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Divisibility of maximum matrices by minimum matrices using near square prime number

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Abstract

We define the near square minimum prime and near square maximum prime matrices separately. We calculate the determinant and inverse of near square maximum prime and near square minimum prime matrices by using arithmetical functions. Also, we discuss the divisibility of the near square maximum prime matrices by near square minimum prime matrices.

Keywords

Minimum matrix, Maximum matrix,Near Square Mersenne Minimum Matrix, Near Square Mersenne Maximum Matrix, Divisibility.

AMS Subject Classification

15A09, 15A15.

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1. Introduction

Let $(P, \leqslant) = (P, \Lambda, V)$ be a lattice, let $S = \{x_1, x_2, x_3, \ldots\ldots, x_n\}$ be a meet-closed subset of P and let $f : P \to Z^+$ be a function. The minimum matrix $(S)_f$ and the maximum matrix $[\mathrm{S}]_f$ on S with respect to f are [1] defined by

$$(S)_f = \min(x_i, x_j) \text{ and } [S]_f = \max[x_i, x_j].$$

It is well known that $(Z_+, |) = (Z_+, \min, \max)$ is a lattice, where $|$ is the usual divisibility relation for the minimum number and the maximum number of integers. Thus minimum and maximum matrices [2] are generalizations of Minimum matrices $\left((\mathrm{S})_f\right)_{ij} = \min(x_i, x_j)$ and Maximum matrices $\left([S]_f\right)_{ij} = \max[x_i, x_j]$.

2. Preliminaries

Let $A, B \in Z^{n\times n}$. We say that A divides B in the ring $Z^{n\times n}$ under addition and multiplication of matrices, written as $A \mid B$, if there exists $M \in Z^{n\times n}$ such that $B = MA$. [6] Note that since $Z^{n\times n}$ is not a commutative ring, it matters on which side of A the matrix Moccurs in [7, Definition 2.4]. If A and B are symmetric, then clearly

$$B = MA \Leftrightarrow B = AM^T.$$

In this paper we consider near square prime minimum and near square prime maximum matrices, and these are symmetric matrices.

Some Basic Definitions:

In this section we discuss some basic definitions of maximum and minimum matrices.

Definition 2.1. *(Minimum and Maximum Matrices) Let us define the matrix [3], operations* $\wedge$ *and* V *by* $(a_{ij})_{n\times m} \wedge (b_{ij})_{n\times m} = \min((a_{ij}), (b_{ij}))$ *and* $(a_{ij})_{n\times m} V (b_{ij})_{n\times m} = \max((a_{ij}), (b_{ij}))$. *Let* C *denote the* $n\times n$ *matrix with* $c_{ij} = x_i$ *for all* $1 \leq i, j \leq n$. *Then* $(T)_{\min} = C \wedge C^T$ *and* $[T]_{\max} = C \vee C^T$.

Definition 2.2. *(Mersenne Matrices) Let* $S = \{x_1, x_2, x_3, \ldots\ldots, x_n\}$ *be a set of distinct positive integers and the* $n\times n$ *matrix and* $[\mathrm{M}] = (m_{ij})$, *where* $m_{ij} = 2^{(x_i x_j)} - 1$, *call it to be Mersenne matrix on* S *[4].*

Definition 2.3. *(Mersenne Minimum Matrices) Let* $S = \{x_1, x_2, x_3, \ldots\ldots, x_n\}$ *be a set of distinct positive integers and the* $n \times n$ *matrix and* $[\mathrm{M}] = (m_{ij})$*, where* $m_{ij} = 2^{\min}(x_i x_j) - 1$*, call it to be Mersenne Minimum matrix on S.*

Definition 2.4. *(Mersenne Maximum Matrices) Let* $S = \{x_1, x_2, x_3, \ldots\ldots, x_n\}$ *be a set of distinct positive integers and the* $n \times n$ *matrix and* $[\mathrm{M}] = (m_{ij})$*, where* $m_{ij} = 2^{\max}(x_i x_j) - 1$*, call it to be Mersenne Maximum matrix on S.*

