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Initial coefficient estimates for subclasses of bi-univalent functions

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Abstract. The purpose of the present paper is to introduce a new subclasses of the function class Σ of normalized analytic and bi-univalent functions in the open disk $\mathbb{U}$. We obtain estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions of this subclasses.

AMS Subject Classifications: 30C45.

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Contents

1. Introduction

Let $\mathcal{A}$ denote the class of all analytic functions $f(z)$ of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \qquad (a_k \geq 0, n \in \mathbb{N} = \{1, 2, 3, ...\}) \tag{1.1}$$

which are analytic and univalent in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C}; |z| < 1\}.$$

We shall denote the class of all functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$ by $\mathcal{S}$, for details (see [9]; see also the work [7], [8], [17]). For $0 < q < 1$, we introduce the family of new functions defined as follows:

$$Q^q_\lambda(\beta) = \left\{ f \in \mathcal{A} : Re\left((1-\lambda)\frac{f(z)}{z} + \lambda D_q f(z) \right) > \beta, \beta < 1, \lambda \geq 0 \right\} \tag{1.2}$$

where D_q stands for q-derivative of the function $f(z)$ introduced by Jackson [14]. For $q \to 1^-$ it reduces to class of analytic function introduced by Ding et al. [6].

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For function $f(z) \in \mathcal{A}$ given by (1.1) and $0 < q < 1$, the q-derivative of a function $f(z)$ is defined by (also refer [12], [21])

$$D_q f(z) = \frac{f(qz) - f(z)}{(q-1)z}; \qquad (z \neq 0, q \neq 0), \tag{1.3}$$

from (1.3), we deduce that

$$D_q f(z) = 1 + \sum_{k=2}^{\infty} [k]_q a_k \, z^{k-1} \tag{1.4}$$

where

$$[k]_q = \frac{1-q^k}{1-q}. \tag{1.5}$$

It is well known that every function $f \in \mathcal{S}$ has a inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z, \quad (z \in \mathbb{U})$$

and

$$f^{-1}(f(w)) = w, \quad \left(|w| < r_0(f); r_0(f) \geq \frac{1}{4}\right)$$

where

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \dots .$$

A function $f(z) \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathbb{U}$. We denote by Σ the class of all functions $f(z)$ which are bi-univalent in $\mathbb{U}$ and are given by the Taylor-Maclaurin series expansion (1.1). The familiar Koebe function is not a member of Σ because it maps the unit disk $\mathbb{U}$ univalently onto the entire complex plane minus a slit along the line $-\frac{1}{4}$ to $-\infty$. Hence image domain does not contain in $\mathbb{U}$. A systematic study of the class Σ of bi-univalent function in $\mathbb{U}$, which is introduced in 1967 by Lewin [17]. Ever since then, several authors investigated various subclasses of the class Σ of bi-univalent functions. By using Grunsky inequalities Lewin showed that $|a_2| < 1.51$. Subsequently, Brannan and Clunie [4] conjectured that $|a_2| \leq \sqrt{2}$. Netanyahu [18], showed that $\max_{f \in \sigma} |a_2| = \frac{4}{3}$. In 1985 Branges [1] proved Bieberbach conjecture which state that, for each $f(z) \in \mathcal{S}$ given by Taylor-Maclaurin expansion (1.1) the following coefficient inequality holds true:

$$|a_n| \leq n; \quad (n \in \mathbb{N} - 1),$$

$\mathbb{N}$ being positive integer.

