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Learning effect on an optimal policy for mathematical inventory model for decaying items under preservation technology with the environment of COVID-19 pandemic

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Abstract

Due to the environment of corona virus pandemic, a huge problem arises in the market, due to which the retailer keeps his items lying, and those items start to deteriorate. He is unable to get the new goods from suppliers and get a shortage. We established an optimal policy for mathematical inventory model for decaying items under preservation technology (PT) with learning effect. This model is starting with partially backlogging shortage. The investment in Preservation technology is used so that the items deposited with the retailer do not deteriorate. In this model learning effect and preservation technology plays a very important role. Which the help of the total cost is reduced and maintains the quality of the environment. We show that the total cost is a convex function. Finally, some figures are presented to highlight the numerical examples and sensitivity results. And performed using the Mathematicia-9.0 software.

Keywords

Covid-19 environment, Decaying items, Learning effect, Preservation technology, Shortage, Ramp type demand.

AMS Subject Classification

90B10, 90Bxx.

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1. Introduction

The corona virus (COVID-19) remains a topic of discussion in the world, which has stopped the pace of life, which has affected the word. Covid-19 came from China in late December 2019; after that, it spread worldwide, which has an impact on the international market. It has also been seen due to which international trade stopped, and the quality of the items also decreased due to a lack of sales. The Replacement policy for deteriorating items has also been given attention over the past few decades. Fruits, vegetables, pharmaceuticals, and other inventory items are included in the category of deterioration items in daily life. The deterioration rate is supposed to be an unbalanced variable, whereas, through investment in PT, it can be balanced up to a certain level.

Learning phenomena are visible everywhere. And dealing with any assignment is involved in some early experience, which is seen in the market nowadays. The basic outcome in the development of EOQ models with declines is that of [1] Ghare and Schrader examined a rapidly decaying inventory for a sustained demand. Yet, as is evident through the chemical and basic sciences, the spoilage rate is seldom, especially in relation to spoiled foods. [2] Mandal and Fazdar (1989) developed an EOQ inventory model, which reflects stock-dependent demand without reducing returns as a linear function of the stock price. [3] Fujiwara and Perera examined an EOQ model for fresh products that consider the continuous deterioration of a product's usefulness. [4] Mandal and Pal established an order level inventory model under decaying items with demand rate a time-dependent function. [5] Chang and Dye developed an EOQ model for deteriorating items with time-varying demand, and the shortages are either completely backlogged or completely lost. [6] Goyal and Giri represented a review of the progress of decaying inventory literature. [7] Papachristos and Skouri examined an inventory model where the backlogging rate is a time-dependent function. [8] Wu et al. developed an inventory model that determines the policy for decaying items with stock-dependent demand. [9] Ouyang et al. joined the phenomenon into the inventory model. They studied that if the retailer was improving the storage facility, they could reduce the item's deteriorating rate, the total yearly relevant inventory cost will be dropped. [10] Panda et al. examined the cyclic decaying items. [11] Skouri et al. developed an inventory model with time-dependent demand and partial backlogging of unsatisfied demand. [12] Scory et al. examined an order level inventory model for decaying items. The demand rate can work at any time until it is stabilized, and using the backlogging rate [13] Hasu et al. derived an inventory model with PT investment to reduce the deteriorating rate of inventory for sustained demand. [14] Bakar et al. referred more information on the subject there. However, current research often assumes that inventory decline as soon as it arrives. In practice, most of the goods will be in the form of a pan, maintaining the quality or original condition, meaning there is no deterioration during that period. [15] Dye and Hesih examined an optimal policy for the deterioration of goods with active PT investment. [16] Glock et al. established an EOQ model with fuzziness demand and learning with the warehouse. [17] Cai et al. have developed an economic sequence model that focuses on the deteriorating rate over time. [18] Kumar et al. developed the learning effect on the inventory policy for decaying items with two warehouses. [19] Hsieh and Dye designed a production inventory model that incorporated PT investment's impact on fluctuations over time. [20] Considering an inventory scheme with a non-instantaneous deteriorating item, Dye aims to study the impact of PT investment on inventory choices. [21] Pal et al. measured a production inventory model for decaying items with RT demand rate and shortages under fuzziness. [22] Sicilia developed a deterministic inventory model for objects with a constant decaying rate, and its demand depends on time. [23] Muriana developed a model, the results of which show that the food recovery strategy is rewarding in the presence of food quality uncertainty, even if it is less than the optimal benefit compared to the deterministic case. [24] Prasad and Mukherjee studied a deterministic inventory model for the deterioration of stocks and time-dependent demand items under the impact of a fall. [25] Zhang et al. examined the inventory problem of perishable goods in which pricing, service investment, and considered PT investment together. [26] Luo examined one of the pioneer's works for the inventory model with ramp type demand. [27] Singh et al. derived the model under the circumstance when the fixed shortage time point is less than the procurement time point, and it is also valid for newly launched high-tech products. [28] Huang et al. developed a supply chain model with manufacturing disruption, which targets learning the best pricing, inventory, and preservation decisions. [29] Suthar and Shukla considered it as a promotional tool to boost demand. Retailers dealing with non-instantaneous deteriorating items offer their customers different price discounts before and after the decline begins. In this article, a policy is presented for non-instantaneous deteriorating commodities with price-sensitive ramp type demand patterns. [30] Liu et al. determined the issue of how to choose supermarket properly and balance the pricing of fresh food in a competitive market environment and PT investment.

