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Ideals and IWI-ideals of residuated lattice Wajsberg algebras

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Abstract
In this paper, we study WI-ideal of residuated lattice Wajsberg algebraand investigate some of their properties. Also, we announce the concept of implicative WI-ideal (IWI-ideal)of residuated lattice Wajsberg algebra. Further, we inspect some of its characterizations and attain some properties of residuated lattice H-Wajsberg algebra.

Keywords
Wajsberg algebra; Lattice Wajsberg algebra; Residuated lattice Wajsberg algebra; Residuated lattice H-Wajsberg algebra; WI-ideal; Lattice ideal; Ideal; Implicative WI-ideal.

AMS Subject Classification
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Contents

1. Introduction

Mordchaj Wajsberg [1] introduced the concept of Wajsberg algebras in 1935 and studied by Font, Rodriguez and Torrens [2]. Residuated lattices were announced by Ward and Dilworth [3]. Ibrahim and Shajitha Begum [4] and [5] introduced the notions of Wajsberg implicative ideal (WI-ideal), ideals and implicative WI-ideals of lattice Wajsberg algebras and also investigated their properties with suitable llustrations. The authors [6],[7] and [8] introduced the notion of Wajsberg implicative ideal (WI-ideal) and Fuzzy Wajsberg Implicative ideal (FWI-ideal) of residuated lattice Wajsberg algebras.

In this paper, we consider ideal of residuated lattice Wajsberg algebra and investigate some related properties. Also, we introduce the notion of IWI-ideal of residuated lattice Wajsberg algebra. Further, we investigate some of its characterizations and obtain some properties of residuated lattice H-Wajsberg algebra.

2. Preliminaries

In this section, we recall some basic definitions and properties which are helpful to develop our main results.

Definition 2.1 ([2]). *Let $(\mathscr{R},\to,*,1)$ bean algebra with a binary operation $''\to''$ and a quasi complement $''*''$ is called a Wajsberg algebra. Then if it satisfied the following axioms for all $x,y,z\in\mathscr{R}$,*

(i) $1\to x=x$

(ii) $(x\to y)\to y=((y\to z)\to(x\to z))=1$

(iii) $(x\to y)\to y=(y\to x)\to x$

(iv) $(x^*\to y^*)\to(y\to x)=1.$

Definition 2.2 ([2]). *A Wajsberg algebra $(\mathscr{R},\to,*,1)$ satisfied the following axioms for all $x,y,z\in\mathscr{R}$,*

(i) $x\to x=1$

(ii) *If* $(x \to y) = (y \to x) = 1$ *then* $x = y$

(iii) $x \to 1 = 1$

(iv) $(x \to (y \to x)) = 1$

(v) *If* $(x \to y) = (y \to z) = 1$ *then* $x \to z = 1$

(vi) $(x \to y) \to ((z \to x) \to (z \to y)) = 1$

(vii) $x \to (y \to z) = y \to (x \to z)$

(viii) $x \to 0 = x \to 1^* = x^*$

(ix) $(x^*)^* = x$

(x) $(x^* \to y^*) = y \to x$.

Definition 2.3 ([2]). *A Wajsberg algebra* $\mathscr{R}$ *is called a lattice Wajsberg algebra, if it satisfied the following conditions for all* $x, y \in \mathscr{R}$,

(i) *The partial ordering "* $\leq$ *" on a lattice Wajsberg algebra, such that* $x \leq y$ *if and only if* $x \to y = 1$

(ii) $x \vee y = (x \to y) \to y$

(iii) $x \wedge y = ((x^* \to y^*) \to y^*)^*$.

Thus, $(\mathscr{R}, \vee, \wedge, *, 0, 1)$ *is a lattice Wajsberg algebra with lower bound 0 and upper bound* 1.

