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Neutrosophic g#S closed sets in neutrosophic topological spaces

A. Atkinswestley[1]* and S. Chandrasekar[2]

Abstract

Aim of this present paper is, we introduced and studied new type of generalized closed sets is called Neutrosophic g#S closed sets and Neutrosophic g#S open sets and followed that its properties and application are also discussed.

Keywords

Neutrosophic g#S closed sets, Neutrosophic g#S open sets, Neutrosophic g#S interior, Neutrosophic g#S interior, Neutrosophic topological spaces.

AMS Subject Classification

03E72.

[1] *Department of Mathematics, Roever College of Engineering and Technology, Elambalur- 621212, Tamil Nadu, India.*

[2] *PG and Research Department of Mathematics, Arignar Anna Government Arts College, Namakkal-637002, Tamil Nadu, India.*

***Corresponding author**: [1] ats.wesly@gmail.com; [2]chandrumat@gmail.com

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Contents

1. Introduction

Neutrosophic sets and system has been developed from intuitionistic fuzzy system with three components T Truth, F-Falsehood, I-Indeterminacy. This concepts developed by Floretin Smarandache [10]. Neutrosophic sets and system have wide range of applications in the field of decision making, Artificial Intelligence, Information Systems, Computer Science, Applied Mathematics, Mechanics, Medicine, Management Science Electrical & Electronic, etc., New kind of open sets and closed sets are introduced every year in Topological spaces, Neutrospohic Topological spaces introduced by A.A. Salama [22] et al., R. Dhavaseelan [7] introduced Neutrosophic generalized closed sets. Neutrosophic semi closed sets are introduced by P. Ishwarya, [11] et al, V.K. Shanthi [23] et al., introduced Neutrosophic generalized semi closed sets. Aim of this paper is we introduce and study about Neutrosophic g#S closed sets and Neutrosophic g#S open sets in Neutrosophic topological spaces and its properties and Characterization are discussed details.

2. Preliminaries

In this section, we recall needed basic definition and operation of Neutrosophic sets and its fundamental Results.

Definition 2.1 ([10]). *Let X be a non-empty fixed set. A Neutrosophic set A_1^* is aobaect having the form*

$$A_1^*=\{<x,\mu_{A_1^*}(x),\sigma_{A_1^*}(x),\gamma_{A_1^*}(x)>: x\in X\},$$

$\mu_{A_1^*}(x)$-represents the degree of membership function
$\sigma_{A_1^*}(x)$-represents degree indeterminacy and then
$\gamma_{A_1^*}(x)$-represents the degree of non-membership function.

Definition 2.2 ([10]). *Neutrosophic set $A_1^*=\{<x,\mu_{A_1^*}(x),\sigma_{A_1^*}(x),\gamma_{A_1^*}(x)>: x\in X\}$, on X and $\forall x\in X$ then complement of A_1^* is,*

$$A_1^{*C}=\{<x,\gamma_{A_1^*}(x),1-\sigma_{A_1^*}(x),\mu_{A_1^*}(x)>: x\in X\}.$$

Definition 2.3 ([10]). *Let A_1^* and A_2^* are two Neutrosophic sets, $\forall x\in X$*

$$A_1^*=\{<x,\mu_{A_1^*}(x),\sigma_{A_1^*}(x),\gamma_{A_1^*}(x)>: x\in X\}$$

$A_2^*=\{<x,\mu_{A_2^*}(x),\sigma_{A_2^*}(x),\gamma_{A_2^*}(x)>: x\in X\}$.

Then

$$A_1^*\subseteq A_2^*\Leftrightarrow \mu_{A_1^*}(x)\leq\mu_{A_2^*}(x),\quad \sigma_{A_1^*}(x)\leq\sigma_{A_2^*}(x)\ \&\ \gamma_{A_1^*}(x)\geq\gamma_{A_2^*}(x)\}.$$

Definition 2.4 ([10]). *Let X be a non-empty set, and Let A_1^* and A_2^* be two Neutrosophic sets are*

$$A_1^*=\{<x,\mu_{A_1^*}(x),\sigma_{A_1^*}(x),\gamma_{A_1^*}(x)>: x\in X\},$$
$$A_2^*=\{<x,\mu_{A_2^*}(x),\sigma_{A_2^*}(x),\gamma_{A_2^*}(x)>: x\in X\}$$

Then

1. $A_1^*\cap A_2^*=\Big\{<x,\mu_{A_1^*}(x)\cap\mu_{A_2^*}(x),\sigma_{A_1^*}(x)\cap\sigma_{A_2^*}(x),\ \gamma_{A_1^*}(x)\cup\gamma_{A_2^*}(x)>: x\in X\Big\}$
2. $A_1^*\cup A_2^*=\Big\{<x,\mu_{A_1^*}(x)\cup\mu_{A_2^*}(x),\sigma_{A_1^*}(x)\cup\sigma_{A_2^*}(x),\ \gamma_{A_1^*}(x)\cap\gamma_{A_2^*}(x)>: x\in X\Big\}$.

Definition 2.5 ([22]). *Let X be non-empty set and τ_N be the collection of Neutrosophic subsets of X satisfying the following properties:*

1. $0_N, 1_N\in\tau_N$
2. $T_1\cap T_2\in\tau_N$ *for any* $T_1,T_2\in\tau_N$
3. $\cup T_i\in\tau_N$ *for every* $\{T_i: i\in J\}\subseteq\tau_N$

Then the space (X,τ_N) is called a Neutrosophic topological space(N-T-S). The element of τ_N are called N.OS (Neutrosophic open set) and its complement is N.CS (Neutrosophic closed set)

Example 2.6. *Let $X=\{x\}$ and $\forall x\in X$*

$$A_1=\left\langle x,\frac{6}{10},\frac{6}{10},\frac{5}{10}\right\rangle, A_2=\left\langle x,\frac{5}{10},\frac{7}{10},\frac{9}{10}\right\rangle$$

$$A_3=\left\langle x,\frac{6}{10},\frac{7}{10},\frac{5}{10}\right\rangle, A_4=\left\langle x,\frac{5}{10},\frac{6}{10},\frac{9}{10}\right\rangle$$

Then the collection $\tau_N=\{0_N,A_1,A_2,A_3,A_4,1_N\}$ is called a N-T-S on X.

