



# Neutrosophic $PRE - \alpha$ , $SEMI - \alpha$ and $PRE - \beta$ irresolute open and closed mappings in neutrosophic topological spaces

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## Abstract

Aim of this present paper is, the notions of Neutrosophic pre- $\alpha$ -irresolute open & closed mappings, Neutrosophic  $\alpha$ -irresolute open & closed mappings, Neutrosophic semi- $\alpha$ - irresolute open & closed mappings and Neutrosophic pre- $\beta$ -irresolute open & closed mappings are introduced and Besides giving characterizations of these mappings and several interesting properties of these mappings are also discussed.

## Keywords

Neutrosophic  $\alpha$  -irresolute, Neutrosophic pre  $\alpha$ -irresolute, Neutrosophic  $\beta$ -irresolute, Neutrosophic  $\alpha$ -closed sets; Neutrosophic topological spaces.

## AMS Subject Classification

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## 1. Introduction

C.L. Chang [7] was introduced fuzzy topological space by using . Zadeh's L.A [23] (uncertain) fuzzy sets. Further Coker [8] was developed the notion of intuitionistic fuzzy topological spaces by using Atanassov's [1] Smarandache [7] was defined the Neutrosophic set of three component  $(t, f, i) = (\text{Truth, Falsehood, Indeterminacy})$ . The Neutrosophic crisp set concept converted to Neutrosophic topological spaces by A.A. Salama [20]. I. Arokiarani [2] et al, introduced Neutrosophic  $\alpha$ -closed sets. K. Bageerathi [11] was developed to the concept of semiopen set and V.

Venkateswara Rao [22] et.al., are introduced by pre open sets in Neutrosophic topological space.

In this paper, the concepts of Neutrosophic pre-  $\alpha$  -irresolute open and closed mappings, Neutrosophic  $\alpha$ -irresolute open and closed mappings, Neutrosophic semi- $\alpha$ -irresolute open and closed mappings and Neutrosophic pre- $\beta$ -irresolute open and closed mappings are introduced and studied. Besides giving characterizations of these mappings, several interesting properties of these mappings are also given.

## 2. Preliminaries

In this section, we introduce the basic definition for Neutrosophic sets and its operations.

**Definition 2.1** ([8]). Let  $\mathcal{S}_N^1$  be a non-empty fixed set. A Neutrosophic set  $A_{\mathcal{S}_N^1}$  is the form

$$A_{\mathcal{S}_N^1} = \{ \langle \xi^*, \mu_{A_{\mathcal{S}_N^1}}(\xi^*), \sigma_{A_{\mathcal{S}_N^1}}(\xi^*), \gamma_{A_{\mathcal{S}_N^1}}(\xi^*) \rangle : \xi^* \in \mathcal{S}_N^1 \}.$$

Where  $\mu_{\mathcal{S}_N^1}(\xi^*) : \mathcal{S}_N^1 \rightarrow [0, 1]$ ,  $\sigma_{\mathcal{S}_N^1}(\xi^*) : \mathcal{S}_N^1 \rightarrow [0, 1]$ ,  $\gamma_{\mathcal{S}_N^1}(\xi^*) : \mathcal{S}_N^1 \rightarrow [0, 1]$  are represent Neutrosophic of the degree of membership function, the degree indeterminacy and

the degree of non membership function respectively of each element  $\xi^* \in \mathcal{S}_N^1$  to the set  $A_{\mathcal{S}_N^1}$  with

$$0 \leq \mu_{A_{\mathcal{S}_N^1}}(\xi^*) + \sigma_{A_{\mathcal{S}_N^1}}(\xi^*) + \gamma_{A_{\mathcal{S}_N^1}}(\xi^*) \leq 1.$$

This is called standard form generalized fuzzy sets. But also Neutrosophic set may be

$$0 \leq \mu_{A_{\mathcal{S}_N^1}}(\xi^*) + \sigma_{A_{\mathcal{S}_N^1}}(\xi^*) + \gamma_{A_{\mathcal{S}_N^1}}(\xi^*) \leq 3.$$

**Definition 2.2** ([8]). Each Intuitionistic fuzzy set  $A_{\mathcal{S}_N^1}$  is a non-empty set in  $\mathcal{S}_N^1$  is obviously on Neutrosophic set having the form

$$A_{\mathcal{S}_N^1} = \{ \langle \xi^*, \mu_{A_{\mathcal{S}_N^1}}(\xi^*), (1 - (\mu_{A_{\mathcal{S}_N^1}}(\xi^*) + \gamma_{A_{\mathcal{S}_N^1}}(\xi^*))), \gamma_{A_{\mathcal{S}_N^1}}(\xi^*) \rangle : \xi^* \in \mathcal{S}_N^1 \}.$$