Proposition 2.4 ([2]). *Alattice Wajsberg algebra* $(\mathscr{R}, \to, *, 1)$ *satisfied the following axioms for all* $x, y, z \in \mathscr{R}$,

(i) *If* $x \leq y$ *then* $x \to z \geq y \to z$ *and* $z \to x \leq z \to y$

(ii) $x \leq y \to z$ *if and only if* $y \leq x \to z$

(iii) $(x \vee y)^* = (x^* \wedge y^*)$

(iv) $(x \wedge y)^* = (x^* \vee y^*)$

(v) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$

(vi) $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$

(vii) $(x \to y) \vee (y \to x) = 1$

(viii) $x \to (y \vee z) = (x \to y) \vee (x \to z)$

(ix) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$

(x) $(x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$

(xi) $(x \wedge y) \to z = (x \to y) \to (x \to z)$.

Definition 2.5 ([3]). *A residuated lattice* $(\mathscr{R}, \vee, \wedge, \otimes, \to, 0, 1)$ *satisfied the following conditions for all* $x, y, z \in \mathscr{R}$,

(i) $(\mathscr{R}, \vee, \wedge, 0, 1)$ *is a bounded lattice*

(ii) $(\mathscr{R}, \otimes, 1)$ *is commutative monoid*

(iii) $x \otimes \mathrm{y} \leq z$ *if and only if* $x \leq y \to z$.

Proposition 2.6 ([3]). Let$(\mathscr{R}, \vee, \wedge, \otimes, \to, 0, 1)$ *be a residuated lattice. Then the following are satisfied for all* $x, y, z \in \mathscr{R}$,

(i) $(x \otimes y) \to z = x \to (y \to z)$

(ii) $(x \otimes y) \otimes z = x \otimes (y \otimes z)$

(iii) $x \otimes y = y \otimes x$.

Definition 2.7 ([2]). Let$(\mathscr{R}, \vee, \wedge, *, \to, 1)$ *be a lattice Wajsberg algebra. If a binary operation "* $\otimes$*" on* $\mathscr{R}$ *satisfied* $x \otimes y = (x \to y^*)^*$ *for all* $x, y \in \mathscr{R}$. *Then* $(\mathscr{R}, \vee, \wedge, \otimes, \to, *, 0, 1)$ *is called a residuated lattice Wajsberg algebra.*

Definition 2.8 ([5]). *The lattice Wajsberg algebra* $\mathscr{R}$ *is called a lattice H - Wajsberg algebra, if it satisfied* $x \vee y \vee ((x \wedge y) \to z) = 1$ *for all* $x, y, z \in \mathscr{R}$.
In a lattice H-Wajsberg algebra $\mathscr{R}$, *the following are hold:*

(i) $x \to (x \to y) = (x \to y)$

(ii) $x \to (y \to z) = (x \to y) \to (x \to z)$.

Definition 2.9 ([7]). *The residuated lattice Wajsberg algebra* $\mathscr{R}$ *is called a residuated lattice H-Wajsberg algebra if it satisfied* $x \vee y \vee ((x \wedge y) \to z) = 1$ *for all* $x, y, z \in \mathscr{R}$.
In a residuated lattice H -Wajsberg algebra $\mathscr{R}$, *the following are hold:*

(i) $x \otimes y \in \mathscr{R}$

(ii) $x \otimes (x \otimes y) = (x \otimes y); x \to (x \to y) = (x \to y)$

(iii) $x \otimes (y \otimes z) = (x \otimes y) \otimes (x \otimes z); x \to (y \to z) = (x \to y) \to (x \to z)$, *for all* $x, y, z \in \mathscr{R}$.

Proposition 2.10 ([5]). *Let* $\mathscr{R}$ *is a lattice H -Wajsberg algebra, then the following equality are hold*

$$(x \to y)^* \to z = (x \to z)^* \to (y \to z)^* \text{ for all } x, y, z \in \mathscr{R}.$$

Definition 2.11 ([2]). *Let I be a non-empty subset of a lattice Wajsberg algebra* $\mathscr{R}$. *Then I is called a WI -ideal* $\mathscr{R}$, *if satisfied for all* $x, y \in \mathscr{R}$,

(i) $0 \in I$

(ii) $(x \to y)^* \in I$ *and* $y \in I$ *imply* $x \in I$.