Definition 2.7 ([2,7,11,12,23,24]). *Let (X,τ_N)be a N-T-S and*

$$A_1^*=\{<x,\mu_{A_1^*}(x),\sigma_{A_1^*}(x),\gamma_{A_1^*}(x)>: x\in X\}$$

be a Neutrosophic set in X. Then A_1^* is said to be

1. *Neutrosophic α-closed set $(N.\alpha CS)$ if $N.cl(N.int(N.cl(A_1^*)))\subseteq A_1^*$,*
2. *Neutrosophic semi closed set (N.SCS) if $N.int(N.cl(A_1^*))\subseteq A_1^*$,*
3. *Neutrosophic β-closed set $(N.\beta CS)$ if $N.int(N.cl(N.int(A_1^*)))\subseteq A_1^*$,*
4. *Neutrosophic generalized closed set (N.GCS) if $N.cl(A_1^*)\subseteq H$ whenever $A_1^*\subseteq H$ and H is a N.OS,*
5. *Neutrosophic α generalized closed set $(N.\alpha GCS)$ if $Neu\ \alpha cl(A_1^*)\subseteq H$ whenever $A_1^*\subseteq H$ and H is a N.OS,*
6. *Neutrosophic generalized semi closed set (N.GSCS) if $N.Scl(A_1^*)\subseteq H$ whenever $A_1^*\subseteq H$ and H is a N.OS,*
7. *Neutrosophic generalized β closed set $(N.G\beta)$ if $N.\beta cl(A_1^*)\subseteq H$ whenever $A_1^*\subseteq H$ and H is a N.OS.*

3. Neutrosophic g#S closed sets

Definition 3.1. *A Neutrosophic set A_1^* of a Neutrosophic topological space (X,τ_N) is called Neutrosophic $g^{\#}$ semi closed set (briefly $\mathscr{N}.g^{\#}SCS$) if $\mathscr{N}.Scl\ (A_1^*)\subseteq U$ whenever $A_1^*\subseteq U$ and U is $\mathscr{N}.\alpha g$-open set in (X,τ_N).*

Example 3.2. *Let $X=\{a_1,a_2\}$, $\tau_N=\{0_N, A_1^*, 1\}$, is a $N.T.$ on X where $A_1^*=\langle x,(\frac{2}{10},\frac{5}{10},\frac{8}{10}),(\frac{3}{10},\frac{5}{10},\frac{7}{10})\rangle$. Then the Neutrosophic set $A_2^*=\langle x,(\frac{7}{10},\frac{5}{10},\frac{3}{10}),(\frac{6}{10},\frac{5}{10},\frac{4}{10})\rangle$is a $\mathscr{N}.g^{\#}S$-in X.*

Theorem 3.3. *Every $\mathscr{N}$ closed set is $\mathscr{N}.g^{\#}SCS$.*

Proof. Let A_1^* be a $\mathscr{N}$ closed set in a $\mathscr{N}\mathscr{T}\mathscr{S}$ X. Let $A_1^*\subseteq U$, where U is $\mathscr{N}.\ \alpha gOS$ in X. Since A_1^* is closed, we have

$$\mathscr{N}.cl(A_1^*)=\ \mathscr{N}.Scl(A_1^*)=A_1^*\subseteq U.$$

That is $\mathscr{N}.Scl(A_1^*)\subseteq U$. Hence A_1^* is $\mathscr{N}.g^{\#}SCS$. □

Example 3.4. *Let $X=\{a_1,a_2\}$, $\tau_N=\{0_N, A_1^*, 1\}$, is a $N.T.$ on X where $A_1^*=\langle x,(\frac{4}{10},\frac{5}{10},\frac{8}{10}),(\frac{6}{10},\frac{5}{10},\frac{7}{10})\rangle$. Then the Neutrosophic set $A_2^*=\langle x,(\frac{7}{10},\frac{5}{10},\frac{8}{10}),(\frac{3}{10},\frac{5}{10},\frac{2}{10})\rangle$ is $\mathscr{N}.g^{\#}S$-in X but not $\mathscr{N}$ closed set.*

Theorem 3.5. *Every $\mathscr{N}SCS$ is $\mathscr{N}.g^{\#}SCS$.*

Proof. Let A_1^* be a $\mathscr{N}SCS$ in a $\mathscr{N}\mathscr{T}\mathscr{S}X$. Let $A_1^*\subseteq U$, where U is $\mathscr{N}.\alpha gOS$ in X. Since A_1^* is $\mathscr{N}.SCS$, we have $\mathscr{N}.Scl(A_1^*)=A_1^*\subseteq U$. That is $\mathscr{N}.Scl(A_1^*)\subseteq U$. Hence A_1^* is $\mathscr{N}.g^{\#}SCS$. □

Example 3.6. *Let $X=\{a_1,a_2\}$, $\tau_N=\{0_N, A_1^*, 1\}$, is a $N.T.$ on X where $A_1^*=\langle x,(\frac{2}{10},\frac{5}{10},\frac{8}{10}),(\frac{3}{10},\frac{5}{10},\frac{7}{10})\rangle$. Then the Neutrosophic set $A_2^*=\langle x,(\frac{7}{10},\frac{5}{10},\frac{3}{10}),(\frac{6}{10},\frac{5}{10},\frac{4}{10})\rangle$ is a $\mathscr{N}.g^{\#}SCS$ in X but not $\mathscr{N}.SCS$.*

Theorem 3.7. *Every $\mathscr{N}.\alpha CS$ is $\mathscr{N}.g^{\#}SCS$.*

Proof. Let A_1^* be a $\mathscr{N}.\alpha CS$ in a $\mathscr{N}\mathscr{T}\mathscr{S}X$. Let $A_1^*\subseteq U$, Where U is $\mathscr{N}.\alpha g$ open set in X. Since A_1^* is $\mathscr{N}.\alpha CS$, we have $\mathscr{N}.Scl(A_1^*)=A_1^*\subseteq U$. That is $\mathscr{N}.Scl(A_1^*)\subseteq U$. Hence A_1^* is $\mathscr{N}.g^{\#}SCS$. □