Brannan and Taha [6](see also [5]) introduce certain subclass of the bi-univalent function class Σ similar to the familiar subclasses $\mathcal{S}^*(\alpha)$ and $\mathcal{K}(\alpha)$ of starlike and convex functions of order $\alpha\,(0 < \alpha \leq 1)$, respectively. According to Brannan and Taha [6] (see also [3]) a function $f(z) \in \mathcal{A}$ is in the class $\mathcal{S}^*_\Sigma(\alpha)$ of strongly bi-univalent functions of order $\alpha\,(0 < \alpha \leq 1)$ if each of the following conditions is satisfied

$$f \in \Sigma \text{ and } \left| arg\left(\frac{zf'(z)}{f(z)}\right)\right| < \frac{\alpha\pi}{2}; \; (0 < \alpha \leq 1, z \in \mathbb{U})$$

and

$$\left| arg\left(\frac{wg'(w)}{g(w)}\right)\right| < \frac{\alpha\pi}{2}; \; (0 < \alpha \leq 1, w \in \mathbb{U}),$$

where g is the extension of f^{-1} in $\mathbb{U}$. Recently, several researchers such as (see [2, 11, 13, 15, 16, 19]) obtained coefficients $|a_2|$ and $|a_3|$ of bi-univalent functions for the various subclasses of the function class Σ. For a further historical amount of functions of class Σ, see the recent pioneering work by Srivastava et al. [22, 23]. The coefficient estimate problem involving the bound of $|a_n|(n \in \mathbb{N} \setminus \{1, 2\})$ for each $f \in \Sigma$ given by (1.1) is still an open problem.

The main aim of the present investigation is to introduce and study two new subclasses of the function class Σ and find estimates on the initial coefficients $|a_2|$ and $|a_3|$ for functions in these new subclasses of the function class Σ using q-differential operator.

2. Coefficient bounds for the function class $\mathcal{H}^q_\Sigma(\alpha, \lambda)$

We now introduce the following class of bi-univalent functions.

Definition 2.1. *: A function $f(z)$ given by (1.1) is said to be in the class $\mathcal{H}^q_\Sigma(\alpha, \lambda)$ if the following conditions satisfied:*

$$f \in \Sigma \text{ and } \left| arg\left((1-\lambda)\frac{f(z)}{z} + \lambda D_q f(z) \right) \right| < \frac{\alpha\pi}{2};\ (0 < \alpha \leq 1, \lambda \geq 1, z \in \mathbb{U}) \tag{2.1}$$

and

$$\left| arg\left((1-\lambda)\frac{g(w)}{w} + \lambda D_q g(w) \right) \right| < \frac{\alpha\pi}{2};\ (0 < \alpha \leq 1, \lambda \geq 1, w \in \mathbb{U}) \tag{2.2}$$

where the function g is given by

$$g(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots . \tag{2.3}$$

We note that for $\lambda = 1$ and $q \to 1^-$, the class $\mathcal{H}^q_\Sigma(\alpha, \lambda)$ reduces to the class $\mathcal{H}^\alpha_\Sigma$ introduced and studied by Srivastava et al. [24] and for $q \to 1^-$, the class $\mathcal{H}^q_\Sigma(\alpha, \lambda)$ reduces to the class $\mathcal{B}_\Sigma(\alpha, \lambda)$ introduced and studied by Frasin and Aouf [11]. We begin by finding the estimates on the coefficients $|a_2|$ and $|a_3|$ for function in the class $\mathcal{H}^q_\Sigma(\alpha, \lambda)$.

In order to derive our main results, we have to recall here the following lemma.

Lemma 2.2. *[9] If $p \in \mathcal{P}$ then $|c_k| \leq 2$ for each k, where $\mathcal{P}$ is the family of all functions p analytic in $\mathbb{U}$ for which $Re\{p(z)\} > 0$*

$$p(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + ... \qquad for\ z \in \mathbb{U}.$$

For functions in the class $\mathcal{H}^q_\Sigma(\alpha, \lambda)$ the following result is obtained.