In this model, we extended the work of Skouri et al. (2009). We have used PT to reduce the rate of deteriorating items due to the environment of Covid-19 and others. Furthermore, the use of the learning effect decreases the cost of the system engaged in repetitive operations. Here, we assumed that the holding cost is decreasing due to the learning effect, and Shortage is allowed, and unsatisfied demand is partially backlogging. The model is prepared as follows. The assumptions and notation used in the models are given in Section 2. The model starting with shortages and mathematical formulation of the Inventory model is studied in Section 3. The optimal solution policy is obtained in Section 4. Numerical examples, figures, and tables highlighting the results obtained are given in Section 5. Sensitivity analysis with tables is provided in Section 6. The paper finishes with a conclusion in Section 7.

2. Assumptions and Notations

2.1 The model is developed under the following assumptions

1. F(t) is the ramp type demand rate function of time t, i.e.

$$\text{F(t)} = \begin{cases} f(t) & t < u \\ f_0 & t > u \end{cases}$$

   wherever $f(t) = x + yt$ is linear function $s, t, x, y > 0$ and f_0 is constant function.

2. Preservation technology (PT) cost investment is controlled deteriorating items due to the environment of covid-19.

3. The shortage is allowed, and unsatisfied demand is partially backlogging,we assume that $\delta(\mathrm{t}) = e^{-\alpha t}$ where $\alpha \geq 0$, and t is the waiting time, $\delta(\mathrm{t})$ the backlogging rate, which is a decreasing function of the waiting time t.

4. $\left(v_1 + \frac{v_L}{z^p}\right)$ is holding cost with learning effect.

5. Decaying rate after investing in PT, $\omega(\theta) = \omega_0 e^{-\sigma\theta}$, Where ω_0 is the decaying rate without PT investment, and σ is the sensitive of investment to the decaying rate.

2.2 The symbols of the different parameters used, the decision variable and the objective functions are explained below.

$\omega(\theta)$: Decaying rate after investment on PT (unit/time unit)

ω_0 : Decaying rate without PT investment (units/time unit)

θ : PT investment cost (\$/unit/time unit)

$\mathrm{v}_1 + \frac{\mathrm{v_L}}{\mathrm{z^p}}$: The holding cost per unit time due to learning effect

v_2 : The shortage cost (/ unit time)

v_3 : The cost gained from the decaying of one unit

v_4 : The opportunity cost per unit due to the lost sales

T : Fixed length of each ordering cycle

$\mathrm{t_s}$: The time when the inventory level reaches zero after completion of the shortage

u : Changing point from linear demand to constant demand (time point)

f(t) : Linear demand function

f_0 : Constant demand function

TC_1 $(\mathrm{t_s}, \theta)$: Total cost in case $(\mathrm{t_s} < \mathrm{u})$

TC_2 $(\mathrm{t_s}, \theta)$: Total cost in case $(\mathrm{t_s} > \mathrm{u})$

ZB(t) : The backlogged level

z(t) : The inventory level at the cycle period $[0, T]$

$\delta(t)$: The backlogging rate

N : The greatest inventory level at the cycle period

3. Mathematical Formulation

The mathematical model starts with shortages first, which occur during the period $[0, t_s]$ and which are partially backlogged. At the time t_s a replenishment brought the inventory level up to maximum N. The inventory level of the items reduces due to demand and deteriorated during the period $[t_s, T]$ until this falls at zero at time T. Here we have discussed the model in two ways.

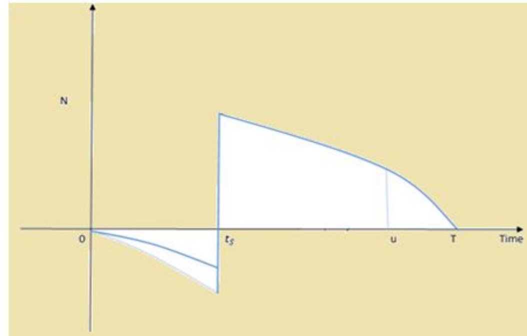

Figure 1. Inventory level model for Case I

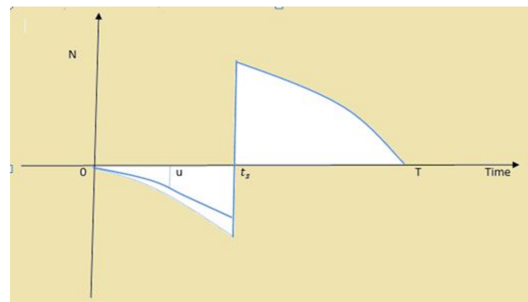

Figure 2. Inventory level model for Case II

3.1 Case I: When $t_s < u$

The behavior of this case l is showed in Fig.1. The differential equations of the inventory model at time t over period a [0, T] can be expressed as follows:

