j) \leq max[\nu_1(v_i), \nu_1(v_j)]$$

and $0 \leq \mu_2(v_i,v_j) + \nu_2(v_i,v_j) \leq 1$
for every $(v_i,v_j) \in E, (i,j = 1,2,\ldots,n)$

Definition 2.2. *[9] Let $G = \langle V,E \rangle$ be an IFG.The neighbourhood of any vertex v is defined as*
$N(v) = (N_\mu(v), N_\nu(v))$ where
$N_\mu(v) = \{w \in V; \mu_2(v,w) = \mu_1(v) \bigwedge \mu_1(w)\}$ and
$N_\nu(v) = \{w \in V; \nu_2(v,w) = \nu_1(v) \bigvee \nu_1(w)\}$ and
$N[V] = N(v) \cup \{v\}$ is called the closed neighbourhood of v.

Definition 2.3. *[9] The neighbourhood degree of a vertex in an IFG G, is defined as $d_N(v) = (d_{N_\mu}(v), d_{N_\nu}(v))$ where $d_{N_\mu}(v) = \sum_{w \in N(v)} \mu_1(w)$ and $d_{N_\nu}(v) = \sum_{w \in N(v)} \nu_1(w)$.*

Definition 2.4. *[9] The closed neighbourhood degree of a vertex v in an IFG G, is defined as $d_N[v] = (d_{N_\mu}[v], d_{N_\nu}[v])$ where $d_{N_\mu}[v] = \sum_{u \in N(v)} \mu_1(u) + \mu_1(v)$ and $d_{N_\nu}[v] = \sum_{u \in N(v)} \nu_1(u) + \nu_1(v)$.*
That is

$$d_{N_{\mu_1}}[v] = \sum_{u \in N(v)} \mu_1(u) + \mu_1(v) = \sum_{u \in N_{\mu_1}[v]} \mu_1(u)$$
$$d_{N_{\nu_1}}[v] = \sum_{u \in N(v)} \nu_1(u) + \nu_1(v) = \sum_{u \in N_{\nu_1}[v]} \nu_1(u)$$

Definition 2.5. *[8] A path P_n in IFG is a sequence of distinct vertices $v_1, v_2, \ldots, v_n$ such that either one of the following conditions is satisfied:*

$\mu_{2ij} > 0$ and $\nu_{2ij} = 0$ for some i and j,
$\mu_{2ij} = 0$ and $\nu_{2ij} > 0$ for some i and j,
$\mu_{2ij} > 0$ and $\nu_{2ij} > 0$ for some i and j $(i,j = 1,2,\ldots,n)$.

Definition 2.6. *[8] The length of a path $P = v_1, v_2, \ldots, v_{n+1} (n > 0)$ is n.*

Definition 2.7. *[8] A path $P = v_1, v_2, \ldots, v_{n+1}$ is called a cycle if $v_1 = v_{n+1}$, and $n \geq 3$.*

Definition 2.8. *[7] The problem of finding the fractional domination number (γ_f) and upper fractional domination number (Γ_f) is equivalent to finding the optimal solution of the following linear programming problem.*

Minimize $z = \sum_{v \in V(G)} f(v)$, *Subject to* $\sum_{x \in N[v]} f(x) \geq 1$ *and* $0 \leq f(v) \leq 1$ *for all* $v \in V(G)$ *and*
Maximize $z = \sum_{v \in V(G)} f(v)$, *Subject to* $\sum_{x \in N[v]} f(x) \leq 1$ *and* $0 \leq f(v) \leq 1$ *for all* $v \in V(G)$

Definition 2.9. *[1] A fuzzy star graph consists of two vertex sets V and U with* $|V| = 1$ *and* $|U| > 1$ *such that* $\mu(v, u_i) > 0$ *and* $\mu(u_i, u_{i+1}) = 0$ *for* $1 \leq i \leq n$.

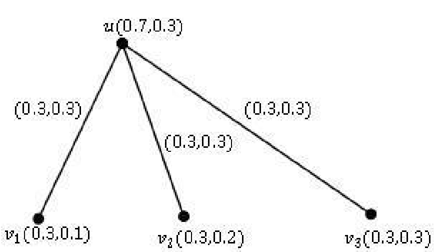
Definition 2.10. *[11] The bistar* $B_{1,n,n}$ *is graph obtained by joining the center(apex) vertices of two copies of* $K_{1,n}$ *by an edge.*

3. Dominating Function in Intuitionistic Fractional Graph

Intuitionistic Fuzzy Graph theory was introduced by Krassimir T Atanassov in [2]. In [8], M.G. Karunambigai and R. Parvathi introduced intuitionistic fuzzy graph as a special case of Atanassov's IFG. In intuitionistic fuzzy graph or intuitionistic fractional graph(IFG), G has a pair (V, E) in which V is a finite set and E is a intuitionistic fuzzy set of 2-element subsets of V. Alternatively, one might allow V to be an intuitionistic fuzzy set as well. In this section, we introduced the definition of intuitionistic fractional star graph, intuitionistic fractional bistar graph, intuitionistic fractional wheel graph, dominating function and minimal dominating function of an intuitionistic fractional graph.

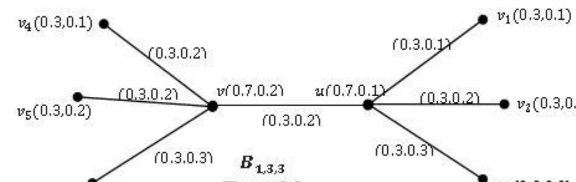
Definition 3.1. *A Star* $S_{1,n}$ *in an intuitionistic fractional graph is a sequence of distinct vertices* $u, v_1, v_2, \ldots, v_n$ *such that the following conditions are holds*

(i) $\mu_2(u, v_i) > 0, \nu_2(u, v_i) \geq 0$ *for some* u, v_i *;*
(ii)$\mu_2(v_i, v_j) = 0, \nu_2(v_i, v_j) = 0$ *for some* v_i, v_j
when $\mu_2(v_i, v_j) = \nu_2(v_i, v_j) = 0$ *for some* v_i, v_j*, there is no edge between* v_i *and* v_j*. Otherwise there exists an edge between u and* v_i *where* $i = 1, 2, \ldots, n$.
Note:If $S_{1,n}$ *be the Star graph, then consider its vertex set V as with* $V = \{u, v_1, v_2, \ldots, v_n\}$ *and edge set E as* $E = \{uv_1, uv_2, \ldots, uv_n\}$.