**Definition 2.3** ([8]). We must introduce the Neutrosophic set  $0_N$  and  $1_N$  in  $\mathcal{S}_N^1$  as follows:

$$0_N = \{ \langle \xi^*, 0, 0, 1 \rangle : \xi^* \in \mathcal{S}_N^1 \}$$

and

$$1_N = \{ \langle \xi^*, 1, 1, 0 \rangle : \xi^* \in \mathcal{S}_N^1 \}.$$

**Definition 2.4** ([8]). Let  $\mathcal{S}_N^1$  be a non-empty set and Neutrosophic sets  $A_{\mathcal{S}_N^1}$  and  $B_{\mathcal{S}_N^1}$  in the form NS

$$A_{\mathcal{S}_N^1} = \{ \langle \xi^*, \mu_{A_{\mathcal{S}_N^1}}(\xi^*), \sigma_{A_{\mathcal{S}_N^1}}(\xi^*), \gamma_{A_{\mathcal{S}_N^1}}(\xi^*) \rangle : \xi^* \in \mathcal{S}_N^1 \}$$

and

$$B_{\mathcal{S}_N^1} = \{ \langle \xi^*, \mu_{B_{\mathcal{S}_N^1}}(\xi^*), \sigma_{B_{\mathcal{S}_N^1}}(\xi^*), \gamma_{B_{\mathcal{S}_N^1}}(\xi^*) \rangle : \xi^* \in \mathcal{S}_N^1 \}$$

defined as:

1.  $A_{\mathcal{S}_N^1} \subseteq B_{\mathcal{S}_N^1} \Leftrightarrow \mu_{A_{\mathcal{S}_N^1}}(\xi^*) \leq \mu_{B_{\mathcal{S}_N^1}}(\xi^*), \sigma_{A_{\mathcal{S}_N^1}}(\xi^*) \leq \sigma_{B_{\mathcal{S}_N^1}}(\xi^*),$  and  $\sigma_{A_{\mathcal{S}_N^1}}(\xi^*) \geq \gamma_{B_{\mathcal{S}_N^1}}(\xi^*)$
2.  $A_{\mathcal{S}_N^1}^C = \{ \langle \xi^*, \gamma_{A_{\mathcal{S}_N^1}}(\xi^*), \sigma_{A_{\mathcal{S}_N^1}}(\xi^*), \mu_{A_{\mathcal{S}_N^1}}(\xi^*) \rangle : \xi^* \in \mathcal{S}_N^1 \}$
3.  $A_{\mathcal{S}_N^1} \cap B_{\mathcal{S}_N^1} = \{ \langle \xi^*, \mu_{A_{\mathcal{S}_N^1}}(\xi^*) \wedge \mu_{B_{\mathcal{S}_N^1}}(\xi^*), \sigma_{A_{\mathcal{S}_N^1}}(\xi^*) \wedge \sigma_{B_{\mathcal{S}_N^1}}(\xi^*), \gamma_{A_{\mathcal{S}_N^1}}(\xi^*) \vee \gamma_{B_{\mathcal{S}_N^1}}(\xi^*) \rangle : \xi^* \in \mathcal{S}_N^1 \}$
4.  $A_{\mathcal{S}_N^1} \cup B_{\mathcal{S}_N^1} = \{ \langle \xi^*, \mu_{A_{\mathcal{S}_N^1}}(\xi^*) \vee \mu_{B_{\mathcal{S}_N^1}}(\xi^*), \sigma_{A_{\mathcal{S}_N^1}}(\xi^*) \vee \sigma_{B_{\mathcal{S}_N^1}}(\xi^*), \gamma_{A_{\mathcal{S}_N^1}}(\xi^*) \wedge \gamma_{B_{\mathcal{S}_N^1}}(\xi^*) \rangle : \xi^* \in \mathcal{S}_N^1 \}$ .

**Proposition 2.5** ([8]). For all  $A_{\mathcal{S}_N^1}$  and  $B_{\mathcal{S}_N^1}$  are two Neutrosophic sets then the following condition are true:

1.  $(A_{\mathcal{S}_N^1} \cap B_{\mathcal{S}_N^1})^c = (A_{\mathcal{S}_N^1})^c \cup (B_{\mathcal{S}_N^1})^c.$
2.  $(A_{\mathcal{S}_N^1} \cup B_{\mathcal{S}_N^1})^c = (A_{\mathcal{S}_N^1})^c \cap (B_{\mathcal{S}_N^1})^c.$

**Definition 2.6** ([18]). A Neutrosophic topology is a non-empty set  $\mathcal{S}_N^1$  is a family  $\tau_{N_{\mathcal{S}_N^1}}$  of Neutrosophic subsets in  $\mathcal{S}_N^1$  satisfying the following axioms:

1.  $0_N, 1_N \in \tau_{N_{\mathcal{S}_N^1}}$
2.  $G_{\mathcal{S}_N^1} \cap H_{\mathcal{S}_N^1} \in \tau_{N_{\mathcal{S}_N^1}}$  for any  $G_{\mathcal{S}_N^1}, H_{\mathcal{S}_N^1} \in \tau_{N_{\mathcal{S}_N^1}}$
3.  $\bigcup_i G_i \in \tau_{N_{\mathcal{S}_N^1}}$  for every  $G_i \in \tau_{N_{\mathcal{S}_N^1}}, i \in J.$

The pair  $(\mathcal{S}_N^1, \tau_{N_{\mathcal{S}_N^1}})$  is called a Neutrosophic topological space.

The element Neutrosophic topological spaces of  $\tau_{N_{\mathcal{S}_N^1}}$  are called Neutrosophic open sets. It is denoted by  $NOS_{\mathcal{S}_N^1}$ .

A Neutrosophic set  $A_{\mathcal{S}_N^1}$  is closed if and only if  $A_{\mathcal{S}_N^1}^C$  is Neutrosophic open. It is denoted by  $NCS_{\mathcal{S}_N^1}$ .