$$Z'B(t) = -\delta(t_s - t) f(t) \quad 0 < t < t_s, \quad ZB(0) = 0 \tag{3.1}$$

$$Z'(t) + \omega(\theta) Z(t) = -f(t) t_s < t < u, \quad Z(u^-) = Z(u^+) \tag{3.2}$$

$$Z'(t) + \omega(\theta) Z(t) = -f_0 \quad u < t < T, \quad Z(T) = 0 \tag{3.3}$$

Solving of the above differential equations, we have

$$\mathrm{ZB(t)} = \frac{e^{-\alpha t_S}}{\alpha^2}\left[\{x + yt)\alpha - y\} e^{\alpha t} - (x\alpha - y)\right] 0 < t < t_s \tag{3.4}$$

$$\begin{aligned} Z(t) = &- \frac{1}{\omega^2(\theta)}\left[\{(x + yt)\omega(\theta) - y\}\right. \\ &+ \left[\left\{f_0 \omega(\theta)\left(1 - e^{-\omega(\theta)(u-T)}\right)\right\}\right. \\ &\left.\left. - \{(x + yu)\omega(\theta) - y\}\right] e^{-\omega(\theta)(t-u)}\right], \quad t_s < t < u \end{aligned} \tag{3.5}$$

$$Z(t) = -\frac{f e^{-\omega(\theta) t}}{\omega(\theta)}\left\{e^{\omega(\theta) t} - e^{\omega(\theta) T}\right\} \quad u < t < T \tag{3.6}$$

The current amount of decayed unit during (t_s, T) is:

$$\begin{aligned} D = &\text{Greatest Inventory level in } [t_s, T] \\ &- \text{ Demand in } [t_s, u] - \text{ Demand in } [u, T] \\ = &-\frac{1}{\omega^2(\theta)}[\{(x+yt_s)\,\omega(\theta)-y\} \\ &+\left[\left\{f_0\omega(\theta)\left(1-e^{-\omega(\theta)(u-T)}\right)\right\}\right. \\ &\left.-\{(x+yu)\omega(\theta)-y\}\right]e^{-\omega(\theta)(t_s-u)}\Big] \\ &-\int_{t_s}^{u} f(t)dt - \int_{u}^{T} f_0 dt \\ = &-\frac{1}{\omega^2(\theta)}[\{(x+yt_s)\,\omega(\theta)-y\}-[\{f_0\omega(\theta) \\ &\left(1-e^{-\omega(\theta)(u-T)}\right)\}+\{(x+yu)\omega(\theta)-y\}]e^{-\omega(\theta)(t_s-u)}\Big] \\ &-\left[x(u-t_s)+\frac{y}{2}\left(u^2-t_s^2\right)\right]-f_0(T-u) \end{aligned}$$

The current rate of holding cost carried for the period of the interval $[t_s, T]$ is:

$$\begin{aligned} Z_1 = &\int_{t_s}^{u} Z(t)dt + \int_{u}^{T} Z(t)dt \\ = &-\frac{1}{\omega^2(\theta)}\left[\left\{\left(x+\frac{y}{2}(u-t_s)\right)(u-t_s)\,\omega(\theta)-y(u-t_s)\right\}\right. \\ &-f_0\left(e^{\omega(\theta)u}-e^{\omega(\theta)T}\right)\left(e^{-\omega(\theta)u}-e^{-\omega(\theta)t_s}\right) \\ &\left.+\left\{(x+yu)-\frac{y}{\omega(\theta)}\right\}\left(1-e^{-\omega(\theta)(t_s-u)}\right)\right] \\ &-\frac{f_0}{\omega^2(\theta)}\left[\left(e^{-\omega(\theta)T}-e^{-\omega(\theta)u}\right)+(T-u)\omega(\theta)\right] \end{aligned}$$

The current rate of the backlogging cost is:

$$\begin{aligned} Z_2 = &\int_0^{t_s} ZB(t)dt \\ = &-\frac{e^{-\alpha t_s}}{\alpha^3}\left[\{(x+yt_s)\,\alpha-2y\}\,e^{\alpha t_s}\right. \\ &\left.-(x\alpha-y)\alpha t_s-(x\alpha-2y)\right] \end{aligned}$$