$S_{1,3}$
Figure 3.1

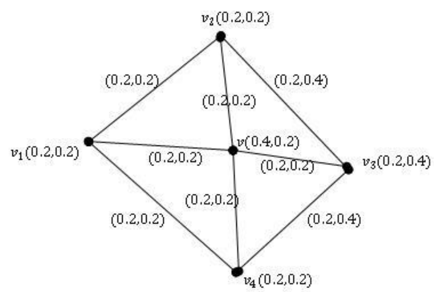
Definition 3.2. *The bistar* $B_{1,n,n}$ *in an intuitionistic fractional graph is a graph obtained by joining the center vertices of two copies of* $S_{1,n}$*(intuitionistic fractional star) by an edge.*



$B_{1,3,3}$
Figure 3.2

Definition 3.3. *A intuitionistic fractional wheel* W_n*, is a graph formed by connecting a single universal vertex to all vertices of a intuitionistic fuzzy cycle* C_n*, where* $V(W_n) = \{v, v_1, v_2, \ldots, v_n\}$ *and edges* $E(W_n) = \{x = vv_i : 1 \leq i \leq n-1\} \cup \{e_i = v_i v_{i+1} : 1 \leq i \leq n-2\} \cup \{e_{n-1} = v_{n-1} v_1\}$ *for* $n \geq 4$*, such that either one of the following conditions is satisfied:*

$\mu_{2ij} > 0$ *and* $\nu_{2ij} = 0$ *for some i and j,*
$\mu_{2ij} = 0$ *and* $\nu_{2ij} > 0$ *for some i and j,*
$\mu_{2ij} > 0$ *and* $\nu_{2ij} > 0$ *for some i and j* $(i, j = 1, 2, \ldots, n)$.



W_5
Figure 3.3

Definition 3.4. *A function* $f_{\mu_1} : V \to [0,1]$ *is called a* μ*-dominating function of* $G(V,E)$ *if the closed neighborhood degree of a vertex* $v \in V$ *such that* $f(d_{N_{\mu_1}}[v]) = \sum_{u \in N_{\mu_1}[v]} \mu_1(u) \geq 1$ *for every* $v \in V$.

Definition 3.5. *A function* $f_{\nu_1} : V \to [0,1]$ *is called a* ν*-dominating function of* $G(V,E)$ *if the closed neighborhood degree of a vertex* $v \in V$ *such that* $f(d_{N_{\nu_1}}[v]) = \sum_{u \in N_{\nu_1}[v]} \nu_1(u) < 1$ *for every* $v \in V$.

Definition 3.6. *A function* $f_{\mu_1,\nu_1} : V \to [0,1]$ *is called a dominating function (DF) if it is* μ*-dominating and* ν*-dominating function of G with* $0 \leq f_{\mu_1}(v) + f_{\nu_1}(v) \leq 1$ *for each* $v \in V$ *or A function* $f = f_{\mu_1,\nu_1} : V \to [0,1]$ *is called a dominating function (DF) of* $G = (V,E)$ *in which V is a intuitionistic fuzzy set and E is a 2-element subsets of V if the closed neighborhood degree of a vertex* $v \in V$ *where* $\mu_1(v) \geq 0, \nu_1(v) \neq 1$ *such that* $\sum_{u \in N_{\mu_1}[v]} \mu_1(u) \geq 1$, $\sum_{u \in N_{\nu_1}[v]} \nu_1(u) < 1$ *for every* $v \in V$ *with* $0 \leq f_{\mu_1}(v) + f_{\nu_1}(v) \leq 1$ *for each* $v \in V$.

Example 3.7. *Consider an IFG,* $G = (V,E)$*, such that* $V = \{v_1, v_2, v_3, v_4\}$ *and* $E = \{(v_1,v_2),(v_2,v_3),(v_3,v_4),(v_1,v_4)\}$.

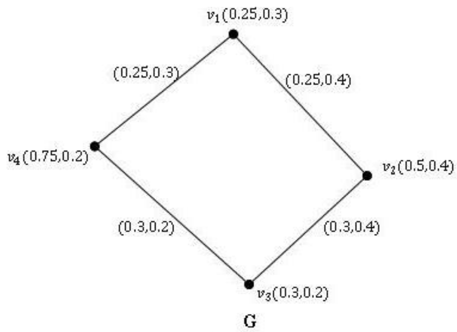


Figure 3.4

Here $N[v_1] = \{v_1, v_2, v_4\}, N[v_2] = \{v_1, v_2, v_3\}$, $N[v_3] = \{v_2, v_3, v_4\}, N[v_4] = \{v_1, v_3, v_4\}$
The closed neighborhood degree of a vertex v_1 *is* $v_1(1.5, 0.9)$, v_2 *is* $v_2(1.05, 0.9), v_3$ *is* $v_3(1.55, 0.8), v_4$ *is* $v_4(1.3, 0.7)$.
Therefore a function $f : V \to [0,1]$ *of an IFG, G is a dominating function. Since*

$$f(d_{N_{\mu_1}}[v]) = \sum_{u \in N_{\mu_1}[v]} \mu_1(u) \geq 1 \text{ and}$$

$$f(d_{N_{\nu_1}}[v]) = \sum_{u \in N_{\nu_1}[v]} \nu_1(u) < 1 \text{ for every } v \in V.$$

Definition 3.8. *A dominating function* $f = f_{\mu_1,\nu_1}$ *of G is called a minimal dominating function (MDF), if for every* $v \in V$*, where* $\nu_1(v) \neq 1$ *such that*

$$\sum_{u \in N_{\mu_1}[v]} \mu_1(u) = 1, \quad \sum_{u \in N_{\nu_1}[v]} \nu_1(u) < 1 \text{ for any } u \in N[v].$$