Theorem 2.3. *Let $f(z)$ be given by (1.1) be in the function class $\mathcal{H}^q_\Sigma(\alpha, \lambda)$, $0 < \alpha \leq 1$; $0 < q < 1$ and $\lambda \geq 1$.Then*

$$|a_2| \leq \frac{2\alpha}{\sqrt{2(1-\lambda+[3]_q\lambda)\alpha + (1-\alpha)(1-\lambda+[2]_q\lambda)^2}} \tag{2.4}$$

and

$$|a_3| \leq \frac{4\alpha^2}{(1-\lambda+[2]_q\lambda)^2} + \frac{2\alpha}{(1-\lambda+[3]_q\lambda)}. \tag{2.5}$$

Proof: It follows from (2.1) and (2.2) that

$$(1-\lambda)\frac{f(z)}{z}+\lambda D_q f(z)=[p(z)]^{\alpha} \tag{2.6}$$

and

$$(1-\lambda)\frac{g(w)}{w}+\lambda D_q g(w)=[q(w)]^{\alpha} \qquad (z,w\in\mathbb{U}), \tag{2.7}$$

respectively, where

$$p(z)=1+p_1z+p_2z^2+...$$

$$q(w)=1+q_1w+q_2w^2+...$$

in $\mathcal{P}$. Now, upon equating the coefficients of z and z^2 in (2.6) and (2.7), we get

$$\left(1-\lambda+[2]_q\lambda\right)a_2=\alpha p_1, \tag{2.8}$$

$$\left(1-\lambda+[3]_q\lambda\right)a_3=\alpha p_2+\frac{\alpha(\alpha-1)}{2}p_1^2, \tag{2.9}$$

$$-\left(1-\lambda+[2]_q\lambda\right)a_2=\alpha q_1 \tag{2.10}$$

and

$$\left(1-\lambda+[3]_q\lambda\right)\left(2a_2^2-a_3\right)=\alpha q_2+\frac{\alpha(\alpha-1)}{2}q_1^2. \tag{2.11}$$

From (2.8) and (2.10), we obtain

$$p_1=-q_1 \tag{2.12}$$

and

$$2\left(1-\lambda+[2]_q\lambda\right)^2a_2^2=\alpha^2\left(p_1^2+q_1^2\right). \tag{2.13}$$

Also, from (2.9), (2.11) and (2.13), we find that

$$2\left(1-\lambda+[3]_q\lambda\right)a_2^2=\alpha\left(p_2+q_2\right)+\frac{(\alpha-1)\left(1-\lambda+[2]_q\lambda\right)^2a_2^2}{\alpha}. \tag{2.14}$$

Therefore, we obtain

$$a_2^2=\frac{\alpha^2}{2(1-\lambda+[3]_q\lambda)\alpha+(1-\alpha)(1-\lambda+[2]_q\lambda)^2}\left(p_2+q_2\right). \tag{2.15}$$

Applying lemma 2.2 for the coefficients p_2 and q_2, yields

$$|a_2|\leq\frac{2\alpha}{\sqrt{2(1-\lambda+[3]_q\lambda)\alpha+(1-\alpha)(1-\lambda+[2]_q\lambda)^2}}, \tag{2.16}$$

which gives desired estimate on $|a_2|$ as asserted in (2.4).

Next, in order to find the bound on $|a_3|$, we subtract (2.11) from (2.9), We thus get

$$2\left(1-\lambda+[3]_q\lambda\right)a_3=\alpha\left(p_2-q_2\right)+\frac{\alpha^2\left(p_1^2+q_1^2\right)\left(1-\lambda+[3]_q\lambda\right)}{\left(1-\lambda+[2]_q\lambda\right)^2}. \tag{2.17}$$

Applying lemma 2.2 for the coefficients p_1, q_1, p_2 and q_2 in above equality, we get

$$|a_3|\leq\frac{4\alpha^2}{(1-\lambda+[2]_q\lambda)^2}+\frac{2\alpha}{(1-\lambda+[3]_q\lambda)}. \tag{2.18}$$

This completes the proof.

If we choose $\lambda=1$ and $q\to1^-$ in Theorem 2.3, we have the following result.

Corollary 2.4. *([24]). Let $f(z)$ given by (1.1) be in the class $\mathcal{H}_{\Sigma}^{\alpha}$, $(0<\alpha\leq 1)$. Then*

$$|a_2| \leq \alpha\sqrt{\frac{2}{2+\alpha}} \tag{2.19}$$

and

$$|a_3| \leq \frac{\alpha(3\alpha+2)}{3}. \tag{2.20}$$

If we take $q \to 1^-$ in Theorem 2.3, we have the following result.