Definition 2.12 ([6]). *Let I be a non-empty subset of a residuated lattice Wajsberg algebras* $\mathscr{R}$. *Then I is called a WI -ideal* $\mathscr{R}$, *if it satisfied the following for all* $x, y \in \mathscr{R}$,

(i) $0 \in I$

(ii) $x \otimes y \in I$ *and* $y \in I$ *imply* $x \in I$

(iii) $(x \to y)^* \in I$ *and* $y \in I$ *imply* $x \in I$.

Definition 2.13 ([2]). *Let* $\mathscr{R}$ *be a lattice. An ideal I of* $\mathscr{R}$ *is a nonempty subset of* $\mathscr{R}$ *is called a lattice ideal, if it satisfied the following axioms for all* $x, y \in \mathscr{R}$,

(i) $x \in I, y \in L$ *and* $y \leq x$ *imply* $y \in I$

(ii) $x, y \in I$ *implies* $x \vee y \in I$.

Definition 2.14 ([4]). *A non-empty subset* I *of a Wajsberg algebra* $\mathscr{R}$ *is an ideal, if it satisfied the following axioms for all* $x, y \in \mathscr{R}$,

(i) $0 \in I$

(ii) $x \in I$ *and* $y \leq x$ *imply* $y \in I$.

Definition 2.15 ([2]). *A non-empty subset of* T *of a residuated lattic* $\mathscr{R}$ *is called an implicative filter if it satisfied the following axioms for all* $x, y \in \mathscr{R}$,

(i) $1 \in \mathrm{T}$

(ii) $x \in T, x \to y \in \mathrm{T}$ *implies* $y \in \mathrm{T}$.

3. Main Results

3.1 Properties of ideals of residuated lattice Wajsberg algebras

In this section, we consider ideal of residuated lattice Wajsberg algebra and explore some properties of ideal.

Proposition 3.1. *Intersection of any two ideals of residuated lattice Wajsberg algebra* $\mathscr{R}$ *is an ideal.*

Proof. Let K_1 and K_2 be two ideals of residuated lattice Wajsberg algebra $\mathscr{R}$.

Since, from (i) of Definition 2.14, $0 \in K_1$ and $0 \in K_2$ imply $0 \in K_1 \cap K_2$.

Therefore $K_1 \cap K_2$ is non-empty, if $x \in K_1 \cap K_2$ and $y \leq x$. Then, from (ii) of Definition 2.14 we have, $x \in K_1$ and $y \leq x$.

Also, from (ii) of Definition 2.14, $x \in K_2$ and $y \leq x$.

Since, from (ii) of Definition 2.14, K_1 and K_2 are ideals of $\mathscr{R}$ imply $y \in K_1$ and $y \in K_2$. Then, $y \in K_1 \cap K_2$ imply

$$x^* \otimes y \in K_1 \cap K_2, x^* \to y \in K_1 \cap K_2.$$

Hence, the intersection of two ideals of residuated lattice Wajsberg algebra $\mathscr{R}$ is an ideal. □

Remark 3.2. *Union of any two ideals of residuated lattice Wajsberg algebra* $\mathscr{R}$ *need not be an ideal of* $\mathscr{R}$.

Proposition 3.3. *Every ideal of residuated lattice Wajsberg algebra* $\mathscr{R}$ *is a lattice ideal.*

Proof. Let T be an ideal of $\mathscr{R}$. From (ii) of Definition 2.15 shows that T satisfies (i) of Definition 2.13. Now,