Definition 3.9. *The intuitionistic fractional domination number of G, denoted by* $\gamma_{if}(G)$ *is defined as,*
$\gamma_{if}(G) = min\{|f| : f \text{ is an MDF of } IFG\}$ *where*
$|f| = \sum_{v \in V} f(v) = (\sum_{v \in V} f_{\mu_1}(v), \sum_{v \in V} f_{\nu_1}(v))$.
or $\gamma_{if}(G) = (\gamma_{if_{\mu_1}}(G), \gamma_{if_{\nu_1}}(G))$ *where* $\gamma_{if_{\mu_1}}$ *is a* f_{μ_1}*-intuitionistic fractional domination number and* $\gamma_{if_{\nu_1}}$ *is a* f_{ν_1}*-intuitionistic fractional domination number of G.*

Definition 3.10. *The upper intuitionistic fractional domination number of G, denoted by* $\Gamma_{if}(G)$ *is defined as,*
$\Gamma_{if}(G) = max\{|f| : f \text{ is an MDF of } IFG\}$ *where*
$|f| = \sum_{v \in V} f(v) = (\sum_{v \in V} f_{\mu_1}(v), \sum_{v \in V} f_{\nu_1}(v))$.
or $\Gamma_{if}(G) = (\Gamma_{if_{\mu_1}}(G), \Gamma_{if_{\nu_1}}(G))$ *where* $\Gamma_{if_{\mu_1}}$ *is a* f_{μ_1}*-upper intuitionistic fractional domination number and* $\Gamma_{if_{\nu_1}}$ *is a* f_{ν_1}*-upper intuitionistic fractional domination number of G.*

Observation 3.1. *The problem of finding the intuitionistic fractional domination number* $\gamma_{if}(G)$ *and upper intuitionistic fractional domination number* $\Gamma_{if}(G)$ *of an IFG which is equivalent to finding the optimal solution of the following linear programming problem.*
For $f = f_{\mu_1,\nu_1} : V(G) \to [0,1]$*, let* X_{if} *be the* μ*-dominating and* ν*-dominating function value of the column vector* $[f_{\mu_1}(v_1), f_{\mu_1}(v_2), \ldots, f_{\mu_1}(v_n)]^t$ *and* $[f_{\nu_1}(v_1), f_{\nu_1}(v_2), \ldots, f_{\nu_1}(v_n)]^t$ *and* $\overline{1}$ *which denotes the column vector of all 1's. Then f is a dominating function if and only if* $N.X_{if} \geq \vec{1}$ *and* $N.X_{if} \leq \vec{1}$ *for the closed neighborhood of* μ*-dominating and* ν*-dominating function value of the matrix N and all* $\vec{1} = [1, 1, \ldots, 1]^t$*. Hence*

$$\gamma_{if}(G) = Min \begin{cases} \sum_{v_i \in V(G)} f_{\mu_1}(v_i) = cX_{if} & if \\ N.X_{if} \geq \vec{1} & if \quad 0 \leq f_{\mu_1}(v_i) \leq 1 \ \forall v_i \in V \\ \sum_{v_i \in (G)} f_{\nu_1}(v_i) = cX_{if} & if \\ N.X_{if} \geq \vec{1} & if \quad 0 \leq f_{\nu_1}(v_i) \leq 1 \ \forall v_i \in V \end{cases} \quad or$$

$\gamma_{if}(G) = Min\{|f| : f \text{ is an MDF of IFG }\}$ *where*
$|f| = \sum_{v_i \in V} f(v_i) = (\sum_{v_i \in V(G)} f_{\mu_1}(v_i), \sum_{v_i \in V(G)} f_{\nu_1}(v_i))$
and

$$\Gamma_{if}(G) = Max \begin{cases} \sum_{v_i \in V(G)} f_{\mu_1}(v_i) = cX_{if} & if \\ N.X_{if} \leq \vec{1} & if \quad 0 \leq f_{\mu_1}(v_i) \leq 1 \ \forall v_i \in V \\ \sum_{v_i \in V(G)} f_{\nu_1}(v_i) = cX_{if} & if \\ N.X_{if} \leq \vec{1} & if \quad 0 \leq f_{\nu_1}(v_i) \leq 1 \ \forall v_i \in V \end{cases} \quad or$$

$\Gamma_{if}(G) = Max\{|f| : f \text{ is an MDF of } IFG\}$ *where*
$|f| = \sum_{v_i \in V} f(v) = (\sum_{v_i \in V(G)} f_{\mu_1}(v_i), \sum_{v \in V(G)} f_{\mu_1}(v_i))$*, and*
$c = \vec{1}_n = [1, 1, ..., 1]$.

The problem of finding the μ_1*-dominating intuitionistic fractional domination number* $\gamma_{if_{\mu_1}}$ *is equivalent to finding the optimal solution of the following linear programming problem.*
Minimize $Z = \sum_{v_i \in V(G)} f_{\mu_1}(v_i)$ *Subject to* $\sum_{x \in N[v_i]} f_{\mu_1}(x) \geq 1$ *and* $0 \leq f_{\mu_1}(v_i) \leq 1 \ \forall v_i \in V(G)$
The problem of finding the μ_1*-dominating upper intuitionistic fractional domination number* $\Gamma_{if_{\mu_1}}$ *is equivalent to finding the optimal solution of the following linear programming problem.*
Maximize $Z = \sum_{v_i \in V(G)} f_{\mu_1}(v_i)$ *Subject to* $\sum_{x \in N[v_i]} f_{\mu_1}(x) \leq 1$ *and* $0 \leq f_{\mu_1}(v_i) \leq 1 \ \forall v_i \in V(G)$
The problem of finding the ν_1*-dominating intuitionistic fractional domination number* $\gamma_{if_{\nu_1}}$ *is equivalent to finding the optimal solution of the following linear programming problem.*
Minimize $Z = \sum_{v_i \in V(G)} f_{\nu_1}(v_i)$ *Subject to* $\sum_{x \in N[v_i]} f_{\nu_1}(x) \geq 1$ *and* $0 \leq f_{\nu_1}(v_i) \leq 1 \ \forall v_i \in V(G)$
The problem of finding the ν_1*-dominating upper intuitionistic fractional domination number* $\Gamma_{if_{\nu_1}}$ *is equivalent to finding the optimal solution of the following linear programming problem.*
Maximize $Z = \sum_{v_i \in V(G)} f_{\nu_1}(v_i)$ *Subject to* $\sum_{x \in N[v_i]} f_{\nu_1}(x) \leq 1$ *and* $0 \leq f_{\nu_1}(v_i) \leq 1 \ \forall v_i \in V(G)$

Theorem 3.11. In any IFG G, the non membership value of intuitionistic fractional domination number $\gamma_{if_{\nu_1}}(G)$ is always zero.