**Definition 2.7** ([20]). Let  $(\mathcal{S}_N^1, \tau_{N_{\mathcal{S}_N^1}})$  be Neutrosophic topological spaces.

$$A_{\mathcal{S}_N^1} = \{ \langle \xi^*, \mu_{A_{\mathcal{S}_N^1}}(\xi^*), \sigma_{A_{\mathcal{S}_N^1}}(\xi^*), \gamma_{A_{\mathcal{S}_N^1}}(\xi^*) \rangle : \xi^* \in \mathcal{S}_N^1 \}$$

be a Neutrosophic set in  $\mathcal{S}_N^1$

1.  $Neu-CI(A_{\mathcal{S}_N^1}) = \cap \{ K_{\mathcal{S}_N^1} : K_{\mathcal{S}_N^1} \text{ is a Neutrosophic closed set in } \mathcal{S}_N^1 \text{ and } A_{\mathcal{S}_N^1} \subseteq K_{\mathcal{S}_N^1} \}.$  It is denoted by  $Neu^{CI}A_{\mathcal{S}_N^1}.$
2.  $Neu-Int(A_{\mathcal{S}_N^1}) = \cup \{ G_{\mathcal{S}_N^1} : G_{\mathcal{S}_N^1} \text{ is a Neutrosophic open set in } \mathcal{S}_N^1 \text{ and } G_{\mathcal{S}_N^1} \subseteq A_{\mathcal{S}_N^1} \}.$  It is denoted by  $Int_{Neu}A_{\mathcal{S}_N^1}.$
3. Neutrosophic semi-open if  $A_{\mathcal{S}_N^1} \subseteq_{Neu}^{CI} (Int_{Neu}A_{\mathcal{S}_N^1}).$  It is denoted by  $N^SOS_{\mathcal{S}_N^1}.$
4. The complement of Neutrosophic semi-open set is called Neutrosophic semi-closed.
5.  $Neus-CI(A_{\mathcal{S}_N^1}) = \cap \{ K_{\mathcal{S}_N^1} / K_{\mathcal{S}_N^1} \text{ is a Neutrosophic semi closed set in } \mathcal{S}_N^1 \text{ and } A_{\mathcal{S}_N^1} \subseteq K_{\mathcal{S}_N^1} \}.$  It is denoted by  $S_{Neu}^{CI}A_{\mathcal{S}_N^1}.$
6.  $Neus-Int(A_{\mathcal{S}_N^1}) = \cup \{ G_{\mathcal{S}_N^1} : G_{\mathcal{S}_N^1} \text{ is a Neutrosophic semi open set in } \mathcal{S}_N^1 \text{ and } G_{\mathcal{S}_N^1} \subseteq A_{\mathcal{S}_N^1} \}.$  It is denoted by  $S_{Neu}^{Int}A_{\mathcal{S}_N^1}.$
7. Neutrosophic  $\alpha$ -open set if  $A_{\mathcal{S}_N^1} \subseteq_{Neu}^{int} (S_{Neu}^{CI} (Int_{Neu}A_{\mathcal{S}_N^1})).$  It is denoted by  $N^{\alpha}OS_{\mathcal{S}_N^1}.$

8. The complement of Neutrosophic  $\alpha$ -open set is called Neutrosophic  $\alpha$ -closed.

9.  $Neu\alpha - CI(A_{\mathcal{S}_N^1}) = \cap \{ K_{\mathcal{S}_N^1} : K_{\mathcal{S}_N^1} \text{ is a Neutrosophic } \alpha\text{-closed set in } \mathcal{S}_N^1 \text{ and } A_{\mathcal{S}_N^1} \subseteq K_{\mathcal{S}_N^1} \}.$  It is denoted by  $Neu^{\alpha}CI A_{\mathcal{S}_N^1}.$



10.  $Neu\alpha - Int(A_{R_N^1}) = \cup\{G_{\mathcal{S}_N^1} : G_{\mathcal{S}_N^1} \text{ is a Neutrosophic } \alpha\text{-open set in } \mathcal{S}_N^1 \text{ and } G_{\mathcal{S}_N^1} \subseteq A_{\mathcal{S}_N^1}\}$ . It is denoted by  $\alpha_{Neq}^{int} A_{\mathcal{S}_N^1}$ .

11. Neutrosophic pre open set if  $A_{\mathcal{S}_N^1} \subseteq \overset{int}{Neu}(\overset{Cl}{Neu} A_{\mathcal{S}_N^1})$ . It is denoted by  $N^P OS_{\mathcal{S}_N^1}$ .

12. The complement of Neutrosophic pre-open set is called Neutrosophic pre-closed.

13.  $Neup-CI(A_{\mathcal{S}_N^1}) = \cap\{K_{A_{\mathcal{S}_N^1}} : K_{A_{\mathcal{S}_N^1}} \text{ is a Neutrosophic } p\text{-closed set in } \mathcal{S}_N^1 \text{ and } A_{\mathcal{S}_N^1} \subseteq K_{\mathcal{S}_N^1}\}$ . It is denoted by  $\overset{pcl}{Neu} A_{\mathcal{S}_N^1}$ .