The current rate of amount loss sale for the period of $[0, t_s]$ is:

$$\begin{aligned} L_s = &\int_0^{t_s}\{1-\delta(t_s-t)\}f(t)dt \\ = &\int_0^{t_s}\left[1-e^{-\alpha(t_s-t)}\right](x+yt)dt \\ = &\left(x+\frac{y}{2}t_s\right)t_s-\frac{1}{a^2}\left[(x+yt_s)\,\alpha\right. \\ &\left.-x\alpha e^{-\alpha t_s}-y\left(1-e^{-\alpha t_s}\right)\right] \end{aligned}$$

Therefore, the total cost is:

$$\begin{aligned} &TC_1(t_s,\theta) \\ =&\frac{1}{T}\left[\left(\mathrm{V}_1+\frac{\mathrm{v_L}}{\mathrm{z^p}}\right)Z_1+v_2Z_2+v_3\mathrm{D}+v_4L_s+\theta\right] \\ =&\frac{1}{T}\Big[-\frac{v_1}{\omega^2(\theta)}\Big[\Big\{\left(x+\frac{y}{2}(u-t_s)\right)(u-t_s)\,\omega(\theta) \\ &-y(y-t_s)\Big\}-f_0\left(e^{\omega(\theta)u}-e^{\omega(\theta)T}\right)\left(e^{-\omega(\beta)u}-e^{-\omega(\theta)t_s}\right) \\ &+\left\{(x+yu)-\frac{y}{\omega(\theta)}\right\}\left(1-e^{-\omega(\theta)(t_s-u)}\right)\Big] \\ &-\frac{v_1f_0}{\omega^2(\theta)}\left[\left(e^{-\omega(\theta)T}-e^{-\omega(\theta)u}\right)+(T-u)\omega(\theta)\right] \\ &-\frac{v_2e^{-\alpha t_s}}{\alpha^3}\Big[\{(x+yt_s)\,\alpha-2y\}\,e^{\alpha t_s}-(x\alpha-y)\alpha t_s \\ &-(x\alpha-2y)\Big]-\frac{v_3}{\omega^2(\theta)}\Big[\{(x+yt_s)\,\omega(\theta)-y\} \\ &-\Big[\left\{y_0\omega(\theta)\left(1-e^{-\omega(\theta)(u-T)}\right)\right\} \\ &+\{(x+yu)\omega(\theta)-y\}\Big]e^{-\omega(\theta)(t_s-u)}\Big]-v_3\Big[x(u-t_s) \\ &+\frac{y}{2}\left(u^2-t_s^2\right)+f_0(T-u)\Big]+\left(x+\frac{y}{2}t_s\right)v_4t_s \\ &-\frac{v_4}{a^2}\left[(x+yt_s)\,\alpha-x\alpha e^{-\alpha t_s}-y\left(1-e^{-\alpha t_s}\right)\right]+\theta\Big] \end{aligned} \tag{3.7}$$

3.2 Case II: When $t_s > u$

The Inventory model behavior of the case II is represented in Fig.2. The inventory level of the case represents the following differential equations:

$$Z'B(t) = -\delta(t_s-t)\,f(t) \quad 0<t<u, \quad ZB(0)=0 \tag{3.8}$$

$$Z'B(t) = -\delta(t_s-t)\,f_0 \quad u<t<t_s, \quad Z\left(u^-\right)=Z\left(u^+\right) \tag{3.9}$$

$$Z'(t)+\omega(\theta)Z(t) = -f_0 \quad t_s<t<T, Z(T)=0 \tag{3.10}$$

And $Z(t_s) = N$.
Solving the above differential equations, we have

$$\mathrm{ZB(t)} = \frac{e^{-\alpha t_s}}{\alpha^2}\left[\{(x+yt)\alpha-y\}e^{\alpha t}-(x\alpha-y)\right], \quad 0<t<u, \tag{3.11}$$

$$\begin{aligned} \mathrm{ZB(t)} = &-\frac{f_0}{\alpha}\left[e^{-\alpha(t_s-t)}-e^{-\alpha(t_s-u)}\right] \\ &-\frac{e^{-\alpha t_s}}{\alpha^2}\left[\{(x+yu)\alpha-y\}e^{\alpha u}-(x\alpha-y)\right], \\ &u<t<t_s, \end{aligned} \tag{3.12}$$

$$Z(t) = -\frac{f_0}{\omega(\theta)}\left[1-e^{\omega(\theta)(T-t)}\right] t_s<t<T, \tag{3.13}$$

$$Z(t_s) = N = -\frac{f_0}{\omega(\theta)}\left[1-e^{\omega(\theta)(T-t_s)}\right] \tag{3.14}$$

The current rate of decayed unit during (t_s, T) is:

$$\begin{aligned} \mathrm{D} &= \text{greatest Inventory level in } [t_s, T] \\ &\quad - \text{Demand in } [t_s, T] \\ &= -\frac{f_0}{\omega(\theta)}\left[1-e^{\omega(\theta)(T-t_s)}\right] - f_0(T-t_s) \end{aligned}$$