Proof. Follows from the definition 3.8 and observation 3.1. □

Theorem 3.12. In any IFG G, the membership value of intuitionistic fractional domination number $\gamma_{if_{\mu_1}}(G)$ and the non membership value of upper intuitionistic fractional domination number $\Gamma_{if_{\nu_1}}(G)$ is always equal. That is
$\gamma_{if_{\mu_1}}(G) = \Gamma_{if_{\nu_1}}(G)$.

Proof. Follows from the observation 3.1. and use the following linear programming algorithm. □

4. A Linear Programming Algorithm for finding out $\gamma_{if}(G)$ and $\Gamma_{if}(G)$ of an IFG using LiPS(Linear Program Solver software):

Step 1: By Observation 3.1, first we formulate the L.P.P. for an intuitionistic fractional graph G.
Step 2: Next we define a number of variables(vertex), number of constraints and number of objective function for an intuitionistic fractional graph G.
Step 3: In Lips, go to $File \rightarrow New \rightarrow TableModel$. Now the Model parameters window is open and here we input the Step 2 values and also select Minimization or Maximization in Optimization direction box and click ok.
Step 4: Now the LiPS Model1 window is open. Enter the values of c, N and 1, if the objective function is minimize click >= symbol or maximize click <= for all the constraints.
Step 5: Finally to press the solve active model icon, we get optimum solution report window for intuitionistic fractional domination number $\gamma_{if}(G)$ or upper intuitionistic fractional domination number $\Gamma_{if}(G)$ of an intuitionistic fractional graph.

Theorem 4.1. *The dominating function $f_{\mu_1,\nu_1} : V \rightarrow [0,1]$ of $C_n(n \geq 3)$ is $(\frac{1}{q}, \frac{1}{s})$ where q and s are integers $(q \neq 0, s \neq 0)$ such that $0 < \frac{1}{q} + \frac{1}{s} \leq 1$ and $1 < q \leq 3, s \geq 4$.*

Proof. Case 1: Let us take q=2 and s=4. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{1}{2}, \frac{1}{4})$ for all $v_i \in V$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \geq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1$ for every $v_i \in V$ and $0 < \frac{1}{2} + \frac{1}{4} \leq 1$. Hence f_{μ_1,ν_1} is an DF and not MDF.
Case 2: Let us take q=2 and $s > 4$. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{1}{2}, \frac{1}{s})$ for all $v_i \in V$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \geq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1$ for every $v_i \in V$ and $0 < \frac{1}{2} + \frac{1}{s} \leq 1$. Hence f_{μ_1,ν_1} is an DF and not MDF.
Case 3: Let us take q=3 and s=4. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{1}{3}, \frac{1}{4})$ for all $v_i \in V$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) = 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1$ for every $v_i \in V$ and $0 < \frac{1}{3} + \frac{1}{4} \leq 1$. Hence f_{μ_1,ν_1} is an DF and also MDF.
Case 4: Let us take q=3 and $s > 4$. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{1}{3}, \frac{1}{s})$ for all $v_i \in V$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) = 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1$ for every $v_i \in V$ and $0 < \frac{1}{3} + \frac{1}{s} \leq 1$. Hence f_{μ_1,ν_1} is an DF and also MDF. □

Theorem 4.2. *If C_n has n vertices where $n \geq 3$, then $\gamma_{if}(C_n) = (\gamma_{if_{\mu_1}}(C_n), \gamma_{if_{\nu_1}}(C_n)) = (\frac{n}{3}, 0)$ and $\Gamma_{if}(C_n) = (\Gamma_{if_{\mu_1}}(C_n), \Gamma_{if_{\nu_1}}(C_n)) = (\frac{n}{3}, \frac{n}{3})$.*

Proof. Follows from Theorem 4.1, by using the observation 3.1 and a linear programming algorithm, to finding out intuitionistic fractional domination number Γ_{if} and upper intuitionistic fractional domination number Γ_{if} of C_n. □

Theorem 4.3. The dominating function $f_{\mu_1,\nu_1} : V \rightarrow [0,1]$ of $P_n(n \geq 4)$ is $(\frac{p}{q}, \frac{1}{s})$ where p,q and s are integers $(q \neq 0, s \neq 0)$ such that $0 < \frac{p}{q} + \frac{1}{s} \leq 1$ and $p \leq 2, 1 < q \leq 3, s \geq 4$.