$$\begin{aligned}
(x \vee y)^* \otimes y &= [(((x \to y) \to y)^* \to y^*)^*] \\
&\quad [\text{ From (ii) of Definition 2.3}] \\
&= [y \to ((x \to y) \to y)]^* [\text{ From (ii) of Definition 2.2}] \\
&= [y \to ((y \to x) \to x)]^* [\text{ From (ii) of Definition 2.1}] \\
&= [y \to (y \vee x)]^* [\text{ From (ii) of Definition 2.3}] \\
&= [(y \to y) v (y \to x)]^* [\text{ From (viii) of Proposition 2.4}] \\
&= [((y \to y) \to (y \to x)) \to (y \to x)]^* \\
&\quad [\text{ From (ii) of Definition 2.3}] \\
&= ((y \to y) \to (y \to x))^* \to (y \to x)^* \\
&= (y \to x) \to [(y \to y) \to (y \to x)] \\
&\quad [\text{ From (x) of Definition 2.2}] \\
&= 1 \in \mathrm{T} [\text{ From (iv) of Definition 2.2}]
\end{aligned}$$

And

$$\begin{aligned}
(x \vee y)^* \to y &= (((x \to y) \to y)^* \to y) \\
&\quad [\text{From (i) of Definition 2.3}] \\
&= ((y^* \to (x \to y)^*) \to y) \\
&= (y^* \to (y^* \to x^*)) \to y \\
&= ((x^* \to (x^* \to y^*)) \to y) [\text{From (iii) of Definition 2.1}] \\
&= ((x \wedge y)^* \to y) [\text{From (iii) of Definition 2.3}] \\
&= ((x^* \vee y^*) \to y) [\text{From (iv) of Proposition 2.4}] \\
&= (x^* \to y) \wedge (y^* \to y) [\text{From (v) of Proposition 2.4}] \\
&= x^* \to y \in \mathrm{T}.
\end{aligned}$$

Thus, we get

$$(x \vee y)^* \otimes y = 1 \in \mathrm{T}, (x \vee y)^* \to y = x^* \to y \in \mathrm{T}.$$

Since T is an ideal,

$$(x \vee y)^* \otimes y \in \mathrm{T}, (x \vee y)^* \to y \in \mathrm{T}$$

imply $x \vee y \in \mathrm{T}$ and $y \in \mathrm{T}$. From Definition 2.13, we have T is a lattice ideal.

Example 3.4. *Consider a set* $\mathscr{R} = \{0, p, q, r, x, y, z, 1\}$. *Define*

Table 1. Complement

x	x^*
0	1
p	x
q	y
r	z
x	p
y	q
z	r
1	0

a partial ordering $'' \leq ''$ *on* $\mathscr{R}$, *such that* $0 \leq a \leq b \leq c \leq$

Table 2. Implication

$\to$	0	p	q	r	x	y	z	1
0	1	1	1	1	1	1	1	1
p	x	1	p	p	z	z	z	1
q	y	1	1	1	p	z	z	1
r	z	1	1	1	1	z	z	1
x	p	1	1	1	1	1	1	1
y	q	1	p	p	p	1	1	1
z	r	1	p	p	p	p	1	1
1	0	p	q	r	x	y	z	1

$d \leq 1$ with a binary operations "$\otimes$" and "$\to$" and a quasi complement "$*$" on $\mathscr{R}$ as in the Tables 1 and 2. Define $\vee$ and $\wedge$ operations on $\mathscr{R}$ as follows:

$$(x \vee y) = (x \to y) \to y$$
$$(x \wedge y) = ((x^* \to y^*) \to y^*)^*; x \otimes y = (x \to y^*)^* \text{ for all } x, y \in \mathscr{R}.$$

Then, $\mathscr{R}$ is a residutaed lattice Wajsberg algebra. It is easy to verify that, $I_1 = \{0, q\}$ is an ideal of $\mathscr{R}$ and also lattice ideal of $\mathscr{R}$.

Proposition 3.5. *Every lattice ideal of residuated lattice H-Wajsberg algebra $\mathscr{R}$ is an ideal.*

Proof. Let $\mathscr{R}$ be a residuated lattice H -Wajsberg algebra.

Let I be a lattice ideal of $\mathscr{R}$ for all $x, y, z \in \mathscr{R}$. Then $x \in I, y \in \mathscr{R}$ and $y \leq x$ imply $y \in I$. [From (i) of Definition 2.13].