Definition 2.5. *(Near Square Mersenne Prime) If* M_p *is a Mersenne prime [8], then* $W_p = 2M_p^2 - 1$ *is called near-square number of Mersenne prime* M_p*. If Mersenne primes* M_p *are infinite then near-square number sequence* $W_p = 2M_p^2 - 1$ *generated from all Mersenne primes* $M_p = 2^p - 1$ *is an infinite sequence.*

Definition 2.6. *(Near Square Mersenne Minimum Matrices) Let* $S = \{x_1, x_2, x_3, \ldots\ldots, x_n\}$ *be a set of distinct positive integers. Then the* $(n \times n)$ *Near Square Mersenne Minimum matrix is* $S_{W_p \min} = (m_{ij})$*, where* $m_{ij} = 2M_p^2 - 1, M_p = 2^{\min(x_i, x_j)} - 1$ *call it to be Mersenne Minimum matrix on S.*

Definition 2.7. *(Near Square Mersenne Maximum Matrices) Let* $S = \{x_1, x_2, x_3, \ldots\ldots, x_n\}$ *be a set of distinct positive integers Then the* $(n \times n)$ *Near Square Mersenne Maximum matrix is* $S_{W_p \max} = (m_{ij})$*, where* $m_{ij} = 2M_p^2 - 1, M_p = 2^{\max}(x_i, x_j) - 1$*, call it to be Mersenne Maximum matrix on S.*

The Infinity of Near-Square Primes of Mersenne Prime Minimum Matrix:
The traditional relation formula between perfect number P_P and Mersenne prime minimum matrix M_P can be expressed as

$$P_P = \frac{\left(M_p^2 + M_P\right)}{2}.$$

From (1) we have

$$W_P = 2\left(2P_P - M_P\right) - 1, \quad \text{where } W_P = 2M_p^2 - 1,$$

is a near-square number of Mersenne prime minimum matrix M_P so that there is a nearsquare number sequence $W_P = 2M_p^2 - 1$ generated from all Mersenne prime minimum matrices M_P. If Mersenne prime minimum matrix M_P are infinite then $W_P = 2M_p^2 - 1$ is an infinite sequence. From $M_p = 2^p - 1$ we get structure of near square number $W_P = 2M_p^2 - 1$ as follows

$$W_P = 2^{2p+1} - 2^{p+2} + 1,$$

where p is exponent of Mersenne minimum matrix $M_p = 2^{\min(x_i, x_j)} - 1$.

Determinants of Near Square Mersenne Minimum and Near Square Mersenne Maximum Matrices:
We consider the determinants of the matrices

$$\det S_{W_p \min} = f(x_1)\{[f(x_2) - f(x_1)][f(x_3) - f(x_2)] \ldots\ldots\ldots [f(x_n) - f(x_{n-1})]\},$$

where $f(x_i) = 2M_{\min}^2 - 1, M_{\min} = 2^{x_i} - 1$

$$\det S_{W_p \max} = [f(x_1) - f(x_2)][f(x_2) - f(x_3)] \ldots\ldots\ldots [f(x_{n-1}) - f(x_n)] f(x_n),$$

where $f(x_i) = 2M_{\max}^2 - 1, M_{\max} = 2^{x_i} - 1$.

3. Inverses of Near Square Mersenne Minimum and Near Square Mersenne Maximum Matrices

Under the assumption that the elements of the set S are distinct and the Minimum and Maximum matrices of the set S are usually invertible[5]. Next, we shall find their inverses. Suppose that the elements of the set S are distinct. If $x_1 \neq 0$, then the Minimum matrix is invertible and the inverse matrix is the $n \times n$ tridiagonal matrix $(S)_f^{-1} = B = (b_{ij})$, where,

$$(S)_f^{-1} = \begin{cases} 0 & \text{if } |i-j| > 1 \\ \frac{f(x_2)}{f(x_1)[f(x_2)-f(x_1)]} & \text{if } i = j = 1 \\ \frac{1}{f(x_i)-f(x_{i-1})} + \frac{1}{f(x_{i+1})-f(x_i)} & \text{if } 1 < i = j < n \\ \frac{1}{f(x_n)-f(x_{n-1})} & \text{if } i = j = n \\ \frac{1}{|f(x_i)-f(x_j)|} & \text{if } |i-j| = 1 \end{cases}$$