Proof. A path P_n is an alternating sequence of vertices and edges $v_1, e_1, v_2, e_2, \ldots, e_{n-2}, v_{n-1}, e_{n-1}$ and v_n.
Case 1: Let us take p=1, q=2 and s=4. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{1}{2}, \frac{1}{4})$ for all $v_i \in V$ where $i = 1, 2, \ldots, n$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \geq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1$ and $0 < \frac{1}{2} + \frac{1}{4} \leq 1$. Hence f_{μ_1,ν_1} is an DF and not MDF.
Case 2: Let us take p=1, q=2 and $s > 4$. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{1}{2}, \frac{1}{s})$ for all $v_i \in V$ where $i = 1, 2, \ldots, n$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \geq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1$ and $0 < \frac{1}{2} + \frac{1}{s} \leq 1$. Hence f_{μ_1,ν_1} is an DF and not MDF.
Case 3: Let us take p=1, q=3 and s=4. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{1}{3}, \frac{1}{4})$ for all $v_i \in V$ where $i = 1, 2, \ldots, n$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \ngeq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1$ and $0 < \frac{1}{3} + \frac{1}{4} \leq 1$. Hence f_{μ_1,ν_1} is not a DF and MDF.
In particular p=1 $\forall v_i \in V$ where $i = 2, 3, \ldots, n-2$ and p=2 for $v_i \in V$ where $i = 1, n-1, q = 3$ and s=4. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{1}{3}, \frac{1}{4})$ for all $v_i \in V$ where $i = 2, 3, \ldots, n-1$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \geq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1 \forall v_i \in V$ and $0 < \frac{p}{3} + \frac{1}{4} \leq 1$. Hence f_{μ_1,ν_1} is a DF and MDF.
Let us take p=1, q=3 and $s > 4$. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{1}{3}, \frac{1}{s})$ for all $v_i \in V$ where $i = 1, 2, \ldots, n$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \ngeq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1$ and $0 < \frac{1}{3} + \frac{1}{s} \leq 1$. Hence f_{μ_1,ν_1} is not a DF and MDF.
In particular p=1 $\forall v_i \in V$ where $i = 2, 3, \ldots, n-1$ and p=2 for $v_i \in V$ where $i = 1, n, q = 3$ and $s > 4$. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{1}{3}, \frac{1}{s}) \forall v_i \in V$ where $i = 2, 3, \ldots, n-1$ and $f_{\mu_1,\nu_1}(v_i) = (\frac{2}{3}, \frac{1}{s})$ for some $v_i \in V$ where i=1,n. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \geq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1 \forall v_i \in V$ and $0 < \frac{p}{q} + \frac{1}{s} \leq 1$. Hence f_{μ_1,ν_1} is an DF and MDF.
Case 4: Let us take p=2, q=2 and s=4. Then $f_{\mu_1,\nu_1}(v_i) = (1, \frac{1}{4})$ for all $v_i \in V$ where $i = 1, 2, \ldots, n$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \geq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1$ but it does not satisfies the inequality and $0 < \frac{p}{q} + \frac{1}{s} \leq 1$. Hence f_{μ_1,ν_1} is not a DF and MDF.
Let us take p=2, q=2 and $s > 4$. Then $f_{\mu_1,\nu_1}(v_i) = (1, \frac{1}{s})$ for all $v_i \in V$ where $i = 1, 2, \ldots, n$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \geq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1$ but it does not satisfies the inequality and $0 < \frac{p}{q} + \frac{1}{s} \leq 1$. Hence f_{μ_1,ν_1} is not a DF and MDF.
Case 5: Let us take p=2, q=3 and s=4. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{2}{3}, \frac{1}{4})$ for all $v_i \in V$ where $i = 1, 2, \ldots, n$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \geq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1$ and $0 < \frac{2}{3} + \frac{1}{4} \leq 1$.

Hence f_{μ_1,ν_1} is a DF and not MDF.
Let us take p=2, q=3 and $s>4$. Then $f_{\mu_1,\nu_1}(v_i)=(\frac{2}{3},\frac{1}{s})$ for all $v_i\in V$ where $i=1,2,\dots,n$. It can be easily verified that $\sum_{u\in N[v_i]} f_{\mu_1}(u)\geq 1$ and $\sum_{u\in N[v_i]} f_{\nu_1}(u)<1$ and $0<\frac{2}{3}+\frac{1}{4}\leq 1$. Hence f_{μ_1,ν_1} is a DF and not MDF. □

Theorem 4.4. *Let P_n be the intuitionistic fuzzy path on n vertices for $n\geq 3$. Then*
$\gamma_{if}(P_n)=(\gamma_{if_{\mu_1}}(P_n),\gamma_{if_{\nu_1}}(P_n))=(\lceil\frac{n}{3}\rceil,0)$ *and*
$\Gamma_{if}(P_n)=(\Gamma_{if_{\mu_1}}(P_n),\Gamma_{if_{\nu_1}}(P_n))=(\lceil\frac{n}{3}\rceil+1$ *or* $\lfloor\frac{n}{3}\rfloor+1,\lceil\frac{n}{3}\rceil)$.
In particular,

$$\Gamma_{if_{\mu_1}}(P_n)=\begin{cases}\lceil\frac{n}{3}\rceil+1 & \text{for } n=3(m-2)+2 \text{ here } m\geq 3\\ \lfloor\frac{n}{3}\rfloor+1 & \text{otherwise}\end{cases}$$

Proof. Follows from Theorem 4.3, by using the observation 3.1 and using a linear programming algorithm, to finding out intuitionistic fractional domination number (γ_{if}) and upper intuitionistic fractional domination number (Γ_{if}) of P_n. □

Theorem 4.5. *The dominating function $f_{\mu_1,\nu_1}:V\to[0,1]$ of $S_{1,n}(n\geq 2)$ is $(\frac{p}{q},\frac{1}{s})$ where p,q and s are integers $(q\neq 0,s\neq 0)$ such that $0<\frac{p}{q}+\frac{1}{s}\leq 1$ and $p\leq 2, 1<q\leq 3, s\geq n+2$.*