For $x, y \in I$ imply $x \vee y \in I$ [From (ii) of Definition 2.13].

Since I is a lattice ideal, it satisfies (ii) of Definition 2.13.

And $x, y \in I$ which imply $x^* \otimes y \in I, x^* \to y \in I$. Hence, we get I is an ideal. □

Proposition 3.6. *Every ideal of residuated lattice H -Wajsberg algebra $\mathscr{R}$ is a WI -ideal.*

Proof. Let T be an ideal of residuated lattice Wajsberg algebra $\mathscr{R}$, then we have $0 \in \mathrm{T}, x \in \mathrm{T}$ and $y \leq x$ imply $y \in \mathrm{T}$ and $x, y \in \mathrm{T}$ imply $x^* \otimes y \in \mathrm{T}, x^* \to y \in \mathrm{T}$. Now,

$((x \otimes y) \to y) = x \to (y \to y)$ [From (i) of Proposition 2.6].
$= (x \to y) \to (x \to y)$ [From (ii) of Definition 2.11].
And
$((x \to y)^* \to y) = (y^* \to (x \to y))$,[From (x) of Definition 2.2]
$= (y^* \to x) \to (y^* \to y)$ [From (ii) of Definition 2.11]
$= (x^* \to y) \to (y^* \to y)$ [From (x) of Definition 2.2].

Since T is an ideal $x^* \otimes y \in \mathrm{T}, x^* \to y \in \mathrm{T}$. We have

$$(x \to y) \to (x \to y) \in \mathrm{T}, (x^* \to y) \to (y^* \to y) \in \mathrm{T}$$

and

$$(x \otimes y) \to y \in \mathrm{T}, (x \to y)^* \to y \in \mathrm{T}.$$

Therefore, $x \otimes y \in \mathrm{T}, (x \to y)^* \in \mathrm{T}$ and $y \in \mathrm{T}$ imply $x \in \mathrm{T}$. Hence, we get T is a WI-ideal. □

3.2 Properties of IWI -ideal of residuated lattice Wajsberg algebras

In this section, we introduce the concept of implicative WI -ideal (IWI -ideal) of residuated lattice Wajsberg algebra and we find some of its properties with illustrations.

Definition 3.7. *Let I be a non-empty subset of residuated lattice wajsberg algebra $\mathscr{R}$. Then, I is said to be a IWI -idealof $\mathscr{R}$, if it satisfies the following axioms for all $x, y \in \mathscr{R}$*

(i) $0 \in I$

(ii) $y \otimes z \in I$ *and* $((x \otimes y) \otimes z) \in I$ *imply* $x \otimes z \in I$

(iii) $(y \to z)^* \in I$ *and* $((x \to y)^* \to z^*)$ *imply* $(x \to z)^*$.

Proposition 3.8. *If I is a IWI -idealof residuated lattice Wajsberg algebra $\mathscr{R}$ then I is a WI -ideal of $\mathscr{R}$.*

Proof. Let I be a IWI-ideal of $\mathscr{R}$, then $0 \in I, y \otimes z \in I, (y \to z)^* \in I$ and $(x \otimes y) \otimes z) \in I, ((x \to y)^* \to z^*) \in I$ imply $x \otimes z \in I, (x \to z)^* \in I$.

If $y \in I$ and $x \otimes y \in I, (x \to y)^* \in I$ for all $x, y \in \mathscr{R}$, we have
$y \otimes 0 = (y \to 0^*)^* = (y \to 1)^* = 1^* = 0 \in I$
[From Definition 2.8]
$(y \to 0)^* = (y^*)^* = y \in I$[From(ii) of Definition 2.9].