where, $f(x_i) = 2M_{\min}^2 - 1, M_{\min} = 2^{x_i} - 1$.
If $x_n \neq 0$, then the inverse of the Maximum matrix is invertible and the inverse matrix is the $n \times n$ tridiagonal matrix $[S]_f^{-1} = C = (c_{ij})$, where,

$$[S]_f^{-1} = \begin{cases} 0 & \text{if } |i-j| > 1 \\ \frac{1}{[f(x_1)-f(x_2)]} & \text{if } i = j = 1 \\ \frac{1}{f(x_{i-1})-f(x_i)} + \frac{1}{f(x_i)-f(x_{i+1})} & \text{if } 1 < i = j < n \\ \frac{1}{f(x_{n-1})-f(x_n)} + \frac{1}{f(x_n)} & \text{if } i = j = n \\ \frac{1}{|f(x_i)-f(x_j)|} & \text{if } |i-j| = 1 \end{cases}$$

where $f(x_i) = 2M_{\max}^2 - 1, M_{\max} = 2^{x_i} - 1$.

Example 3.1. *If* $S = \{2, 4, 6\}$ *is a lower closed set. Then* 3×3 *Near Square Mersenne Minimum matrix on S is*

$$S_{mermin} = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 15 & 15 \\ 3 & 15 & 63 \end{bmatrix} \quad S_{W_p \min} = \begin{bmatrix} 17 & 17 & 17 \\ 17 & 449 & 449 \\ 17 & 449 & 7937 \end{bmatrix}$$

Here, $f(x_1) = 17, f(x_2) = 449$ *and* $f(x_3) = 7937$

$$\det S_{W_p \min} = f(x_1)\{[f(x_2) - f(x_1)][f(x_3) - f(x_2)]\} = 17 \times 432 \times 7488 = 54991872.$$

(S) *is a Near Square Mersenne Minimum matrix on lower closed set* $S = \{2, 4, 6\}$*. Then by definition*

$$(S_{mer\,min})^{-1} = \begin{bmatrix} \frac{5}{12} & \frac{-1}{12} & 0 \\ \frac{-1}{12} & \frac{5}{48} & \frac{-1}{48} \\ 0 & \frac{-1}{48} & \frac{1}{48} \end{bmatrix},$$

$$\left(S_{W_p\,\min}\right)^{-1}=\left[\begin{array}{ccc}\frac{449}{7344} & \frac{-1}{432} & 0\\ \frac{-1}{432} & \frac{55}{2246} & \frac{-1}{7488}\\ 0 & \frac{-1}{7488} & \frac{1}{7488}\end{array}\right].$$

Also,3 $\times$ 3 Near Square Mersenne Maximum matrix on S is

$$S_{mer\,max}=\left[\begin{array}{ccc}3 & 15 & 63\\ 15 & 15 & 63\\ 63 & 63 & 63\end{array}\right]$$

$$S_{W_p\,\max}=\left[\begin{array}{ccc}17 & 449 & 7937\\ 449 & 449 & 7937\\ 7937 & 7937 & 7937\end{array}\right]$$

$$\begin{aligned}\det S_{W_p\,\max}&=[f(x_1)-f(x_2)]\,[f(x_2)-f(x_3)]\,f(x_3)\\&=(-432)\times(-7488)\times 7937\\&=25674734592.\end{aligned}$$

(S) is a Near Square Mersenne Maximum matrix on lower closed set $S=\{2,4,6\}$. Then by definition

$$\left(S_{mermax}\right)^{-1}=\left[\begin{array}{ccc}\frac{-1}{12} & \frac{1}{12} & 0\\ \frac{1}{12} & \frac{-5}{48} & \frac{1}{48}\\ 0 & \frac{1}{48} & \frac{-5}{1008}\end{array}\right]$$

$$\left(S_{W_p\,\max}\right)^{-1}=\left[\begin{array}{ccc}\frac{-1}{432} & \frac{1}{432} & 0\\ \frac{1}{432} & \frac{-55}{22464} & \frac{1}{7488}\\ 0 & \frac{1}{7488} & \frac{-449}{59432256}\end{array}\right].$$