Proof. Let $G=S_{1,n}$ be the intuitionistic fractional star graph. It has $n+1$ vertices and n edges. Denote the vertices as, $V=\{u,v_1,v_2,\dots,v_n\}$ and edges $E=\{uv_1,uv_2,\dots,uv_n\}$.
Case 1: Let us take $p=1,q=2$ and $s\geq n+2$. Then $f_{\mu_1,\nu_1}(v_i)=(\frac{1}{2},\frac{1}{n+2})$ for all $v_i\in V$ where $i=1,2,\dots,n$. It can be easily verified that $\sum_{u\in N[v_i]} f_{\mu_1}(u)\geq 1$ and $\sum_{u\in N[v_i]} f_{\nu_1}(u)<1\forall v_i\in V$ and $0<\frac{1}{2}+\frac{1}{n+2}\leq 1$. Hence f_{μ_1,ν_1} is an DF and not MDF.
Case 2: Let us take $p=1,q=3$ and $s\geq n+2$. Then $f_{\mu_1,\nu_1}(v_i)=(\frac{1}{3},\frac{1}{n+2})$ for all $v_i\in V$ where $i=1,2,\dots,n$. It can be easily verified that $\sum_{u\in N[v_i]} f_{\mu_1}(u)\ngeq 1$ and $\sum_{u\in N[v_i]} f_{\nu_1}(u)<1\forall v_i\in V$ and $0<\frac{1}{3}+\frac{1}{n+2}\leq 1$. Hence f_{μ_1,ν_1} is not a DF and MDF.
Case 3: Let us take p=2,q=3 and $s\geq n+2$. Then $f_{\mu_1,\nu_1}(v_i)=(1,\frac{1}{n+2})$ for all $v_i\in V$ where $i=1,2,\dots,n$. It can be easily verified that $\sum_{u\in N[v_i]} f_{\mu_1}(u)\geq 1$ and $\sum_{u\in N[v_i]} f_{\nu_1}(u)<1\forall v_i\in V$ but it does not satisfies the inequality $0<\frac{p}{q}+\frac{1}{s}\leq 1$. Hence f_{μ_1,ν_1} is not a DF and MDF.
Case 4: Let us take p=2,q=3 and $s\geq n+2$. Then $f_{\mu_1,\nu_1}(v_i)=(\frac{2}{3},\frac{1}{n+2})$ for all $v_i\in V$ where $i=1,2,\dots,n$. It can be easily verified that $\sum_{u\in N[v_i]} f_{\mu_1}(u)\geq 1$ and $\sum_{u\in N[v_i]} f_{\nu_1}(u)<1\forall v_i\in V$ and $0<\frac{2}{3}+\frac{1}{n+2}\leq 1$. Hence f_{μ_1,ν_1} is a DF and not MDF.
Case 5: Let us take p=2,q=3 for $u\in V$ and p=1,q=3 where $v_i\in V, i=1,2,\dots,n$ and $s\geq n+2$. Then $f_{\mu_1,\nu_1}(u)=(\frac{2}{3},\frac{1}{n+2})$ for $u\in V$ and $f_{\mu_1,\nu_1}(v_i)=(\frac{1}{3},\frac{1}{n+2})$ for $v_i\in V$ where $i=1,2,\dots,n$. It can be easily verified that $\sum_{u\in N[v_i]} f_{\mu_1}(u)\geq 1$ and $\sum_{u\in N[v_i]} f_{\nu_1}(u)<1\forall u,v_i\in V$ and $0<\frac{p}{q}+\frac{1}{s}\leq 1$. Hence f_{μ_1,ν_1} is a DF and MDF.
In this same manner let us take p=1,q=3 for $u\in V$ and p=2,q=3 where $v_i\in V$ where i=1,2,...,n and $s\geq n+2$. Then $f_{\mu_1,\nu_1}(u)=(\frac{1}{3},\frac{1}{n+2})$ for $u\in V$ and $f_{\mu_1,\nu_1}(v_i)=(\frac{2}{3},\frac{1}{n+2})$ for $v_i\in V$ where $i=1,2,\dots,n$. It can be easily verified that $\sum_{u\in N[v_i]} f_{\mu_1}(u)\geq 1$ and $\sum_{u\in N[v_i]} f_{\nu_1}(u)<1\forall u,v_i\in V$ and $0<\frac{p}{q}+\frac{1}{s}\leq 1$. Hence f_{μ_1,ν_1} is a DF and MDF. □

Theorem 4.6. *If a intuitionistic fractional star $S_{1,n}$ has n vertices then*
$\gamma_{if}(S_{1,n})=(\gamma_{if_{\mu_1}}(S_{1,n}),\gamma_{if_{\nu_1}}(S_{1,n}))=(1,0)$ *and*
$\Gamma_{if}(S_{1,n})=(\Gamma_{if_{\mu_1}}(S_{1,n}),\Gamma_{if_{\nu_1}}(S_{1,n}))=(n,1)$, *where* $n\geq 2$.

Proof. Follows from Theorem 4.5, by using the observation 3.1 and using a linear programming algorithm, to finding out intuitionistic fractional domination number (γ_{if}) and upper intuitionistic fractional domination number (Γ_{if}) of $S_{1,n}$ □

Corollary 4.7. *If an intuitionistic fractional bistar $B_{1,n,n}$ has $2n$ vertices then*
$\gamma_{if}(B_{1,n,n})=(\gamma_{if_{\mu_1}}(B_{1,n,n}),\gamma_{if_{\nu_1}}(B_{1,n,n}))=(2,0)$ *and*
$\Gamma_{if}(B_{1,n,n})=(\Gamma_{if_{\mu_1}}(B_{1,n,n}),\Gamma_{if_{\nu_1}}(B_{1,n,n}))=(2n,2)$
where $n\geq 2$

Corollary 4.8.
Let the intuitionistic fractional bistar $B_{1,n,m}$ graph obtained by joining the center(apex) vertices of two intuitionistic fractional star graphs $S_{1,n}$ and $S_{1,m}$ by an edge where $n\neq m$. Then
$\gamma_{if}(B_{1,n,m})=(\gamma_{if_{\mu_1}}(B_{1,n,m}),\gamma_{if_{\nu_1}}(B_{1,n,m}))=(2,0)$ *and*
$\Gamma_{if}(B_{1,n,m})=(\Gamma_{if_{\mu_1}}(B_{1,n,m}),\Gamma_{if_{\nu_1}}(B_{1,n,m}))=(n+m,2)$ *where* $n,m\geq 2$

Theorem 4.9.
The dominating function $f_{\mu_1,\nu_1}:V\to[0,1]$ of $W_n(n\geq 4)$ is $(\frac{1}{p},\frac{1}{q})$ where p and q are integers $(p\neq 0,q\neq 0)$ such that $0<\frac{1}{p}+\frac{1}{q}\leq 1$ and $1<p\leq 4, q\geq n+2$.