14.  $Neup-Int(A_{R_N^1}) = \cup\{G_{A_{\mathcal{S}_N^1}} : G_{A_{\mathcal{S}_N^1}} \text{ is a Neutrosophic } p\text{-open set in } \mathcal{S}_N^1 \text{ and } G_{\mathcal{S}_N^1} \subseteq A_{\mathcal{S}_N^1}\}$ . It is denoted by  $\overset{pint}{Neu} A_{\mathcal{S}_N^1}$ .

**Definition 2.8** ([9]). Take  $\Sigma'_1, \Sigma'_2, \Sigma'_3$  are belongs to real numbers 0 to 1 such that  $0 \leq \Sigma'_1 + \Sigma'_2 + \Sigma'_3 \leq 1$ . An Neutrosophic point  $f(\Sigma'_1, \Sigma'_2, \Sigma'_3)$  is Neutrosophic set defined by

$$f(\Sigma'_1, \Sigma'_2, \Sigma'_3) = \begin{cases} (\Sigma'_1, \Sigma'_2, \Sigma'_3), & \text{if } \Sigma = f \\ (0, 0, 1), & \text{if } \Sigma \neq f. \end{cases}$$

Take  $(\Sigma_1, \Sigma_2, \Sigma_3) = \langle f_{\Sigma_1}, f_{\Sigma_2}, f_{\Sigma_3} \rangle$ , where  $f_{\Sigma_1}, f_{\Sigma_2}, f_{\Sigma_3}$  are represent Neutrosophic the degree of membership function, the degree indeterminacy and the degree of non-membership function respectively of each element  $\Sigma^* \in \mathcal{S}_N^1$  to the set  $A_{\mathcal{S}_N^1}$ .

**Definition 2.9** ([9]). Let  $\mathcal{S}_N^1$  and  $\mathcal{S}_N^2$  be two finite sets. Define  $\psi_1 : \mathcal{S}_N^1 \rightarrow \mathcal{S}_N^2$ . If

$$A_{\mathcal{S}_N^2} = \{ \langle \theta, \mu_{A_{\mathcal{S}_N^2}}(\theta), \sigma_{A_{\mathcal{S}_N^2}}(\theta), \gamma_{A_{\mathcal{S}_N^2}}(\theta) \rangle : \theta \in \mathcal{S}_N^2 \}$$

is an NS in  $\mathcal{S}_N^2$ , then the inverse image(pre image)  $A_{\mathcal{S}_N^2}$  under  $\psi_1$  is an NS defined by

$$\psi_1^{-1}(A_{\mathcal{S}_N^2}) = \langle \xi^*, \psi_1^{-1} \mu_{A_{\mathcal{S}_N^2}}(\xi^*), \psi_1^{-1} \sigma_{A_{\mathcal{S}_N^2}}(\xi^*), \psi_1^{-1} \gamma_{A_{\mathcal{S}_N^2}}(\xi^*) : \xi^* \in \mathcal{S}_N^1 \rangle.$$

Also define image NS

$$U = \langle \xi^*, \mu_U(\xi^*), \sigma_U(\xi^*), \gamma_U(\xi^*) : \xi^* \in \mathcal{S}_N^1 \rangle :$$

under  $\psi_1$  is an NS defined by

$$\psi_1(U) = \langle \theta, \psi_1(\mu_{A_{\mathcal{S}_N^2}}(\theta)), \psi_1(\sigma_{A_{\mathcal{S}_N^2}}(\theta)), \psi_1(\gamma_{A_{\mathcal{S}_N^2}}(\theta)) : \theta \in \mathcal{S}_N^2 \rangle$$

where

$$\psi_1(\mu_{A_{\mathcal{S}_N^2}}(\theta)) = \begin{cases} \sup \mu_{A_{\mathcal{S}_N^2}}(\xi^*), & \text{if } \psi_1^{-1}(\theta) \neq \phi, \\ & \xi^* \in \psi_1^{-1}(\theta) \\ 0, & \text{elsewhere} \end{cases}$$

$$\psi_1(\sigma_{A_{\mathcal{S}_N^2}}(\theta)) = \begin{cases} \sup \sigma_{A_{\mathcal{S}_N^2}}(\xi^*), & \text{if } \psi_1^{-1}(\theta) \neq \phi, \\ & \xi^* \in \psi_1^{-1}(\theta) \\ 0, & \text{elsewhere} \end{cases}$$

$$\psi_1(\gamma_{A_{\mathcal{S}_N^2}}(\theta)) = \begin{cases} \inf \gamma_{A_{\mathcal{S}_N^2}}(\xi^*), & \text{if } \psi_1^{-1}(\theta) \neq \phi, \\ & \xi^* \in \psi_1^{-1}(\theta) \\ 0, & \text{elsewhere} \end{cases}$$

**Definition 2.10** ([2]). A mapping  $\psi_1 : (\mathcal{S}_N^1, \tau_{N_{\mathcal{S}_N^1}}) \rightarrow (\mathcal{S}_N^2, \tau_{N_{\mathcal{S}_N^2}})$  is called a

1. Neutrosophic continuous (Neu-continuous) if  $\psi_1^{-1}(A_{\mathcal{S}_N^2}) \in NCS_{\mathcal{S}_N^1}$  whenever  $A_{\mathcal{S}_N^2} \in NCS_{\mathcal{S}_N^2}$ .
2. Neutrosophic  $\alpha$ -continuous (Neu  $\alpha$ -continuous) if  $\psi_1^{-1}(A_{\mathcal{S}_N^2}) \in N^\alpha CS_{\mathcal{S}_N^1}$  whenever  $A_{\mathcal{S}_N^2} \in NCS_{\mathcal{S}_N^2}$ .
3. Neutrosophic semi-continuous (Neu semi-continuous) if  $\psi_1^{-1}(A_{\mathcal{S}_N^2}) \in N^s CS_{\mathcal{S}_N^1}$  whenever  $A_{\mathcal{S}_N^2} \in NCS_{\mathcal{S}_N^2}$ .