Example 3.8. *Let $X=\{a_1, a_2\}$, $\tau_N=\{0_N, A_1^*, 1\}$, is a N.T. on X where $A_1^*=\langle x, (\frac{4}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{5}{10})\rangle$. Then the Neutrosophic set $A_2^*=\langle x, (\frac{8}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{4}{10})\rangle$ is $\mathcal{N}.g^{\#}SCS$ in X but not $\mathcal{N}.\alpha CS$.*

Theorem 3.9. *Every $\mathcal{N}.g^{\#}SCS$ is $\mathcal{N}.gSCS$ in $\mathcal{N}\mathcal{T}\mathcal{S}X$.*

Proof. Let A_1^* be a $\mathcal{N}.g^{\#}SCS$ in X. Let $A_1^*\subseteq U$, where U is $\mathcal{N}$ open and so $\mathcal{N}\alpha g$ open set. Since A_1^* is $\mathcal{N}.g^{\#}SCS$, we have

$$\mathcal{N}.Scl(A_1^*)\subseteq\mathcal{N}.Scl(A_1^*)\subseteq A_1^*\subseteq U.$$

That is $\mathcal{N}.Scl(A_1^*)\subseteq U$ and hence A_1^* is $\mathcal{N}.gSCS$. □

Example 3.10. *Let $X=\{a_1, a_2\}$, $\tau_N=\{0_N, A_1^*, 1_N1\}$, is a N.T. on X where $A_1^*=\langle x, (\frac{2}{10}, \frac{5}{10}, \frac{8}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{7}{10})\rangle$. Then the Neutrosophic set $A_2^*=\langle x, (\frac{7}{10}, \frac{5}{10}, \frac{3}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{4}{10})\rangle$ is a $\mathcal{N}.gSCS$ in X. But not $\mathcal{N}.g^{\#}SCS$.*

Theorem 3.11. *Every $\mathcal{N}.g^{\#}SCS$ is $\mathcal{N}.g\beta CS$ in $\mathcal{N}\mathcal{T}\mathcal{S}X$.*

Proof. Let A_1^* be a $\mathcal{N}.g^{\#}SCS$ in X. Let $A_1^*\subseteq U$, where U is $\mathcal{N}$ open set and so $\mathcal{N}.\alpha g$ open set. Since A_1^* is $\mathcal{N}.g^{\#}SCS$, we have $\mathcal{N}.Scl(A_1^*)\subseteq U$. Therefore

$$\mathcal{N}.Scl(A_1^*)\subseteq\mathcal{N}.g\beta cl(A_1^*)\subseteq A_1^*\subseteq U.$$

That is $\mathcal{N}.g\beta cl(A_1^*)\subseteq U$. And hence A_1^* is $\mathcal{N}.g\beta CS$. □

Example 3.12. *Let $X=\{a_1, a_2, a_3\}$, $\tau_N=\{0_N, A_1^*, 1_N\}$, is a N.T. on X where*

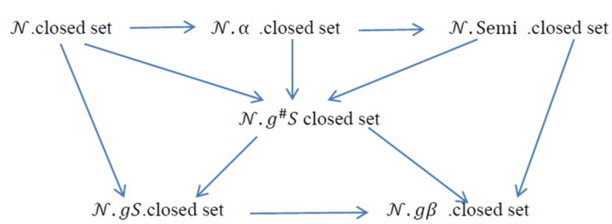
$$A_1^*=\left\langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right)\right\rangle.$$

Then the Neutrosophic set

$$A_2^*=\left\langle x, \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right)\right\rangle$$

is a $\mathcal{N}.g\beta CS$ in X. But not $\mathcal{N}.g^{\#}SCS$.

Remark 3.13. *The following diagram shows the relationships of $g^{\#}$ semi closed sets with some other Neutrosophic sets.*



Theorem 3.14. *In a $\mathcal{N}\mathcal{T}\mathcal{S}$ X, if a Neutrosophic set A_1^* is both $\mathcal{N}.\alpha gOS$ and $\mathcal{N}.g^{\#}SCS$, then A_1^* is $\mathcal{N}.SCS$.*

Proof. Suppose a Neutrosophic set A_1^* of a $\mathcal{N}\mathcal{T}\mathcal{S}$ X is both $\mathcal{N}.\alpha gOS$ and $\mathcal{N}.g^{\#}SCS$. Now $A_1^*\subseteq A_1^*$, where A_1^* is $\mathcal{N}.$ open set and so A_1^* is $\mathcal{N}.\alpha gOS$. This implies that $\mathcal{N}.Scl(A_1^*)\subseteq A_1^*$, since A_1^* is $\mathcal{N}.g^{\#}SCS$. Also, we have $A_1^*\subseteq\mathcal{N}.Scl(A_1^*)$, which implies $\mathcal{N}.Scl(A_1^*)=A_1^*$. Hence A_1^* is $\mathcal{N}.SCS$. □

Theorem 3.15. *If a Neutrosophic set A_1^* is $\mathcal{N}.g^{\#}SCS$ in X such that $A_1^*\subseteq A_2^*\subseteq\mathcal{N}.Scl(A_1^*)$, then A_2^* is also a $\mathcal{N}.g^{\#}SCS$ in X.*

Proof. Let U be an $\mathcal{N}.\alpha gOS$ in X, such that $A_2^*\subseteq U$, then $A_1^*\subseteq U$. Since A_1^* is $\mathcal{N}.g^{\#}SCS$, then by definitions $\mathcal{N}.Scl(A_1^*)\subseteq U$. Now $A_2^*\subseteq\mathcal{N}.Scl(A_1^*)$,

$$\mathcal{N}.Scl(A_2^*)\subseteq\mathcal{N}.Scl(A_1^*))=\mathcal{N}.Scl(A_1^*)\subseteq U.$$