Proof. Let $G=W_n(V,E)$ be the intuitionistic fractional wheel graph and the vertex set $V(W_n)=\{v,v_1,v_2,\dots,v_n\}$ and the edges set $E(W_n)=\{x=vv_i:1\leq i\leq n-1\}\cup\{e_i=v_iv_{i+1}:1\leq i\leq n-2\}\cup\{e_{n-1}=v_{n-1}v_1\}$.
Case 1: Let us take p=2 and $q\geq n+2$. Then $f_{\mu_1,\nu_1}(v_i)=(\frac{1}{2},\frac{1}{n+2})$ for all $v,v_i\in V$ where $i=1,2,\dots,n$. It can be easily verified that $\sum_{u\in N[v_i]} f_{\mu_1}(u)\geq 1$ and $\sum_{u\in N[v_i]} f_{\nu_1}(u)<1\forall v,v_i\in V$ and $0<\frac{1}{2}+\frac{1}{n+2}\leq 1$. Hence f_{μ_1,ν_1} is an DF and not MDF.
Case 2: Let us take p=3 and $q\geq n+2$. Then $f_{\mu_1,\nu_1}(v_i)=(\frac{1}{3},\frac{1}{q})$ for all $v,v_i\in V$ where $i=1,2,\dots,n$. It can be easily verified that $\sum_{u\in N[v_i]} f_{\mu_1}(u)\geq 1$ and $\sum_{u\in N[v_i]} f_{\nu_1}(u)<1\forall v,v_i\in V$ and $0<\frac{1}{3}+\frac{1}{q}\leq 1$. Hence f_{μ_1,ν_1} is an DF and not MDF.
Case 3: Let us take p=4 and $q\geq n+2$. Then $f_{\mu_1,\nu_1}(v_i)=(\frac{1}{4},\frac{1}{q})$ for all $v,v_i\in V$ where $i=1,2,\dots,n$. It can be easily verified that $\sum_{u\in N[v_i]} f_{\mu_1}(u)\geq 1$ and $\sum_{u\in N[v_i]} f_{\nu_1}(u)<1\forall v,v_i\in V$

and $0 < \frac{1}{4} + \frac{1}{q} \leq 1$. Hence f_{μ_1,ν_1} is an DF and MDF.
In particular $p \geq 5$ and $q \geq n+2$. Then $f_{\mu_1,\nu_1}(v_i) = (\frac{1}{p}, \frac{1}{q})$ for all $v, v_i \in V$ where $i = 1, 2, \ldots, n$. It can be easily verified that $\sum_{u \in N[v_i]} f_{\mu_1}(u) \ngeq 1$ and $\sum_{u \in N[v_i]} f_{\nu_1}(u) < 1 \forall v, v_i \in V$ and $0 < \frac{1}{3} + \frac{1}{q} \leq 1$. Hence f_{μ_1,ν_1} is a not DF and MDF.

Theorem 4.10. *If a intuitionistic fuzzy wheel W_n has n vertices then $\gamma_{if}(W_n) = (\gamma_{if_{\mu_1}}(W_n), \gamma_{if_{\nu_1}}(W_n)) = (1,0)$ and $\Gamma_{if}(W_n) = (\Gamma_{if_{\mu_1}}(W_n), \Gamma_{if_{\nu_1}}(W_n)) = (\frac{n}{3}, 1)$ where $n \geq 4$*

Proof.
Follows from Theorem 4.9, by using the observation 3.1 and using a linear programming algorithm, to finding out intuitionistic fractional domination number γ_{if} and upper intuitionistic fractional domination number Γ_{if} of W_n.

Example 4.11. *Consider a cycle C_4 has 4 vertices, it also satisfies the definition 3.6 and definition 3.8. Now we formulate L.P.P. for the membership and non membership value of the above graph,to finding the intuitionistic fractional domination number $\gamma_{if}(C_4)$ and upper intuitionistic fractional domination number $\Gamma_{if}(C_4)$ of an intuitionistic fractional graph G, which is equivalent to finding the optimal solution of the L.P.P. which has been formulate for the above graph using Figure 4.1*

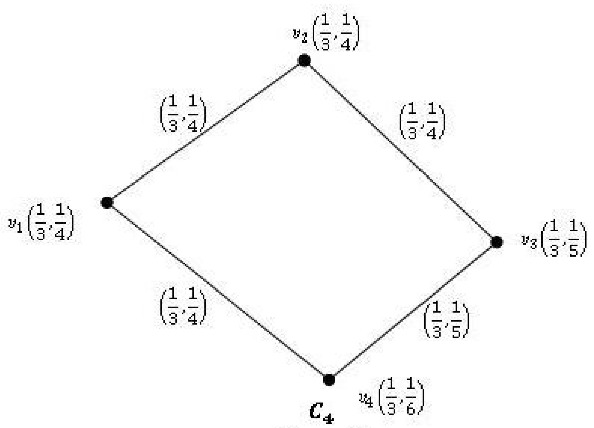


Figure 4.1

Step 1:Consider the graph in figure 4.1, we formulate the following L.P.P.
Minimize $Z = \sum_{v \in V(G)} f_{\mu_1}(v)$
Subject to
$f_{\mu_1}(v_1) + f_{\mu_1}(v_2) + f_{\mu_1}(v_4) \geq 1$
$f_{\mu_1}(v_1) + f_{\mu_1}(v_2) + f_{\mu_1}(v_3) \geq 1$
$f_{\mu_1}(v_2) + f_{\mu_1}(v_3) + f_{\mu_1}(v_4) \geq 1$
$f_{\mu_1}(v_1) + f_{\mu_1}(v_3) + f_{\mu_1}(v_4) \geq 1$
and $0 \leq f_{\mu_1}(v) \leq 1$ *for all* $v \in V(G)$. *It can be written as*
Minimize $Z = f_{\mu_1}(v_1) + f_{\mu_1}(v_2) + f_{\mu_1}(v_3) + f_{\mu_1}(v_4)$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} f_{\mu_1}(v_1) \\ f_{\mu_1}(v_2) \\ f_{\mu_1}(v_3) \\ f_{\mu_1}(v_4) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$0 \leq f_{\mu_1}(v) \leq 1$ *for all* $v \in V(G)$ *where* $c = \vec{1_4} = [1,1,1,1]$,

$$N = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix},$$

$X_{if} = \begin{bmatrix} f_{\mu_1}(v_1) \\ f_{\mu_1}(v_2) \\ f_{\mu_1}(v_3) \\ f_{\mu_1}(v_4) \end{bmatrix}, \vec{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ *be the column vector with respect to the constraint part of the L.P.P. The above L.P.P. is to be solved by Linear Program Solver software(LiPS).*