**Definition 2.11** ([2]). A mapping  $\psi_1 : (\mathcal{S}_N^1, \tau_{N_{\mathcal{S}_N^1}}) \rightarrow (\mathcal{S}_N^2, \tau_{N_{\mathcal{S}_N^2}})$  is called a

1. Neutrosophic open map if  $\psi_1(A_{\mathcal{S}_N^1}) \in NOS_{\mathcal{S}_N^2}$  whenever  $A_{\mathcal{S}_N^1} \in NOS_{\mathcal{S}_N^1}$ .
2. Neutrosophic  $\alpha$ -open map if  $\psi_1(A_{\mathcal{S}_N^1}) \in N^\alpha OS_{\mathcal{S}_N^2}$  whenever  $A_{\mathcal{S}_N^1} \in NOS_{\mathcal{S}_N^1}$ .
3. Neutrosophic pre -open map if  $\psi_1(A_{\mathcal{S}_N^1}) \in N^P OS_{\mathcal{S}_N^2}$  whenever  $A_{\mathcal{S}_N^1} \in NOS_{\mathcal{S}_N^1}$ .
4. Neutrosophic  $\beta$ -open map if  $\psi_1(A_{\mathcal{S}_N^1}) \in N^\beta OS_{\mathcal{S}_N^2}$  whenever  $A \in NOS_{\mathcal{S}_N^1}$ .

### 3. Neutrosophic PRE- $\alpha$ , SEMI- $\alpha$ and PRE- $\beta$ Irresolute Open Mappings

In this section, we introduce the Neutrosophic PRE- $\alpha$ , SEMI- $\alpha$  and PRE- $\beta$  Irresolute Open Mappings and its properties.

**Definition 3.1.** A mapping  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  and  $\lambda_{-1}^N$  is said to be

1. Neutrosophic Pre- $\alpha$  irresolute open (Neutrosophic pre- $\alpha$  irresolute closed.) mapping if  $\lambda_{-1}^N(V_1)$  is an  $N^\alpha OS(N^\alpha CS)$  in  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  for every  $N^P OS(N^P CS)V_1$  in  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$ .



2. Neutrosophic  $\alpha$ -irresolute open mapping (Neutrosophic  $\alpha$ -irresolute closed) mapping if  $\lambda_{-1}^N(V_1)$  is an  $N^\alpha OS(N^\alpha CS)$  in  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  for every  $N^\alpha OS(CS)V_1$  in  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$ .
3. Neutrosophic semi-  $\alpha$ -irresolute open (Neutrosophic semi- $\alpha$  -irresolute closed.) mapping. if  $\lambda_{-1}^N(V_1)$  is an  $N^\alpha OS(N^\alpha CS)$  in  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  for every  $N^\mathcal{S} OS(N^\mathcal{S} CS)V_1$  in  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$ .
4. Neutrosophic pre- $\beta$ -irresolute open (Neutrosophic pre- $\beta$  -irresolute closed.) mapping if  $\lambda_{-1}^N(V_1)$  is an  $N^\beta OS(N^\beta CS)$  in  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  for every  $N^P OS(N^P CS)V_1$  in  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$ .

**Proposition 3.2.** Every Neutrosophic Pre- $\alpha$  (Neutrosophic  $\alpha$  and Neutrosophic semi- $\alpha$ ) irresolute open mapping is Neutrosophic  $\alpha$ -open mapping.

*Proof.* Let  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$ . Assume that  $\lambda_{-1}^N$  is Neutrosophic pre- $\alpha$  (Neutrosophic  $\alpha$  and Neutrosophic semi- $\alpha$ ) -irresolute open mapping. Let  $V_1$  be  $NOS\mathfrak{R}_N^1$ . Since every  $NOS\mathfrak{R}_N^1$  is an  $N^P OS\mathfrak{R}_N^1(N^\alpha OS\mathfrak{R}_N^1$  and  $N^\mathcal{S} OS\mathfrak{R}_N^1)$ . Take  $V_1$  is an  $N^P OS\mathfrak{R}_N^1(N^\mathcal{S} OS\mathfrak{R}_N^1$  and  $N^\mathcal{S} OS\mathfrak{R}_N^1)$ . As  $\lambda_{-1}^N$  is an Neutrosophic pre- $\alpha$  (Neutrosophic-  $\alpha$  and Neutrosophic semi- $\alpha$ , resp.) irresolute open mapping.  $\lambda_{-1}^N(V_1)$  is an  $N^\alpha OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is Neutrosophic  $\alpha - open$  (Neutrosophic  $\alpha$  and Neutrosophic semi- $\alpha$ ) mapping.  $\square$