Corollary 2.5. *([11]). Let $f(z)$ given by (1.1) be in the class $\mathcal{B}_{\Sigma}(\alpha,\lambda)$, $(0<\alpha\leq 1)$ and $(\lambda \geq 1)$. Then*

$$|a_2| \leq \frac{2\alpha}{\sqrt{(\lambda+1)^2+\alpha(1+2\lambda-\lambda^2)}} \tag{2.21}$$

and

$$|a_3| \leq \frac{4\alpha^2}{(\lambda+1)^2} + \frac{2\alpha}{2\lambda+1}. \tag{2.22}$$

3. Coefficient bounds for the function class $\mathcal{H}_{\Sigma}^{q}(\beta,\lambda)$

We now introduce the following class of bi-univalent functions.

Definition 3.1. *: A function $f(z)$ given by (1.1) is said to be in the class $\mathcal{H}_{\Sigma}^{q}(\beta,\lambda)$ if the following conditions satisfied:*

$$f \in \Sigma \text{ and } Re\left((1-\lambda)\frac{f(z)}{z} + \lambda D_q f(z)\right) > \beta;\ (0 \leq \beta < 1, \lambda \geq 1, z \in \mathbb{U}) \tag{3.1}$$

and

$$Re\left((1-\lambda)\frac{g(w)}{w} + \lambda D_q g(w)\right) > \beta;\ (0 \leq \beta < 1, \lambda \geq 1, w \in \mathbb{U}). \tag{3.2}$$

For functions in the class $\mathcal{H}_{\Sigma}^{q}(\beta,\lambda)$ the following result is obtained.

Theorem 3.2. *Let $f(z)$ be given by (1.1) be in the function class $\mathcal{H}_{\Sigma}^{q}(\beta,\lambda)$, $0 \leq \beta < 1$; $0 < q < 1$ and $\lambda \geq 1$.Then*

$$|a_2| \leq \min\left\{\frac{2(1-\beta)}{(1-\lambda+[2]_q\lambda)}, \sqrt{\frac{2(1-\beta)}{(1-\lambda+[3]_q\lambda)}}\right\} \tag{3.3}$$

and

$$|a_3| \leq \min\left\{\frac{2(1-\beta)}{(1-\lambda+[3]_q\lambda)}, \frac{4(1-\beta)^2}{(1-\lambda+[2]_q\lambda)^2} + \frac{2(1-\beta)}{(1-\lambda+[3]_q\lambda)}\right\}. \tag{3.4}$$

Proof: It follows from (3.1) and (3.2) that

$$(1-\lambda)\frac{f(z)}{z}+\lambda D_q f(z)=\beta+(1-\beta)p(z) \tag{3.5}$$

and

$$(1-\lambda)\frac{g(w)}{w}+\lambda D_q g(w)=\beta+(1-\beta)q(w) \qquad (z,w\in\mathbb{U}), \tag{3.6}$$

respectively, where

$$p(z)=1+p_1z+p_2z^2+...$$
$$q(w)=1+q_1w+q_2w^2+...$$

in $\mathcal{P}$. Now, upon equating the coefficients of (3.5) and (3.6), we obtain

$$(1-\lambda+[2]_q\lambda)\,a_2=(1-\beta)p_1, \tag{3.7}$$

$$(1-\lambda+[3]_q\lambda)\,a_3=(1-\beta)p_2, \tag{3.8}$$

$$-(1-\lambda+[2]_q\lambda)\,a_2=(1-\beta)q_1 \tag{3.9}$$

and

$$(1-\lambda+[3]_q\lambda)\left(2a_2^2-a_3\right)=(1-\beta)q_2. \tag{3.10}$$

From (3.7) and (3.9), we obtain

$$p_1=-q_1 \tag{3.11}$$

and

$$2\left(1-\lambda+[2]_q\lambda\right)^2a_2^2=(1-\beta)^2\left(p_1^2+q_1^2\right). \tag{3.12}$$