Step 2: In Figure 4.1, number of variables is 4, number of constraints is 4 and number of objective function is 1.
Step 3: In Lips, we input the Step 2 values in Step 3 and select Minimization in Optimization direction box and click ok.
Step 4: Enter the values of $c, N and \vec{1}$ *in LiPS Model1 window and also click >= symbol for all the constraints.*
Step 5: Press the solve active model icon, we get optimum solution report window for intuitionistic fractional domination number $\gamma_{if_{\mu_1}}(C_4) = \frac{4}{3}$ *where* $f_{\mu_1}(v_1) = \frac{1}{3}, f_{\mu_1}(v_2) = \frac{1}{3}, f_{\mu_1}(v_3) = \frac{1}{3}, f_{\mu_1}(v_4) = \frac{1}{3}$
Similarly we have to find the upper intuitionistic fractional domination function number $\Gamma_{if_{\mu_1}}(C_4) = \frac{4}{3}$ *and* $\Gamma_{if_{\nu_1}}(C_4) = \frac{4}{3}$ *which implies* $\Gamma_{if}(C_4) = (\Gamma_{if_{\mu_1}}(C_4), \Gamma_{if_{\nu_1}}(C_4)) = (\frac{4}{3}, \frac{4}{3})$. *That is*
Maximize $Z = \sum_{v \in V(G)} f_{\mu_1}(v)$
Subject to
$f_{\mu_1}(v_1) + f_{\mu_1}(v_2) + f_{\mu_1}(v_4) \leq 1$
$f_{\mu_1}(v_1) + f_{\mu_1}(v_2) + f_{\mu_1}(v_3) \leq 1$
$f_{\mu_1}(v_2) + f_{\mu_1}(v_3) + f_{\mu_1}(v_4) \leq 1$
$f_{\mu_1}(v_1) + f_{\mu_1}(v_3) + f_{\mu_1}(v_4) \leq 1$
and $0 \leq f_{\mu_1}(v) \leq 1$ *for all* $v \in V(G)$. *and*
Maximize $Z = \sum_{v \in V(G)} f_{\nu_1}(v)$
Subject to
$f_{\nu_1}(v_1) + f_{\nu_1}(v_2) + f_{\nu_1}(v_4) \leq 1$
$f_{\nu_1}(v_1) + f_{\nu_1}(v_2) + f_{\nu_1}(v_3) \leq 1$
$f_{\nu_1}(v_2) + f_{\nu_1}(v_3) + f_{\nu_1}(v_4) \leq 1$
$f_{\nu_1}(v_1) + f_{\nu_1}(v_3) + f_{\nu_1}(v_4) \leq 1$
and $0 \leq f_{\nu_1}(v) \leq 1$ *for all* $v \in V(G)$.
We get $\Gamma_{if_{\mu_1}}(C_4) = \frac{4}{3}$ *where* $f_{\mu_1}(v_1) = \frac{1}{3}, f_{\mu_1}(v_2) = \frac{1}{3}, f_{\mu_1}(v_3) = \frac{1}{3}, f_{\mu_1}(v_4) = \frac{1}{3}$ *and* $\Gamma_{if_{\nu_1}}(C_4) = \frac{4}{3}$ *where* $f_{\nu_1}(v_1) = \frac{1}{3}, f_{\nu_1}(v_2) = \frac{1}{3}, f_{\nu_1}(v_3) = \frac{1}{3}, f_{\nu_1}(v_4) = \frac{1}{3}$
In this similar manner we have to calculate the non membership value of an intuitionistic fractional domination number of an IFG and it is found to be $\gamma_{if_{\nu_1}}(C_4) = 0$. *Hence we get* $\gamma_{if}(C_4) = (\gamma_{if_{\mu_1}}(C_4), \gamma_{if_{\nu_1}}(C_4)) = (\frac{4}{3}, 0)$ *and* $\Gamma_{if}(C_4) = (\Gamma_{if_{\mu_1}}(C_4), \Gamma_{if_{\nu_1}}(C_4)) = (\frac{4}{3}, \frac{4}{3})$.

- $\gamma_{if_{\mu_1}}(C_4) = \frac{4}{3}$ *where* $f_{\mu_1}(v_1) = \frac{1}{3}, f_{\mu_1}(v_2) = \frac{1}{3}, f_{\mu_1}(v_3) = \frac{1}{3}, f_{\mu_1}(v_4) = \frac{1}{3}$ *and* $|f| = \sum_{v \in V} f(v) = \sum_{v \in V(G)} f_{\mu_1}(v) = \frac{4}{3}$.

- $\gamma_{if_{\nu_1}}(C_4) = 0$ *where* $f_{\nu_1}(v_1) = 0, f_{\nu_1}(v_2) = 0,$

$f_{\nu_1}(v_3) = 0, f_{\nu_1}(v_4) = 0$ *and*
$|f| = \sum_{v \in V} f(v) = \sum_{v \in V(G)} f_{\nu_1}(v) = 0.$

- $\Gamma_{if_{\mu_1}}(C_4) = \frac{4}{3}$ *where* $f_{\mu_1}(v_1) = \frac{1}{3}, f_{\mu_1}(v_2) = \frac{1}{3},$
  $f_{\mu_1}(v_3) = \frac{1}{3}, f_{\mu_1}(v_4) = \frac{1}{3}$ *and*
  $|f| = \sum_{v \in V} f(v) = \sum_{v \in V(G)} f_{\mu_1}(v) = \frac{4}{3}.$

- $\Gamma_{if_{\nu_1}}(C_4) = \frac{4}{3}$ *where* $f_{\nu_1}(v_1) = \frac{1}{3}, f_{\nu_1}(v_2) = \frac{1}{3},$
  $f_{\nu_1}(v_3) = \frac{1}{3}, f_{\nu_1}(v_4) = \frac{1}{3}$ *and*
  $|f| = \sum_{v \in V} f(v) = \sum_{v \in V(G)} f_{\nu_1}(v) = \frac{4}{3}.$

5. Conclusion

In this paper, we introduce the concept of intuitionistic fractional domination number, upper intuitionistic fractional domination number for a path, cycle, star, bistar and wheel intuitionistic fractional graphs have been discussed and we found these two parameters by using LiPS(Linear Program Solver software).We further extended this study on some special classes of graphs in future.

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