**Remark 3.3.** The above converse of the Proposition necessity not be true as shown by the following below examples.

**Example 3.4.** Let  $\mathfrak{R}_N^1 = \{a_n, b_n, c_n\} = \mathfrak{R}_N^2$  then  $\mathfrak{S}_N^1 = \{0_N, A_{\mathfrak{R}_N^1}, 1_N\}$ ,  $\mathfrak{S}_N^2 = \{0_N, B_{\mathfrak{R}_N^2}, C_{\mathfrak{R}_N^2}, B_{\mathfrak{R}_N^2} \cup C_{\mathfrak{R}_N^2}, B_{\mathfrak{R}_N^2} \cap C_{\mathfrak{R}_N^2}, 1_N\}$  are Neutrosophic Ts on  $\mathfrak{R}_N^1$  and  $\mathfrak{R}_N^2$  where

$$A_{\mathfrak{R}_N^1} = \{ \langle x, (a_n, 0.5, 0.5, 0.5), (b_n, 0.4, 0.5, 0.6), (c_n, 0.6, 0.5, 0.4) \rangle; x \in \mathfrak{R}_N^1 \}$$

$$B_{\mathfrak{R}_N^2} = \{ \langle y, (a_n, 0.5, 0.5, 0.5), (b_n, 0.3, 0.5, 0.7), (c_n, 0.6, 0.5, 0.4) \rangle; y \in \mathfrak{R}_N^1 \}$$

$$C_{\mathfrak{R}_N^2} = \{ \langle y, (a_n, 0.2, 0.5, 0.7), (b_n, 0.4, 0.5, 0.6), (c_n, 0.3, 0.5, 0.7) \rangle; y \in \mathfrak{R}_N^1 \}$$

$$D_{\mathfrak{R}_N^1} = \{ \langle x, (a_n, 0.5, 0.5, 0.4), (b_n, 0.4, 0.5, 0.5), (c_n, 0.6, 0.5, 0.4) \rangle; x \in \mathfrak{R}_N^1 \}.$$

Define an Neutrosophic mapping  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  by  $\lambda_{-1}^N(a_n) = a_n$ ,  $\lambda_{-1}^N(b_n) = b_n$ ,  $\lambda_{-1}^N(c_n) = c_n$ . Then  $\lambda_{-1}^N(D_{\mathfrak{R}_N^1})$  is an  $N^\alpha OS\mathfrak{R}_N^2$  and  $\lambda_{-1}^N(D_{\mathfrak{R}_N^1})$  is an  $N^\beta OS\mathfrak{R}_N^2$ . Therefore  $\lambda_{-1}^N$  is Neutrosophi  $\alpha - open$  mapping and Neutrosophic-open mapping.  $D_{\mathfrak{R}_N^1}$  is an Neutrosophic in  $\mathfrak{R}_N^1$ . Also  $D_{\mathfrak{R}_N^1}$  is  $N^\alpha OS\mathfrak{R}_N^1$ . Thus  $D_{\mathfrak{R}_N^1}$  is  $N^\beta OS\mathfrak{R}_N^1$  and  $N^\mathcal{S} OS\mathfrak{R}_N^1$ .  $\lambda_{-1}^N(D_{\mathfrak{R}_N^1})$  is not  $N^\alpha OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is not Neutrosophic  $\alpha$ -irresolute open mapping, not Neutrosophic pre-  $\alpha$  -irresolute open mapping and not Neutrosophic Semi-  $\alpha - irresolute$  open mapping.

**Proposition 3.5.** Every Neutrosophic Pre- $\alpha$  Neutrosophic and Neutrosophic semi- $\alpha$ , resp.)-irresolute open mapping is Neutrosophic-open mapping.

*Proof.* Let  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$ . Assume that  $\lambda_{-1}^N$  is Neutrosophic pre-  $\alpha$  (Neutrosophic  $\alpha$  and Neutrosophic semi- $\alpha$ .)-irresolute open mapping. Let  $V_1$  be  $NOS\mathfrak{R}_N^1$ . Since every  $NOS\mathfrak{R}_N^1$  is an  $N^P OS\mathfrak{R}_N^1(N^\alpha OS\mathfrak{R}_N^1$  and  $N^\mathcal{S} OS\mathfrak{R}_N^1$ , resp)  $V_1$  is an  $N^P OS\mathfrak{R}_N^1(N^\alpha OS\mathfrak{R}_N^1$  and  $N^\mathcal{S} OS\mathfrak{R}_N^1)$ . As  $\lambda_{-1}^N$  is an Neutrosophic pre- $\alpha$  (Neutrosophic  $\alpha -$  and Neutrosophic semi- $\alpha$ , resp.) irresolute open mapping.  $\lambda_{-1}^N(V_1)$  is an  $N^\alpha OS$  in  $\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N(V_1)$  is an  $N^\beta OS$  in  $\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is Neutrosophic  $\beta$ -open( Neutrosophic  $\alpha$  and Neutrosophic semi- $\alpha$ ) mapping.  $\square$