That is $\mathcal{N}.Scl(A_2^*)\subseteq U$. Hence A_2^* is a $\mathcal{N}.g^{\#}SCS$. □

Theorem 3.16. *A Finite union of $\mathcal{N}.g^{\#}SCS$ is a $\mathcal{N}.g^{\#}SCS$.*

Proof. Let A_1^* and A_2^* be $\mathcal{N}.g^{\#}SCS$s in a $\mathcal{N}\mathcal{T}\mathcal{S}X$. To prove that $A_1^*\cup A_2^*$ is $\mathcal{N}.g^{\#}SCS$. Let $A_1^*\cup A_2^*\subseteq U$, where U be $\mathcal{N}.\alpha gOS$. Then $A_1^*\subseteq U$, $A_2^*\subseteq U$ and so $\mathcal{N}.Scl(A_1^*)\subseteq U$ and $\mathcal{N}.Scl(A_2^*)\subseteq U$ since A_1^* and A_2^* are $\mathcal{N}.g^{\#}SCS$s. This implies that

$$\mathcal{N}.Scl(A_1^*)\cup\mathcal{N}.Scl(A_2^*)\subseteq U,$$

and so $\mathcal{N}.Scl(A_1^*\cup A_2^*)\subseteq U$. Hence $A_1^*\cup A_2^*$ is $\mathcal{N}.g^{\#}SCS$. Thus a finite union of $\mathcal{N}.g^{\#}SCS$ is a $\mathcal{N}.g^{\#}SCS$. We introduce $\mathcal{N}.g^{\#}SOS$. □

4. Neutrosophic g#S open sets

Definition 4.1. *A Neutrosophic set A_1^* of a $\mathcal{N}\mathcal{T}\mathcal{S}(X,\tau_N)$ is called$\mathcal{N}.g^{\#}S$ open set (briefly $\mathcal{N}.g^{\#}SOS$) if its complement A_1^{*C} is $\mathcal{N}.g^{\#}SCS$.*

Theorem 4.2. *A Neutrosophic set A_1^* of a$\mathcal{N}\mathcal{T}\mathcal{S}$ X is $\mathcal{N}.g^{\#}SOS$ iff $\mathcal{F}\subseteq\mathcal{N}.Sint\ (A_1^*)$, whenever $\mathcal{F}$ is $\mathcal{N}.\alpha gCS$ and $\mathcal{F}\subseteq A_1^*$.*

Proof. Suppose A_1^* is $\mathcal{N}.g^{\#}SOS$. Then A_1^{*C} is $\mathcal{N}.g^{\#}SCS$. Let $\mathcal{F}$ be $\mathcal{N}.\alpha gCS$ in X and $\mathcal{F}\subseteq A_1^*$. Then $A_1^{*C}\subseteq\mathcal{F}^C$, $\mathcal{F}^C$ is $\mathcal{N}.\alpha gOS$. Since A_1^{*C} is $\mathcal{N}.g^{\#}SCS$, we have $\mathcal{N}.Scl(A_1^{*C})\subseteq\mathcal{F}^C$, which implies $\mathcal{F}\subseteq\mathcal{N}.Sint(A_1^*)$. As $\mathcal{N}.Scl(A_1^{*C})=(\mathcal{N}.Sint\ (A_1^*))^C$.

Conversely, assume that $\mathcal{F}\subseteq\mathcal{N}.Sint(A_1^*)$, whenever $\mathcal{F}\subseteq A_1^*$ and $\mathcal{F}$ is $\mathcal{N}.\alpha gCS$ in a $\mathcal{N}\mathcal{T}\mathcal{S}X$. Let $A_1^{*C}\subseteq G$, where G is $\mathcal{N}.\alpha gOS$ in X. Then $G^C\subseteq A_1^*$, where G^C is $\mathcal{N}.\ \alpha gCS$, which implies that $G^C\subseteq\mathcal{N}.Sint(A_1^*)$, implies that $(\mathcal{N}.Sint\ (A_1^*))^C\subseteq(G^C)^C$. That is $\mathcal{N}.Scl(A_1^{*C})\subseteq G$. Hence A_1^{*C} is $\mathcal{N}.g^{\#}SCS$ and so A_1^* is $\mathcal{N}.g^{\#}SOS$. □

Theorem 4.3. *Every $\mathcal{N}.$ open set is a$\mathcal{N}.g^{\#}SOS$.*

Proof. Let A_1^* be a$\mathcal{N}$.open set in a $\mathcal{N}\mathcal{T}\mathcal{S}$ X. Then$A_1^{*C}$is $\mathcal{N}.$ closed set. And so A_1^{*C} is $\mathcal{N}.g^{\#}$SCS. Hence A_1^* is $\mathcal{N}.g^{\#}$SOS. □

Example 4.4. *Let $X=\{a_1, a_2\}$, $\tau_N=\{0_N, A_1^*, 1_N\}$, is a N.T. on X where*

$$A_1^*=\left\langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{3}{10}\right)\right\rangle.$$

Then the Neutrosophic set

$$A_2^*=\left\langle x,\left(\frac{0}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{1}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{2}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle,$$

the Neutrosophic set A_2^* is $\mathscr{N}.g^{\#}SOS$ but not a $\mathscr{N}$ open set in X.

Theorem 4.5. *Every $\mathscr{N}.S$ open set is a $\mathscr{N}.g^{\#}SOS$.*

Proof. Let A_1^* be a $\mathscr{N}.S$ open set in a $\mathscr{N}\mathscr{T}\mathscr{S}X$. Then A_1^{*C} is $\mathscr{N}.SCS$. And so A_1^{*C} is $\mathscr{N}.g^{\#}SCS$. Hence A_1^* is $\mathscr{N}.g^{\#}SOS$. ☐

Example 4.6. *Let $X=\{a_1,\ a_2,a_3\}$, $\tau_N=\{0_N,\ A_1^*,\ 1_N\}$, is a $N.T.$ on X where*

$$A_1^*=\left\langle x,\left(\frac{3}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{2}{10}\right)\right\rangle.$$

Then the Neutrosophic set

$$A_2^*=\left\langle x,\left(\frac{0}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{1}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{2}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$

$\mathscr{N}.g^{\#}SOS$. But not a $\mathscr{N}.SOS$ in X.