The current rate of holding cost carried during the interval $[t_s, T]$ is:

$$Z_1 = \int_{t_s}^{T} Z(t)dt = \frac{f_0\left(-1+e^{\omega(T-t_s)} - T\omega + \omega t_s\right)}{\omega^2(\theta)}$$

The current rate of the backlogging cost is:

$$\begin{aligned} Z_2 &= \int_0^{t_s} ZB(t)dt = \int_0^{u} -ZB(t)dt + \int_u^{t_s} -ZB(t)dt \\ &= -\frac{e^{-t_s a}\left(x\alpha\left(1-e^{u\alpha}+u\alpha\right)-y\left(2+u\alpha+e^{u\alpha}(-2+u\alpha)\right)\right)}{\alpha^3} \\ &\quad - \frac{e^{-t_w\alpha}D_0\left(-e^{\alpha t_w}+e^{v\alpha}\left(1-v\alpha+\alpha t_w\right)\right)}{\alpha^2} \\ &\quad + \frac{e^{-t_s\alpha}\left(y-x\alpha+e^{u\alpha}\left(-y+(x+yu)\alpha\right)\right)(-u+t_s)}{\alpha^2} \end{aligned}$$

The current amount loss sale during in $[0, t_s]$ is:

$$\begin{aligned} L_s &= \int_0^{u}\{1-\delta(t_s-t)\} f(t)dt \\ &\quad + \int_u^{t_s}\{1-\delta(t_s-t)\} f_0 dt \\ &= xu + \frac{yu^2}{2} - \frac{xe^{-t_s^2\alpha}\left(-1+e^{ua}\right)}{\alpha} \\ &\quad - \frac{ye^{-t_s a}\left(1+e^{ua}(-1+u\alpha)\right)}{\alpha^2} \\ &\quad + \frac{\left(-1+e^{(-t_s+u)\alpha}+t_s\alpha-u\alpha\right)f_0}{\alpha} \end{aligned}$$

Therefore, the total cost is:

$$TC_2(t_s,\theta) = \frac{1}{T}\left[\left(\mathrm{v}_1+\frac{\mathrm{v_L}}{\mathrm{z^p}}\right)Z_1 + v_2 Z_2 + v_3 D + v_4 L_s + \theta\right]$$

$$\begin{aligned} &TC_2(t_s) \\ &= \frac{1}{T}\left[\frac{v_1 f_0\left(-1+e^{\omega(\theta)(T-t_s)} - T\omega(\theta)+\omega(\theta)t_s\right)}{\omega(\theta)^2}\right. \\ &\quad - \frac{v_2 e^{-t_s\alpha}\left(x\alpha\left(1-e^{y\alpha}+u\alpha\right)-y\left(2+u\alpha+e^{u\alpha}(-2+u\alpha)\right)\right)}{\alpha^3} \\ &\quad - \frac{v_2 e^{-t_s\alpha} f_0\left(-e^{\alpha t_s}+e^{u\alpha}\left(1-u\alpha+\alpha t_s\right)\right)}{\alpha^2} \\ &\quad + \frac{v_2 e^{-t_s\alpha}\left(b-x\alpha+e^{u\alpha}\left(-y+(x+yu)\alpha\right)\right)(-u+t_s)}{\alpha^2} \\ &\quad - \frac{v_3 f_0}{\omega(\theta)}\left(1-\mathrm{Exp}[\omega(\theta)(T-t_s)]\right) - v_3 f_0 (T-t_s) \\ &\quad + v_4\left(xu + \frac{yu^2}{2} - \frac{ae^{-t_s^2\alpha}\left(-1+e^{u\alpha}\right)}{\alpha}\right. \\ &\quad - \frac{ye^{-t_s\alpha}\left(1+e^{u\alpha}(-1+u\alpha)\right)}{\alpha^2} \\ &\quad \left.\left. + \frac{\left(-1+e^{(-t_s+u)\alpha}+t_s\alpha-u\alpha\right)f_0}{\alpha}\right) + \theta\right] \end{aligned} \tag{3.15}$$

The total cost function of the system over $[0, T]$ is:

$$TC(t_s,\theta) = \begin{cases} TC_1(t_s,\theta), & \text{if } t_s \le u \\ TC_2(t_s,\theta), & \text{if } u < t_s \end{cases} \tag{3.16}$$

4. Optimal Solution Method

We derive the optimal solution, i.e., we calculate the value t_s, θ^*, which minimizes the Total cost function. Therefore, from (3.7),

$$\frac{\partial TC_1}{\partial t_s} = 0 \quad \& \quad \frac{\partial TC_1}{\partial \theta} = 0 \tag{4.1}$$