**Remark 3.6.** The above converse of the Proposition necessity not be true as shown by the following below examples.

**Example 3.7.** Let  $\mathfrak{R}_N^1 = \{a_n, b_n, c_n\} = \mathfrak{R}_N^2$  and let  $\mathfrak{S}_N^1 = \{0_N, A_{\mathfrak{R}_N^1}, 1_N\}$ ,  $\mathfrak{S}_N^2 = \{0_N, B_{\mathfrak{R}_N^2}, 1_N\}$  be a two Neutrosophic Topological spaces on  $\mathfrak{R}_N^1$  and  $\mathfrak{R}_N^2$ . Where

$$A_{\mathfrak{R}_N^1} = \{ \langle x, (a_n, 0.5, 0.5, 0.3), (b_n, 0.3, 0.5, 0.5), (c_n, 0.4, 0.5, 0.6) \rangle; x \in \mathfrak{R}_N^1 \},$$

$$B_{\mathfrak{R}_N^2} = \{ \langle y, (a_n, 0.4, 0.5, 0.6), (b_n, 0.4, 0.5, 0.3), (c_n, 0.2, 0.5, 0.6) \rangle; y \in \mathfrak{R}_N^1 \},$$

Define an Neutrosophic mapping  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  by  $\lambda_{-1}^N(a_n) = a_n$ ,  $\lambda_{-1}^N(b_n) = b_n$ ,  $\lambda_{-1}^N(c_n) = c_n$ . Take  $A_{\mathfrak{R}_N^1}$  is Neutrosophic Open set in  $\mathfrak{R}_N^1$ . Then  $\lambda_{-1}^N(A_N)$  is an  $N^\beta OS\mathfrak{R}_N^2$ . Thus  $\lambda_{-1}^N$  is Neutrosophic-open mapping. But not Neutrosophic Pre- $\alpha$  (Neutrosophic and Neutrosophic semi- $\alpha$ )-irresolute open mapping.

**Proposition 3.8.** Every Neutrosophic pre- $\alpha$  (Neutrosophic semi- $\alpha$ ) irresolute open mapping is Neutrosophic- irresolute open mapping.

*Proof.* Consider,  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  is Neutrosophic pre- $\alpha$  (Neutrosophic semi- $\alpha$ .) - irresolute open mapping. Let  $V_1$  be  $N^\alpha OS\mathfrak{R}_N^1$ . Since every  $N^\alpha OS\mathfrak{R}_N^1$  is an  $N^P OS\mathfrak{R}_N^1(N^\mathcal{S} OS\mathfrak{R}_N^1)$ . Then  $V_1$  is an  $N^P OS\mathfrak{R}_N^1(N^\mathcal{S} OS\mathfrak{R}_N^1)$ . As  $\lambda_{-1}^N$  is an Neutrosophic pre- $\alpha$  (Neutrosophic semi- $\alpha$  resp.) -irresolute open mapping,  $\lambda_{-1}^N(V_1)$  is an  $N^\alpha OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is Neutrosophic-irresolute open mapping.  $\square$

**Remark 3.9.** The above converse of the Proposition necessity not be true as shown by the following below examples.

**Example 3.10.** Let  $\mathfrak{R}_N^1 = \{a_n, b_n, c_n\} = \mathfrak{R}_N^2$  and take,  $\mathfrak{S}_N^1 = \{0_N, A_{\mathfrak{R}_N^1}, 1_N\}$ ,  $\mathfrak{S}_N^2 = \{0_N, B_{\mathfrak{R}_N^2}, 1_N\}$  are a two Neutrosophic





Topological spaces on  $\mathfrak{R}_N^1$  and  $\mathfrak{R}_N^2$  where

$$\begin{aligned} A_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.5, 0.5, 0.3), (b_n, 0.3, 0.5, 0.5), \\ &\quad (c_n, 0.4, 0.5, 0.6) \rangle; x \in \mathfrak{R}_N^1 \}, \\ B_{\mathfrak{R}_N^2} &= \{ \langle y, (a_n, 0.4, 0.5, 0.6), (b_n, 0.4, 0.5, 0.3), \\ &\quad (c_n, 0.2, 0.5, 0.6) \rangle; y \in \mathfrak{R}_N^1 \}, \\ C_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.4, 0.5, 0.5), (b_n, 0.6, 0.5, 0.3), \\ &\quad (c_n, 0.7, 0.5, 0.3) \rangle; x \in \mathfrak{R}_N^1 \} \end{aligned}$$

Define an Neutrosophic mapping  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$ ,  $\lambda_{-1}^N(a_n) = a_n$ ,  $\lambda_{-1}^N(b_n) = b_n$ ,  $\lambda_{-1}^N(c_n) = c_n$ .  $A_{\mathfrak{R}_N^1}$  is an Neutrosophic Open set and  $N^\alpha OS\mathfrak{R}_N^1$ . Hence  $\lambda_{-1}^N(A_{\mathfrak{R}_N^1})$  is an  $N^\alpha OS\mathfrak{R}_N^2$ . Thus  $\lambda_{-1}^N$  is Neutrosophic-irresolute open mapping. Then  $C_{\mathfrak{R}_N^1}$  is  $N^P OS\mathfrak{R}_N^1$  and  $\lambda_{-1}^N(C_{\mathfrak{R}_N^1})$  is an  $N^\beta OS\mathfrak{R}_N^2$ . Thus  $\lambda_{-1}^N$  is Neutrosophic pre- $\beta$ -irresolute open mapping. But  $\lambda_{-1}^N(C_{\mathfrak{R}_N^1})$  is not  $N^\alpha OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is not Neutrosophic pre- $\alpha$ -irresolute open mapping.