Now, $(x \otimes y) \otimes z = \left((x \to y^*)^* \to 0^*\right)^*$ [From Definition 2.8]
$= ((x \to x)^* \to 1)^* = (1^* \to 1)^* = (0 \to 1)^* = 1^* = 0 \in I$

and

$$((x \to y)^* \to 0)^* = \left(((x \to y)^*)^*\right)^* = (x \to y)^* \in I$$

[From(ii) of Definition 2.9].
Since I is a IWI -ideal of $\mathscr{R}$. Which follows that
$x = (x^*)^* = x \otimes 0 = (x \to 0^*)^* = (x \to 1)^* = 1^* = 0 \in I$,
[From Definition 2.8]
$x = (x^*)^* = (x \to 0)^* = y^* = x \in I$.
[From(ii) of Definition 2.9]
Hence, I is a WI -ideal of $\mathscr{R}$. □

Example 3.9. *Consider a set $\mathscr{R} = \{0, p, q, r, s, t, 1\}$. Define a partial ordering "$\leq$" on $\mathscr{R}$, such that $0 \leq a \leq b \leq c \leq d \leq 1$ with a binary operations" $\otimes$ "and" $\to$ "and a quasi complement " $*$ "on $\mathscr{R}$ as in following tables 3 and 4.*

Define $\vee$ and $\wedge$ operations on $\mathscr{R}$ as follows:

$$(x \vee y) = (x \to y) \to y$$
$$(x \wedge y) = (x^* \to y^*) \to y^*)^*; x \otimes y = (x \to y^*)^*$$

for all $x, y \in \mathscr{R}$. Then, $\mathscr{R}$ is a residutaed lattice Wajsberg algebra. It is easy to verify that, $I_2 = \{0, q, s, 1\}$ is a IWI -ideal of $\mathscr{R}$.

Table 3. Complement

x	x^*
0	1
p	s
q	s
r	q
s	q
t	0
1	0

Table 4. Complement

$\rightarrow$	0	p	q	r	s	t	1
0	1	1	1	1	1	1	1
p	s	1	1	s	s	1	1
q	s	t	1	s	s	1	1
r	q	q	q	1	1	1	1
s	q	q	q	t	1	1	1
t	0	q	q	s	s	1	1
1	0	p	q	r	s	t	1

Proposition 3.10. *Every WI -ideal of a residuated lattice H -Wajsberg algebra is a IWI -ideal of $\mathscr{R}$.*

Proof. Let $\mathscr{R}$ be a residuated lattice H -Wajsberg algebra and let I be a WI-ideal of $\mathscr{R}$ for all $x,y,z \in \mathscr{R}$. Then we have

$$y \otimes z, (x \otimes y) \otimes z \in I, (y \rightarrow z)^*, ((x \rightarrow y)^* \rightarrow z)^* \in I$$

and

$$(x \otimes z) \otimes (y \otimes z) = ((x \otimes y) \otimes z) \in I$$

$$((x \rightarrow z)^* \rightarrow (y \rightarrow z)^*)^* = ((x \rightarrow y)^* \rightarrow z)^* \in I$$

[From Proposition 2.6]

Since I is a WI -ideal of $\mathscr{R}, (x \rightarrow z)^* \in I$ Hence, I is a IWI -ideal of $\mathscr{R}$. □

Proposition 3.11. *If $\mathscr{R}$ is a residuated lattice H -Wajsberg algebra if and only if every WI -ideal of $\mathscr{R}$ is a IWI -idealof $\mathscr{R}$.*

Proof. We can easily prove from Proposition 3.10. □

Proposition 3.12. *Let $\mathscr{R}$ be a residuated lattice Wajsberg algebra and I be a subset of A. Define $I^* = \{x^*/x \in I\}$ is a IWI -ideal of $\mathscr{R}$ if and only if I^* is an implicative filter of $\mathscr{R}$.*

Proof. Let I be aIWI -ideal of $\mathscr{R}$, then $1 = 0^* \in I^*$, since $0 \in I$ for all $x,y,z \in \mathscr{R}$. If $x \otimes y, x \rightarrow y$ and $x \otimes (y \otimes z), x \rightarrow (y \rightarrow z) \in I^*$, then we have

$$y^* \otimes x^* = x \otimes y \in I, y^* \rightarrow x^* = x \rightarrow y \in I$$

and

$((z^* \otimes y^*) \otimes x^*) = (z \otimes x) \otimes y$
$= (x \otimes z) \otimes y$, [From (ii) of Proposition 2.6]
$= x \otimes (z \otimes y)$, [From (i) of Proposition 2.6]
$= x \otimes (y \otimes z) \in I^*$, [From (ii) of Proposition 2.6]