Equation (4.1) can be solved easily by using Mathematica9.0 software and find the value of $t_s{}^*, \theta^*$. And for minimizes the total cost conditions, i.e,

$$Det\begin{pmatrix} r_1 & s_1 \\ s_1 & \mathrm{t}_1 \end{pmatrix} > 0, \mathrm{r}_1 = \frac{\partial^2 \mathrm{TC}_1}{\partial \mathrm{t_s^2}} > 0, \mathrm{t}_1 = \frac{\partial^2 \mathrm{TC}_1}{\partial^2 \theta} > 0 \tag{4.2}$$

for the optimal value $\mathrm{t_s^*}, \theta^*$ Consequently, if (4.2) holds, then the Order level N is:

$$\begin{aligned} N^* &= \frac{1}{\omega^2(\beta)}\left[\{(x+yt_s)\,\omega(\theta) - y\}\right. \\ &\quad + \left[\left\{f_0\omega(\theta)\left(1-e^{-\omega(\theta)(u-T)}\right)\right\}\right. \\ &\quad \left. + \{(x+yu)\omega(\theta)-y\}\right] e^{-\omega(\theta)(t_s-u)}\Big] \end{aligned} \tag{4.3}$$

Order quantity $= N^* + \int_0^{t_s}\delta(t_s-t) f(t)dt$

$$= N^* + \frac{e^{-\alpha t_s}}{\alpha^3}\left[\{(x+yt_s)\,\alpha - 2y\}\, e^{\alpha t_s} - (x\alpha - 2y)\right] \tag{4.4}$$

Now from (3.15) an optimal solution, it is considered that

$$\frac{\partial TC_2}{\partial \mathrm{t_s}} = 0 \quad \& \quad \frac{\partial TC_2}{\partial \theta} = 0 \tag{4.5}$$

Equation (4.5) can be solved easily by using Mathematica9.0 software and find the value of $t_s{}^*, \theta^*$. And for minimizes the total cost conditions, i.e.,

$$\begin{pmatrix} r_2 & s_2 \\ s_2 & \mathrm{t}_2 \end{pmatrix} > 0 \text{ and } \mathrm{r}_2 = \frac{\partial^2 \mathrm{TC}_2}{\partial \mathrm{t_s^2}} > 0, \mathrm{t}_2 = \frac{\partial^2 \mathrm{TC}_2}{\partial^2 \theta} > 0 \tag{4.6}$$

for the optimal value, t_s^*, θ^*. Hence, if (4.6) holds then the Order level N is

$$N^* = -\frac{f_0}{\omega}(1 - \text{Exp}[\omega(T-t)]) \tag{4.7}$$

$$\text{Order quantity } = N^* + \int_0^u \delta(t_s - t) f(t) dt + \int_u^{t_s} \delta(t_s - t) f_0 dt \tag{4.8}$$

5. Numerical Examples

Example 5.1. *For an inventory model, let the values of parameters be as follows:* $v_1 = \$51$ *per unit,* $v_L = \$0.5$*per unit,* $v_2 = \$71$ *per unit,* $v_3 = \$65$ *per unit,* $v_4 = \$11$ *per unit,* $\sigma = 0.2, z = 3, p = 0.4$ $\omega_0 = 0.1$ *units / year,* $x = 2132$ *units,* $y = 11$ *units,* $f_0 = 655$ *units,* $\alpha = 0.01$, $u = 9$ *year,* $T = 10$ *year The optimal solution is solved with the Mathematica9.0 software after some iterations,* $t_s^* = 4.49, \theta^* = 3.08$. *The minimum total cost is* $TC_1(t_s^*, \theta) = 314326$ *and* $N^* = 83648.4$ *and order quantity* $= 547691.7$*in case I and satisfied conditions; Det* $\begin{pmatrix} r_1 & s_1 \\ s_1 & t_1 \end{pmatrix} > 0, r_1 = \frac{\partial^2 TC_1}{\partial t_s^2} > 27611.5 > 0, t_1 = \frac{\partial^2 TC_1}{\partial^2 \theta} > 1813.27 > 0, s_1 = 2529.85$ *for the optimal value* t_s^*, θ^*.