4. Divisibility of Near Square Maximum Matrices by Near Square Minimum Matrices

Let S be a minimum-closed or maximum-closed set with n elements, where $n\leq 3$ and let f be a semi multiplicative function[7] satisfying $f(x_i)\neq 0$ for all $x_i,x_j\in S$. Then$[S]_f \mid (S)_f$.

Proof. Suppose first that S is a ged-closed set with n elements. If $n=1$, then $(S)_f=[S]_f$. Let $n=2$. Then $x_1\mid x_2$ and we have $f(x_1)\mid f(x_2)$ and further

$$\begin{aligned}[S]_f(S)_f^{-1}&=\left[\begin{array}{cc}f(x_1) & f(x_2)\\ f(x_2) & f(x_2)\end{array}\right]\left[\begin{array}{cc}f(x_1) & f(x_1)\\ f(x_1) & f(x_2)\end{array}\right]^{-1}\\&=\left[\begin{array}{cc}0 & 1\\ \frac{f(x_2)}{f(x_1)} & 0\end{array}\right]\in\mathcal{M}_3(\mathbb{Z})\end{aligned}$$

□

Example 4.1. *Let $S=\{2,3\}$ is a lower closed set. By definition of Maximum and Minimum matrices,*

$$[S]_f=\left[\begin{array}{cc}2 & 3\\ 3 & 3\end{array}\right] \text{ and } (S)_f=\left(\begin{array}{cc}2 & 2\\ 2 & 3\end{array}\right)$$
$$\Rightarrow (S)_f^{-1}=\left(\begin{array}{cc}\frac{3}{2} & -1\\ -1 & 1\end{array}\right)$$

Then the divisibility of Maximum matrices by Minimum matrices is,

$$[S]_f \mid (S)_f=[S]_f(S)_f^{-1}=\left(\begin{array}{cc}0 & 1\\ \frac{3}{2} & 0\end{array}\right).$$

If $S=\{2,3\}$ is a lower closed set. Then2 $\times$2 Near Square Mersenne Minimum matrix on S is

$$S_{mermin}=\left[\begin{array}{cc}3 & 3\\ 3 & 7\end{array}\right]\quad S_{W_p\,\min}=\left[\begin{array}{cc}17 & 17\\ 17 & 97\end{array}\right]$$

$$\det S_{W_p\,\min}=f(x_1)\{[f(x_2)-f(x_1)]\}=17\times 80=1360.$$

And 2×2 Near Square Mersenne Maximum matrix on S is

$$S_{\text{mer}\max}=\left[\begin{array}{cc}3 & 7\\ 7 & 7\end{array}\right]\quad S_{W_p\,\max}=\left[\begin{array}{cc}17 & 97\\ 97 & 97\end{array}\right]$$

$$\det S_{W_p\,\max}=[f(x_1)-f(x_2)]\,f(x_2)=(-80)\times 97=-7760.$$

Then the divisibility of Near Square Mersenne Maximum matrix by Near Square Mersenne Minimum matrix is,

$$S_{W_p\,\max}\mid S_{W_p\,\min}=S_{W_p\,\max}S_{W_p\,\min}^{-1}=\left(\begin{array}{cc}0 & 1\\ \frac{97}{17} & 0\end{array}\right).$$

5. Conclusion

In this paper, the different properties of Near Square Minimum and Near Square Maximum matrices of the set S with $\min(x_i,x_j)$ and $\max(x_i,x_j)$ as their (i,j)entries like determinant value, inverse and divisibility of Near Square Minimum and Near Square Maximum matrices have been studied. The study is carried out by applying known results of meet and joins matrices to Mersenne minimum and Mersenne maximum matrices.

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