$(z^* \rightarrow y^*) \rightarrow x^* = (x \rightarrow (y \rightarrow z)) \in I^*$, which implies that $(y^* \otimes x^*)^* \in I, (y^* \otimes x^*)^* \in I$, $((z^* \otimes y^*) \otimes x^*)^* \in I$, $((z^* \otimes y^*) \rightarrow x^*)^* \in I$. It follows that,

$$x \otimes z = (x \rightarrow z^*)^* \in I, (x \rightarrow z)^* = (z^* \rightarrow x^*)^* \in I.$$

Since I is a IWI -ideal of $\mathscr{R}$. Consequently, $(x \otimes z) \in I^*, x \rightarrow z \in I^*$. Thus, I^* is an implicative filter of $\mathscr{R}$.

Conversely, I^* is an implicative filter of $I, 0 = I^* \in I$ since $1 \in I^*$ for all $x,y,z \in I$ If $y \otimes z, (y \rightarrow z)^*$ and $(x \otimes y) \otimes z \in I, ((x \rightarrow y)^* \rightarrow z)^* \in I$. Then we have

$$\begin{aligned} z^* \otimes y^* &= \left(z^* \rightarrow (y^*)^*\right)^* = (z \rightarrow y^*) = (z \rightarrow y^*) \\ &= (y \rightarrow z^*) \in I, [\text{ From Definition 2.8}] \\ (z^* \rightarrow y^*)^* &= (y \rightarrow z)^* \in I, [\text{ From (x) of Definition 2.2 }] \end{aligned}$$

And

$$\begin{aligned} &(z^* \otimes (y^* \otimes x^*)) = \left(z^* \rightarrow (y^* \otimes x^*)^*\right)^*, [\text{ From Definition 2.8}] \\ &= \left(z^* \rightarrow \left(\left(y^* \rightarrow (x^*)^*\right)^*\right)^*\right)^*, [\text{ From Definition 2.8}] \\ &= \left(z^* \rightarrow (y^* \rightarrow x^*)^*\right)^*, [\text{ From (ix) of Definition 2.2}] \\ &= (z^* \rightarrow (y^{**} \rightarrow x^{**}))^*, [\text{ From (ix) of Definition 2.2}] \\ &= (z^* \rightarrow (y \rightarrow x))^*, [\text{ From (ix) of Definition 2.2}] \\ &= ((y \rightarrow x)^* \rightarrow z)^* \in I, [\text{ From (ix) of Definition 2.2}] \\ &(z^* \rightarrow (y^* \rightarrow x^*))^* = ((x \rightarrow y)^* \rightarrow z)^* \in I \end{aligned}$$

Now, $z^* \rightarrow x^* \in I^*, z^* \rightarrow y^* \in I^*$ and $z^* \otimes (y^* \otimes x^*) \in I^*, z^* \rightarrow (y^* \rightarrow x^*) \in I^*$.

And from Definition 2.8 we have, $x \otimes z = (x \rightarrow z^*)^* = (x^* \rightarrow z) \in I^*$. Also, from (x) of Definition 2.2, $x \rightarrow z = (z^* \rightarrow x^*) \in I^*$.

Since I^* is an implicative filter of $\mathscr{R}$, equivalently $(x \otimes z) \in I, (x \rightarrow z)^* \in I$. Therefore, I is a IWI -ideal of $\mathscr{R}$. □

4. Conclusion

In this paper, we have studied WI-ideal of residuated lattice Wajsberg algebra and investigated some of their properties. Also, we have announced the concept of implicative WI-ideal (IWI-ideal) of residuated lattice Wajsberg algebra. Further, we have inspected some of its characterizations and attained some properties of residuated lattice H-Wajsberg algebra.

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