Theorem 4.7. *Every $\mathscr{N}.\alpha OS$ is a $\mathscr{N}.g^{\#}SOS$.*

Proof. Let A_1^* be a $\mathscr{N}.\alpha OS$ in a $\mathscr{N}\mathscr{T}\mathscr{S}X$. Then A_1^{*C} is $\mathscr{N}.\alpha CS$. And so A_1^{*C} is $\mathscr{N}.g^{\#}SCS$. Hence A_1^* is $\mathscr{N}.g^{\#}SOS$. ☐

Example 4.8. *Let $X=\{a_1,\ a_2,a_3\}$, $\tau_N=\{0_N,\ A_1^*,\ 1_N\}$, is a $N.T.$ on X where*

$$A_1^*=\left\langle x,\left(\frac{4}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{7}{10},\frac{5}{10},\frac{3}{10}\right)\right\rangle.$$

Then the Neutrosophic set

$$A_2^*=\left\langle x,\left(\frac{0}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{1}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{2}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$

is $\mathscr{N}.g^{\#}SOS$ but not a $\mathscr{N}.\alpha OS$ in X.

Theorem 4.9. *In a $\mathscr{N}\mathscr{T}\mathscr{S}X$, every $\mathscr{N}.g^{\#}SOS$ is $\mathscr{N}.gSOS$.*

Proof. Let A_1^* be$\mathscr{N}.g^{\#}SOS$ in a $\mathscr{N}\mathscr{T}\mathscr{S}X$. Then A_1^{*C} is $\mathscr{N}.g^{\#}SCS$ in X. And so A_1^{*C} is $\mathscr{N}.gSCS$. That is A_1^* is $\mathscr{N}.gSOS$ in X. ☐

Example 4.10. *Let $X=\{a_1,\ a_2\}$, $\tau_N=\{0_N, A_1^*,\ 1_N\}$, is a $N.T.$ on X where*

$$A_1^*=\left\langle x,\left(\frac{2}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{7}{10}\right)\right\rangle.$$

Then the Neutrosophic set

$$A_2^*=\left\langle x,\left(\frac{3}{10},\frac{5}{10},\frac{7}{10}\right),\left(\frac{4}{10},\frac{5}{10},\frac{6}{10}\right)\right\rangle$$

is a $\mathscr{N}.gSOS$ in X. But not $\mathscr{N}.g^{\#}SOS$.

Theorem 4.11. *In a $\mathscr{N}\mathscr{T}\mathscr{S}$ X, every $\mathscr{N}.g^{\#}SOS$ is $\mathscr{N}.g\beta OS$.*

Proof. Let A_1^* be $\mathscr{N}.g^{\#}SOS$ in a $\mathscr{N}\mathscr{T}\mathscr{S}X$. Then A_1^{*C} is $\mathscr{N}.g^{\#}SCS$ in X. And so A_1^{*C} is $\mathscr{N}.g\beta CS$. That is A_1^* is $\mathscr{N}.g\beta OS$ in X. ☐

Example 4.12. *Let $X=\{a_1,\ a_2,a_3\}$, $\tau_N=\{0_N,\ A_1^*,\ 1_N\}$, is a $N.T.$ on X where*

$$A_1^*=\left\langle x,\left(\frac{2}{10},\frac{5}{10},\frac{7}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{7}{10}\right)\right\rangle.$$

Then the Neutrosophic set

$$A_2^*=\left\langle x,\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{2}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{6}{10}\right)\right\rangle$$

is $\mathscr{N}.g\beta OS$ but not $\mathscr{N}.g^{\#}SOS$.

Theorem 4.13. *If $\mathscr{N}.Sint(A_1^*)\subseteq A_2^*\subseteq A_1^*$ and if A_1^* is $\mathscr{N}.g^{\#}SOS$, then A_2^* is $\mathscr{N}.g^{\#}SOS$ in a $\mathscr{N}\mathscr{T}\mathscr{S}X$.*

Proof. We have $\mathscr{N}.Sint(A_1^*)\subseteq A_2^*\subseteq A_1^*$. Then

$$A_1^{*C}\subseteq A_2^{*C}\subseteq\mathscr{N}.Scl(A_1^{*C})$$

and since (A_1^{*C}) is $\mathscr{N}.g^{\#}SCS$ and then, we have A_2^{*C} is $\mathscr{N}.g^{\#}SCS$ in X. And hence A_2^* is $\mathscr{N}.g^{\#}SOS$ in a $\mathscr{N}\mathscr{T}\mathscr{S}$ X. ☐

Theorem 4.14. *A Neutrosophic set A_1^* is $\mathscr{N}.g^{\#}SCS$ and $\mathscr{N}.Scl(A_1^*)\cap(\mathscr{N}.Scl\ (A_1^*))^C$=$0_N$, then $\mathscr{N}.Scl(A_1^*)\cap(A_1^{*C})$ is $\mathscr{N}.g^{\#}SOS$ in X.*

Proof. Let A_1^* be $\mathscr{N}.g^{\#}SCS$ in a $\mathscr{N}\mathscr{T}\mathscr{S}X$. Let $\mathscr{F}\subseteq\mathscr{N}.Scl(A_1^*)\cap(A_1^{*C})$, $\mathscr{F}$ is $\mathscr{N}.\ \alpha gCS$ in X. Then $\mathscr{F}$ is zero and so $\mathscr{F}\subseteq\ \mathscr{N}.Sint(\mathscr{N}.Scl(A_1^*)\cap(A_1^{*C})$. Then, $\mathscr{N}.Scl(A_1^*)\cap(A_1^{*C})$ is $\mathscr{N}.g^{\#}SOS$ in $\mathscr{N}\mathscr{T}\mathscr{S}X$. ☐