Also, from (3.8) and (3.10), we have

$$2\left(1-\lambda+[3]_q\lambda\right)a_2^2=(1-\beta)\left(p_2+q_2\right). \tag{3.13}$$

Applying lemma 2.2 for (3.12) and (3.13), we get

$$|a_2|\le\min\left\{\frac{2(1-\beta)}{(1-\lambda+[2]_q\lambda)},\sqrt{\frac{2(1-\beta)}{(1-\lambda+[3]_q\lambda)}}\right\}, \tag{3.14}$$

we get desired estimate on $|a_2|$ as asserted in (3.3).

Next, in order to find the bound on $|a_3|$, we subtract (3.10) and (3.8), we get

$$2(1-\lambda+[3]_q\lambda)\,a_3=(1-\beta)\left(p_2-q_2\right)+2(1-\lambda+[3]_q\lambda)\,a_2^2, \tag{3.15}$$

which, upon substitution of the value of a_2^2 from (3.12), yields

$$|a_3|=\frac{(1-\beta)^2}{2(1-\lambda+[2]_q\lambda)^2}(p_1^2+q_1^2)+\frac{(1-\beta)}{(1-\lambda+[3]_q\lambda)}(p_2-q_2). \tag{3.16}$$

On the other hand, by using (3.13) into (3.15), it follows that

$$|a_3|=\frac{(1-\beta)}{2(1-\lambda+[3]_q\lambda)^2}\,p_2. \tag{3.17}$$

Applying lemma 2.2 for (3.16) and (3.17), yields

$$|a_3|\le\min\left\{\frac{2(1-\beta)}{(1-\lambda+[3]_q\lambda)},\frac{4(1-\beta)^2}{(1-\lambda+[2]_q\lambda)^2}+\frac{2(1-\beta)}{(1-\lambda+[3]_q\lambda)}\right\}. \tag{3.18}$$

This completes the proof.

The next Corollary can be easily obtained from Theorem 3.2.

Corollary 3.3. . *Let $f(z)$ given by (1.1) be in the class $\mathcal{H}_{\Sigma}^{\alpha}$, $0 \leq \beta < 1$. Then*

$$|a_2| = \begin{cases} \sqrt{\frac{2(1-\beta)}{3}}, & \text{for } 0 \leq \beta \leq 1/3 \\ 1-\beta, & \text{for } 1/3 \leq \beta < 1 \end{cases} \tag{3.19}$$

and

$$|a_3| \leq \frac{2(1-\beta)}{3}. \tag{3.20}$$

Remark 3.4. *Corollary (3.3) provides an improvement for the estimates obtained by Srivastava et al. ([24]).*

Corollary 3.5. *([24]). Let $f(z)$ given by (1.1) be in the class $\mathcal{H}_{\Sigma}^{\beta}$, $(0 \leq \beta < 1)$. Then*

$$|a_2| \leq \sqrt{\frac{2(1-\beta)}{3}} \tag{3.21}$$

and

$$|a_3| \leq \frac{(1-\beta)(5-3\beta)}{3}. \tag{3.22}$$

If we choose $q \to 1^-$ in Theorem 3.2, we have the following result.

Corollary 3.6. *([11]). Let $f(z)$ given by (1.1) be in the class $\mathcal{B}_{\Sigma}(\beta, \lambda)$, $(0 \leq \beta < 1)$ and $(\lambda \geq 1)$. Then*

$$|a_2| \leq \sqrt{\frac{2(1-\beta)}{2\lambda+1}} \tag{3.23}$$

and

$$|a_3| \leq \frac{4(1-\beta)^2}{(\lambda+1)^2} + \frac{2(1-\beta)}{2\lambda+1}. \tag{3.24}$$

Remark 3.7. . *For $\lambda = 1$ the results obtained in this paper are coincides with the results discussed in ([2]).*

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