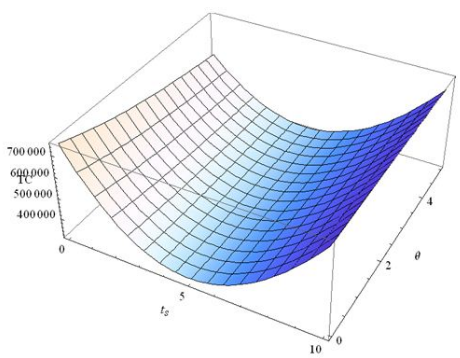

Figure 3. TC_1 with respect the t_s and θ

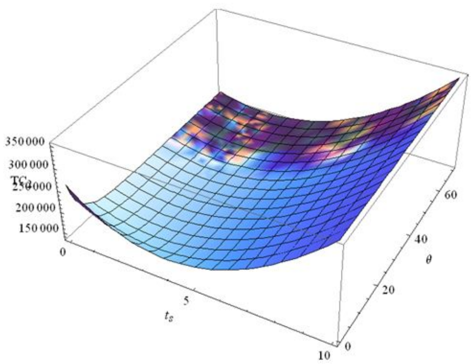

Figure 4. TC_2 with respect the t_s and θ

Example 5.2. *Consider all parameters are same as the example 1 except* $u = 1.5$ *year. The optimal solution is solved with the Mathematica9.0 software after several iterations,* $t_s{}^* = 2.28$, $\theta^* = 69.89$. *The minimum total cost is* $TC_2(t_s^*, \theta) = 135928$ and $N^* = 5056.6$and *order quantity=8727.2 in case II* $(u < t_s)$ *and satisfied condition; Det* $\begin{pmatrix} r_2 & s_2 \\ s_2 & t_2 \end{pmatrix} > 528501 > 0$ *and* $r_2 = \frac{\partial^2 TC_2}{\partial t_s^2} > 7561.9 > 0, t_2 = \frac{\partial^2 TC_2}{\partial^2 \theta} > 69.89 > 0 \& S_2 = \frac{\partial^2 TC_2}{\partial t_s \partial \theta} = 0.0226$ *for the optimal value* t_s^*, θ^* *Hence, Figures 3 and 4 of the above examples establish the convexity of* TC_1 *and* TC_2 *with respect the* t_s *and* θ.

Table 1. Effect learning by holding cost on TC for the changed values of shipments (z) and parameter (p),

Number of shipments	0.2	0.4	0.6	0.8
1	314894	314894	314894	314894
2	314688	314508	314351	314214
3	314579	314326	314122	313959
4	314508	314214	313991	313822
5	314455	314135	313903	313735
6	314413	314076	313840	313675
7	314379	314029	313792	313630
8	314351	313951	313753	313596
9	314326	313959	313722	313569
10	314305	313931	313695	313546

The following corollaries give below are created with the help of Table 1 for the case I and Table 2 for case II:

(a) Increasing the value of p means that is the rate of learning effect increases, and noticed that the total Inventory cost decreases.

(b) Increasing the number of shipments, the value of total inventory cost decreases. Furthermore, we have illustrated below in Fig. 5 and Fig. 6.

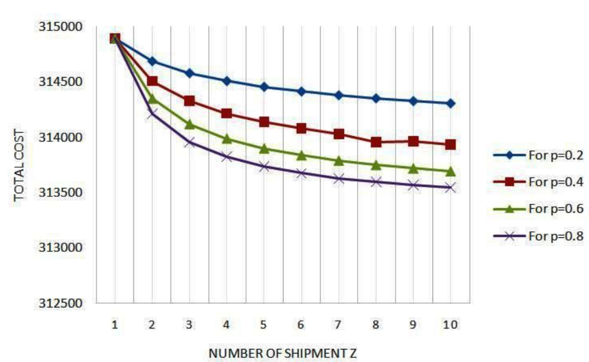

Figure 5. Effect learning of holding cost on TC

Table 2. Effect learning by holding cost on TC for the changed values of shipments (z) and parameter (p),

Number of shipments	0.2	0.4	0.6	0.8
1	136358	136358	136358	136358
2	136243	136132	135943	135943
3	136175	135928	135899	135800
4	136132	135943	135819	135716
5	136101	135902	135766	135714
6	135919	135871	135727	135627
7	135900	135843	135698	135599

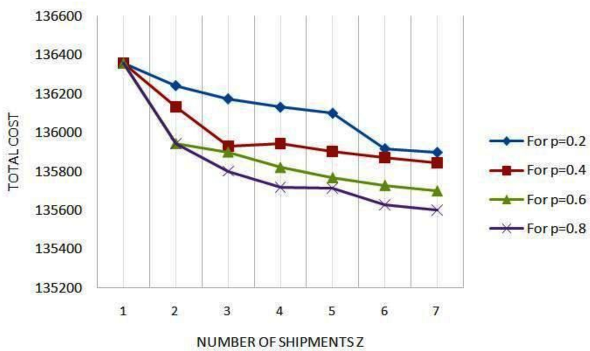

Figure 6. Effect learning of holding cost on TC

Table 3. Optimal results for the model with and without preservation technology for case I

	t_s^*	θ^*	$TC_1(t_s^*, \theta^*)$	N^*
With Preservation technology	4.49	3.08	314326	t 83648.4
Without Preservation technology	4.84	-	321686	52929.6

Table 4. Optimal results for the model with and without preservation technology for case II

	t_s^*	θ^*	$TC_1(t_s^*, \theta^*)$	N^*
With Preservation technology	2.28	69.89	135928	t 5056.6
Without Preservation technology	3.83	-	170437	7624.79

6. Sensitivity Analysis

In this section, we have seen some changes in the total cost by changing the parameters up and down slightly. This investigation performs a sensitivity analysis by changing each of the parameters by -15% $-10\%, -5\%, 0\%, +5\%, +10\%$, and $+15\%$. We have done a sensitivity analysis, for example 1 in this. Similarly, we can also provide sensitivity analysis for example 2. We have changed the percentage in one parameter in it and left the rest unchanged. And we have studied the effect of the total cost. The results are presented in Table (5) for Example 5.1.