Theorem 4.15. *Finite intersection of $\mathscr{N}.g^{\#}SOS$ is a$\mathscr{N}.g^{\#}SOS$.*

Proof. Let A_1^* and A_2^* be $\mathscr{N}.g^{\#}$SOSsets in a $\mathscr{N}\mathscr{T}\mathscr{S}$ X. To prove that $A_1^*\cap A_2^*$ is $\mathscr{N}.g^{\#}$SOS. Let $\mathscr{F}\subseteq A_1^*\cap A_2^*$ where $\mathscr{F}$ be $\mathscr{N}.\ \alpha g$CS. Then $\mathscr{F}\subseteq A_1^*$, $\mathscr{F}\subseteq A_2^*$. Then $\mathscr{F}\subseteq\mathscr{N}$.Sint (A_1^*), $\mathscr{F}\subseteq\ \mathscr{N}$.Sint$(A_2^*)$ as A_1^* and A_2^* are $\mathscr{N}.g^{\#}$SOS.Then $\mathscr{F}\subseteq\mathscr{N}$.Sint $(A_1^*)\cap\mathscr{N}$.Sint (A_2^*)= $\mathscr{N}$.Sint$(A_1^*\cap A_2^*)$. That is $\mathscr{F}\subseteq\mathscr{N}$.Sint $(A_1^*\cap A_2^*)$. Hence $A_1^*\cap A_2^*$ is $\mathscr{N}.g^{\#}$SOS. Thus finite intersection of $\mathscr{N}.g^{\#}$SOS is a$\mathscr{N}.g^{\#}$SOS. ☐

Definition 4.16. *For any Neutrosophic set a in any $\mathscr{N}\mathscr{T}\mathscr{S}$,*
$\mathscr{N}.g^{\#}Scl(A_1^*)=\cap\{U:U$ is $\mathscr{N}.g^{\#}SCS$ and $A_1^*\subseteq U$.
$\mathscr{N}.g^{\#}Sint(A_2^*)=\cup V:V$ is $\mathscr{N}.g^{\#}SOS$ and $A_1^*\supseteq U$.

Theorem 4.17. *In a $\mathscr{N}\mathscr{T}\mathscr{S}(X,T)$, a Neutrosophic set A_1^* is $\mathscr{N}.g^{\#}S$ closed iff A_1^*=$\mathscr{N}.g^{\#}Scl(A_1^*)$.*

Proof. Let A_1^* be a $\mathcal{N}.g^{\#}SCS$ in a $\mathcal{N}\mathcal{T}\mathcal{S}(X,T)$. Since $A_1^* \subseteq A_1^*$ and A_1^* is $\mathcal{N}.g^{\#}SCS$, $A_1^* \in \{\mathcal{F} : \mathcal{F}$ is $\mathcal{N}.g^{\#}SCS$ and $A_1^* \subseteq \mathcal{F}\}$ and $A_1^* \subseteq \mathcal{F}$ implies that $A_1^* = \cap\{\mathcal{F} : \mathcal{F}$ is $\mathcal{N}.g^{\#}SCS$ and $A_1^* \subseteq \mathcal{F}\}$ that is $A_1^* = \mathcal{N}.g^{\#}\mathrm{Scl}(A_1^*)$.

Conversely, Suppose that $A_1^* = \mathcal{N}.g^{\#}\mathrm{Scl}(A_1^*)$, that is $A_1^* = \cap\{\mathcal{F} : \mathcal{F}$ is a $\mathcal{N}.g^{\#}SCS$ and $A_1^* \subseteq \mathcal{F}\}$. This implies that $A_1^* \in \{\mathcal{F} : \mathcal{F}$ is a $\mathcal{N}.g^{\#}SCS$ and $A_1^* \subseteq \mathcal{F}\}$. Hence A_1^* is $\mathcal{N}.g^{\#}SCS$. □

Theorem 4.18. *In a $\mathcal{N}\mathcal{T}\mathcal{S}X$ the following results hold for Neutrosophic $\mathcal{N}.g^{\#}S$-closure.*
1) $\mathcal{N}.g^{\#}Scl(0_N) = 0_N$.
2) $\mathcal{N}.g^{\#}Scl(A_1^*)$ is $\mathcal{N}.g^{\#}SCS$ in X.
3) $\mathcal{N}.g^{\#}Scl(A_1^*) \subseteq \mathcal{N}.g^{\#}Scl(A_2^*)$ if $A_1^* \subseteq A_2^*$.
4) $\mathcal{N}.g^{\#}Scl(\mathcal{N}.g^{\#}Scl(A_1^*)) = \mathcal{N}.g^{\#}Scl(A_1^*)$.
5) $\mathcal{N}.g^{\#}Scl(A_1^* \cup A_2^*) \supseteq \mathcal{N}.g^{\#}Scl(A_1^*) \cup \mathcal{N}.g^{\#}Scl(A_2^*)$.
6) $\mathcal{N}.g^{\#}Scl(A_1^* \cap A_2^*) \subseteq \mathcal{N}.g^{\#}Scl(A_1^*) \cap \mathcal{N}.g^{\#}Scl(A_2^*)$.

Proof. The easy verification is omitted. □

Theorem 4.19. *In a $\mathcal{N}\mathcal{T}\mathcal{S}X$, a Neutrosophic set A_1^* is $\mathcal{N}.g^{\#}SOS$ iff $A_1^* = \mathcal{N}.g^{\#}Sint(A_1^*)$.*

Proof. Let A_1^* be a $\mathcal{N}.g^{\#}SOS$ in X. Since $A_1^* \subseteq A_1^*$ and A_1^* is $\mathcal{N}.g^{\#}SOS$, $A_1^* \in \{\mathcal{F} : \mathcal{F}$ is $\mathcal{N}.g^{\#}SOS$ and $A_1^* \supseteq \mathcal{F}$ implies that $A_1^* = \cup\{\mathcal{F} : \mathcal{F}$ is $\mathcal{N}.g^{\#}SOS$ and $A_1^* \supseteq \mathcal{F}$. That is $A_1^* = \mathcal{N}.g^{\#}Sint(A_1^*)$.