Table 5. Percentage change in parameter

Varia -tion par -ameters	-15	-10	-5	0	5	10	15
v_1	287498	296927	305858	314326	322366	330008	337280
v_2	289734	298445	306629	314326	321574	328407	334855
v_3	314498	314444	314388	314324	314127	314045	313960
v_4	314291	314303	314314	314324	314338	314350	314361
ω_0	314324	314324	314324	314324	314324	314324	314324
α	314956	314747	314537	314324	314115	313904	313692
σ	314324	314324	314324	314324	314324	314324	314324
x	272542	286508	300435	314326	315186	342014	355816
y	310089	311533	312945	314326	315679	317006	318309
f_0	312894	313373	313851	314324	314800	315272	315743

The following corollaries can be prepared with the support of Table (5)

(a) Percentage change (PC) in holding cost, shortage cost, demand variable x, y and constant demand leads a positive change in the current value of the total cost.

(b) Percentage change (PC) in deterioration cost, opportunity cost and α variable lead a negative change in the current value of the total cost.

Furthermore, we have illustrated below in Fig. 7, Fig. 8 and Fig.9.

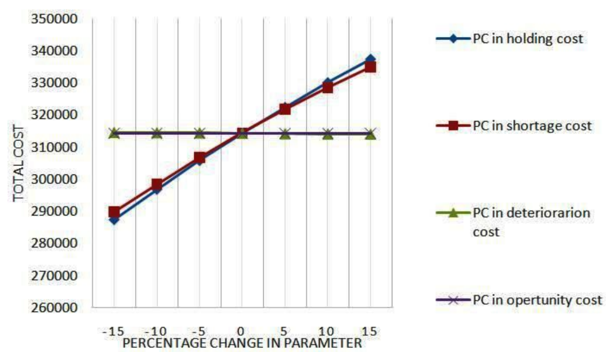

Figure 7. Effects of parameters on TC

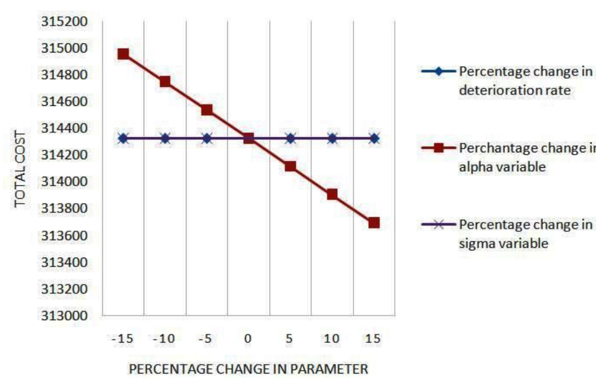

Figure 8. Effects of parameters on TC

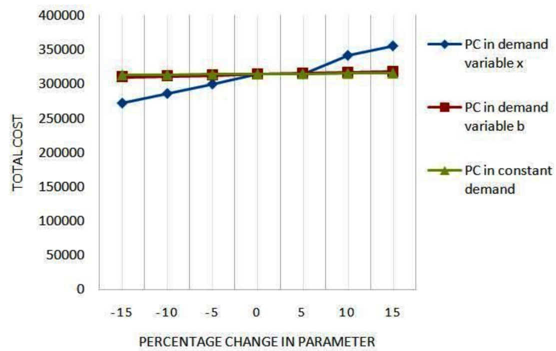

Figure 9. Effects of parameters on TC

7. Conclusion

In this model, we have shown how to protect the quality of the items spoiled in the international market due to COVID-19 and any other disaster environment with the help of investment in Preservation Technology. The perishable items in the market, like fruits, vegetables, and medicines, etc. can be curbed. These items are essential for human life. Due to not being spoiled by these items, the purity of the atmosphere also remains. We have used the learning effect in its holding cost, due to which total cost reduction was observed. In this, we have seen that as the rate of learning effect increases, also as the number of shipments increases, the total inventory cost decreases, shown in fig. 5 and fig. 6. Moreover, the convexity of the total cost functions has been established graphically with the help of Mathematica9.0 software. We invested in preservation technology in this model; with its use, the quality of deteriorating items has been saved. In view of this, the total cost of retailer decrease,which is shown in tables 3 and 4. Many avenues are also open for further research. In further, the model can be extended as follows, such as the use of various types of flexible demand, trade credit, and green supply chain.
In real life, the presented model can be used to control deteriorating items such as food items, medicines, and others.

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