Conversely, suppose that $A_1^* = \mathcal{N}.g^{\#}Sint(A_1^*)$, that is $A_1^* = \cup\{\mathcal{F} : \mathcal{F}$ is a $\mathcal{N}.g^{\#}SOS$ and $A_1^* \supseteq \mathcal{F}\}$. This implies that $A_1^* \in \{\mathcal{F} : \mathcal{F}$ is a $\mathcal{N}.g^{\#}SOS$ and $A_1^* \supseteq \mathcal{F}\}$. Hence A_1^* is $\mathcal{N}.g^{\#}SOS$. □

Theorem 4.20. *In a $\mathcal{N}\mathcal{T}\mathcal{S}X$, the following results hold for Neutrosophic $\mathcal{N}.g^{\#}S$ interior.*
1) $\mathcal{N}.g^{\#}Sint(0_N) = 0_N$.
2) $\mathcal{N}.g^{\#}Sint(A_1^*)$ is $\mathcal{N}.g^{\#}SOS$ in X.
3) $\mathcal{N}.g^{\#}Sint(A_1^*) \subseteq \mathcal{N}.g^{\#}Sint(A_2^*)$ if $A_1^* \subseteq A_2^*$.
4) $\mathcal{N}.g^{\#}Sint(\mathcal{N}.g^{\#}Sint(A_1^*)) = \mathcal{N}.g^{\#}Sint(A_1^*)$.
5) $\mathcal{N}.g^{\#}Sint(A_1^* \cup A_2^*) \supseteq \mathcal{N}.g^{\#}Sint(A_1^*) \cup \mathcal{N}.g^{\#}Sint(A_2^*)$.
6) $\mathcal{N}.g^{\#}Sint(A_1^* \cap A_2^*) \subseteq \mathcal{N}.g^{\#}Sint(A_1^*) \cap \mathcal{N}.g^{\#}Sint(A_2^*)$.

Proof. The routine proof is omitted. □

Definition 4.21. *A $\mathcal{N}\mathcal{T}\mathcal{S}(X,\tau_N)$ is called a Neutrosophic-$TS^{\#}$ space if every $\mathcal{N}.g^{\#}SCS$ is a $\mathcal{N}$ closed set.*

Theorem 4.22. *A $\mathcal{N}\mathcal{T}\mathcal{S}(X,\tau_N)$ is called a $\mathcal{N}.TS^{\#}$ space if every $\mathcal{N}.g^{\#}SOS$ is a $\mathcal{N}$ open set in X.*

Proof. Suppose X is Neutrosophic-$TS^{\#}$ space. Let V be $\mathcal{N}.g^{\#}SOS$ in X. Then V^C is $\mathcal{N}.g^{\#}SCS$ in X. Since X is Neutrosophic-$TS^{\#}$ space, V^C is $\mathcal{N}$. closed set in X. Therefore V is $\mathcal{N}$ open set in X.

Conversely, assume that every $\mathcal{N}.g^{\#}SOS$ in X is $\mathcal{N}$ open set in X. Let $\mathcal{F}$ be $\mathcal{N}.g^{\#}SCS$ in X, then $\mathcal{F}^C$ is $\mathcal{N}.g^{\#}SOS$ in X. By hypothesis, $\mathcal{F}^C$ is $\mathcal{N}$ open set in X. Therefore $\mathcal{F}$ is $\mathcal{N}$ closed set in X. Hence X is $\mathcal{N}.TS^{\#}$ space. □

Theorem 4.23. *Every $\mathcal{N}.T_{\frac{1}{2}}$ space is $\mathcal{N}.TS^{\#}$ space.*

Proof. Let X be $\mathcal{N}.T_{\frac{1}{2}}$ space. Let $\mathcal{F}$ be $\mathcal{N}.g^{\#}SCS$ in X. Then $\mathcal{F}$ is $\mathcal{N}.gCS$ in X. Since X is $\mathcal{N}.T_{\frac{1}{2}}$ space, $\mathcal{F}$ is $\mathcal{N}$ closed set in X. Hence X is $\mathcal{N}.TS^{\#}$ space. □

Definition 4.24. *A $\mathcal{N}\mathcal{T}\mathcal{S}(X,\tau_N)$ is called a Neutrosophic $^{\#}ST$ space if every $\mathcal{N}.gCS$ is a $\mathcal{N}.g^{\#}SCS$.*

Theorem 4.25. *Every $\mathcal{N}.T_{\frac{1}{2}}$ space is $\mathcal{N}^{\#}ST$ space.*

Proof. Let X be a $\mathcal{N}.T_{\frac{1}{2}}$ space. Let A_1^* be $\mathcal{N}.gCS$ in X. Since X is $\mathcal{N}.T_{\frac{1}{2}}$ space, the set A_1^* is $\mathcal{N}$ closed set in X and so A_1^* is $\mathcal{N}.g^{\#}SCS$. Hence X is $\mathcal{N}^{\#}ST$ space. □

Theorem 4.26. *A $\mathcal{N}\mathcal{T}\mathcal{S}X\mathcal{N}.\ T_{\frac{1}{2}}$ space iff it is $\mathcal{N}^{\#}ST$ space and $\mathcal{N}.TS^{\#}$ space.*

Proof. Suppose X is $\mathcal{N}.T_{\frac{1}{2}}$ then. It follows that X is $\mathcal{N}.TS^{\#}$ space and $\mathcal{N}^{\#}ST$ space. Conversely, suppose X is both $\mathcal{N}.TS^{\#}$ space and $\mathcal{N}^{\#}ST$ space. Let A_1^* be $\mathcal{N}.gCS$ in X. Since X is $\mathcal{N}^{\#}ST$ space, A_1^* is $\mathcal{N}.g^{\#}SCS$. Also, since X is $\mathcal{N}.TS^{\#}$ space, A_1^* is $\mathcal{N}$ closed set. Thus X is $.T_{\frac{1}{2}}\mathcal{N}\mathcal{T}\mathcal{S}$. □

Theorem 4.27. *A $\mathcal{N}\mathcal{T}\mathcal{S}X$ is a Neutrosophic $^{\#}ST$ space iff every $\mathcal{N}.\alpha gOS$ in X is a $\mathcal{N}.g^{\#}SOS$ in X.*

Proof. Assume that X is $\mathcal{N}^{\#}ST$ space. Let V be $\mathcal{N}.\alpha gOS$ in X. Then V^C is $\mathcal{N}.\alpha gCS$ in X. Since X is $\mathcal{N}.^{\#}ST$ space. Therefore V is $\mathcal{N}.g^{\#}SOS$ in X.

Conversely, assume that every $\mathcal{N}.\alpha gOS$ in X is $\mathcal{N}.g^{\#}SOS$ in X. Let $\mathcal{F}$ be $\mathcal{N}.\alpha gCS$ in X, then $\mathcal{F}^C$ is $\mathcal{N}.\alpha gOS$ in X. By hypothesis, $\mathcal{F}^C$ is $\mathcal{N}$ open set in X. Therefore $\mathcal{F}$ is $\mathcal{N}$ closed set in X. □

References

[1] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20(1986), 87–95.

[2] I. Arokiarani, R. Dhavaseelan, S. Jafari, M. Parimala, On Some New Notions and Functions in Neutrosophic topological space, *Neutrosophic Sets and Systems*, 16(2017), 16–19.

[3] A. Atkinswestley and S. Chandrasekar, Neutrosophic g*-Closed Sets and its maps, *Neutrosophic Sets and Systems*, 36(2020), 96–107.

[4] A. Atkinswestley S. Chandrasekar, Neutrosophic Weakly G*closed sets, *Advances in Mathematics: Scientific Journal*, 9(5), (2020), 2853–2861.

[5] V. Banupriya S.Chandrasekar: Neutrosophic αgs Continuity and Neutrosophic αgs Irresolute Maps, *Neutrosophic Sets and Systems*, 28(2019), 162–170.

[6] V. BanuPriya, S. Chandrasekar, and M. Suresh, Neutrosophic Strongly α-Generalized Semi Closed Sets, *Advances in Mathematics: Scientific Journal* 9(10), (2020), 8605–8613.

[7] R. Dhavaseelan, and S.Jafari, Generalized Neutrosophic closed sets, *New Trends in Neutrosophic Theory and Applications*, II(2018), 261–273.

[8] R. Dhavaseelan, S.Jafari, and Hanifpage, Neutrosophic generalized α-contra-continuity, *Creat. Math. Inform.,* 27(2), (2018), 133–139,

[9] Florentin Smarandache, Neutrosophic and NeutrosophicLogic, *First International Conference On Neutrosophic, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup*, NM 87301, USA, 2002.

[10] Floretin Smarandache, Neutrosophic Set: A Generalization of Intuitionistic Fuzzy set, *Journal of Defense Resources Management* 1(2010), 107–114.

[11] P. Ishwarya and K. Bageerathi, On Neutrosophic semi open sets in Neutrosophic Topological Spaces, *International Jour. Of Math. Trends and Tech.*, (2016), 214–223.

[12] D.Jayanthi, α Generalized closed Sets in NTSs, *International Journal of Mathematics Trends and Technology (IJMTT)- Special Issue ICRMIT* March (2018).

[13] S. Narmatha, E. Glory Bebina, R. Vishnu Priyaa, On $\pi g\beta$–Closed Sets and Mappings in Neutrosophic Topological Spaces, *International Journal of Innovative Technology and Exploring Engineering (IJITEE)*, 8(125), (2019), 505–510.

[14] C. Maheswari, M.Sathyabama, S.Chandrasekar, Neutrosophic generalized *b* closed Sets In Neutrosophic Topological Spaces, *Journal of Physics Conf. Series* 1139(2018) 12–65.

[15] C. Maheswari, S. Chandrasekar , Neutrosophic gb closed Sets and Neutrosophic gb-Continuity, *Neutrosophic Sets and Systems*, 29(2019), 89–99.

[16] C. Maheswari and S. Chandrasekar, Neutrosophic bg-closed Sets and its Continuity, *Neutrosophic Sets and Systems*, 36(2020), 108–120.

[17] T. Rajesh kannan, S. Chandrasekar, Neutosophic $\omega\alpha$ closed sets in Neutrosophic topological space, *Journal Of Computer And Mathematical Sciences*, 9(10), (2018), 1400–1408.

[18] T. Rajesh kannan ,S. Chandrasekar, Neutosophic α-continuous multifunction in Neutrosophic topological space, *The International Journal of Analytical and Experimental Modal Analysis*, XI(IX), (2019), 1360–1367.

[19] T. Rajesh Kannan, and S. Chandrasekar, Neutrosophic α-Irresolute Multifunction In Neutrosophic Topological Spaces, *Neutrosophic Sets and Systems* , 32(1), (2020), 390–400.

[20] T. Rajesh kannan, S. Chandrasekar, Neutrosophic Pre-α, Semi-α & Pre-β Irresolute Functions(Communicated).

[21] A.A. Salama and S.A. Alblowi, Generalized Neutrosophic Set and Generalized Neutrosophic topological spaces , *Journal computer Sci. Engineering*, 2(7), (2012).

[22] A.A. Salama, and S.A. Alblowi, Neutrosophic set and Neutrosophic topological spaces, *ISOR J. Mathematics*, 3(4), (2012), 31–35.

[23] V.K. Shanthi, S. Chandrasekar, K. Safina Begam, Neutrosophic Generalized Semi-closed Sets In neutrosophic topological spaces, *International Journal of Research in Advent Technology*, 6(7), (2018), 1739–1743.

[24] V. Venkateswara Rao, Y. Srinivasa Rao, Neutrosophic Pre-open Sets and Pre-closed Sets in Neutrosophic Topology, *International Journal of Chem Tech Research*, 10(10), (2017), 449–458.

[25] L. A. Zadeh, Fuzzy sets, *Information and Control*, 8(3), (1965), 338–353.

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