**Example 3.11.** Let  $\mathfrak{R}_N^1 = \{a_n, b_n\}$ ,  $\mathfrak{R}_N^2 = \{c_n, d_n\}$  and take,

$$\mathfrak{S}_N^1 = \{0_N, A_{\mathfrak{R}_N^1}, 1_N\}, \quad \mathfrak{S}_N^2 = \{0_N, B_{\mathfrak{R}_N^2}, 1_N\}$$

are two Neutrosophic Topological spaces on  $\mathfrak{R}_N^1$  and  $\mathfrak{R}_N^2$  where

$$\begin{aligned} A_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.4, 0.5, 0.5), (b_n, 0.3, 0.5, 0.6) \rangle; \\ &\quad x \in \mathfrak{R}_N^1 \}, \\ B_{\mathfrak{R}_N^2} &= \{ \langle y, (c_n, 0.3, 0.5, 0.6), (d_n, 0.4, 0.5, 0.5) \rangle; \\ &\quad y \in \mathfrak{R}_N^1 \}, \\ C_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.4, 0.5, 0.5), (b_n, 0.6, 0.5, 0.3) \rangle; \\ &\quad x \in \mathfrak{R}_N^1 \} \end{aligned}$$

is in Neutrosophic  $\mathfrak{R}_N^1$ . Define an Neutrosophic mapping  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$ ,  $\lambda_{-1}^N(a_n) = c_n$ ,  $\lambda_{-1}^N(b_n) = d_n$ . Then  $A_{\mathfrak{R}_N^1}$  is an  $NOS\mathfrak{R}_N^1$  and  $N^\alpha OS\mathfrak{R}_N^1$ . Therefore  $\lambda_{-1}^N$  is Neutrosophic  $\alpha$ -irresolute open mapping. and also  $C_{\mathfrak{R}_N^1}$  is an  $N^\beta OS\mathfrak{R}_N^1$ . Hence  $\lambda_{-1}^N(C_{\mathfrak{R}_N^1})$  is not in  $N^\alpha OS\mathfrak{R}_N^2$  and  $\lambda_{-1}^N$  is not Neutrosophic semi- $\alpha$ -irresolute open mapping.

**Proposition 3.12.** Every Neutrosophic pre- $\alpha$  irresolute open mapping is Neutrosophic pre- $\beta$ -irresolute open mapping.

*Proof.* Consider,  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  is Neutrosophic pre- $\alpha$  irresolute open mapping. Let  $V_1$  be  $N^P OS\mathfrak{R}_N^1$ . As  $\lambda_{-1}^N$  is an Neutrosophic pre- $\alpha$  irresolute open mapping,  $\lambda_{-1}^N(V_1)$  is an  $N^\alpha OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N(V_1)$  is an  $N^\beta OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is Neutrosophic pre  $\beta$ -irresolute open mapping.  $\square$

**Remark 3.13.** The above converse of the Proposition necessity not be true as shown by the following below examples.

**Example 3.14.** Let  $\mathfrak{R}_N^1 = \{a_n, b_n, c_n\} = \mathfrak{R}_N^2$  and take  $\mathfrak{S}_N^1 = \{0_N, A_{\mathfrak{R}_N^1}, 1_N\}$ ,  $\mathfrak{S}_N^2 = \{0_N, B_{\mathfrak{R}_N^2}, 1_N\}$  are two Neutrosophic Topological spaces on  $\mathfrak{R}_N^1$  and  $\mathfrak{R}_N^2$  where

$$\begin{aligned} A_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.5, 0.5, 0.3), (b_n, 0.3, 0.5, 0.5), \\ &\quad (c_n, 0.4, 0.5, 0.6) \rangle; x \in \mathfrak{R}_N^1 \}, \\ B_{\mathfrak{R}_N^2} &= \{ \langle y, (a_n, 0.4, 0.5, 0.6), (b_n, 0.4, 0.5, 0.3), \\ &\quad (c_n, 0.2, 0.5, 0.6) \rangle; y \in \mathfrak{R}_N^1 \}, \\ C_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.4, 0.5, 0.5), (b_n, 0.6, 0.5, 0.3), \\ &\quad (c_n, 0.7, 0.5, 0.3) \rangle; x \in \mathfrak{R}_N^1 \}. \end{aligned}$$

is in neutrosophic  $\mathfrak{R}_N^1$ . Define an Neutrosophic mapping  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  by  $\lambda_{-1}^N(a_n) = b_n$ ,  $\lambda_{-1}^N(b_n) = c_n$ ,  $\lambda_{-1}^N(c_n) = a_n$ . Here  $A_{\mathfrak{R}_N^1}$  is an Neutrosophic Open set and,  $N^\alpha OS\mathfrak{R}_N^1$ . We get,  $\lambda_{-1}^N(A_{\mathfrak{R}_N^1})$  is an  $N^\alpha OS\mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Thus  $\lambda_{-1}^N$  is Neutrosophic- irresolute open mapping. Finally  $C_{\mathfrak{R}_N^1}$  is  $N^P OS\mathfrak{R}_N^1$ . We get  $\lambda_{-1}^N(C_{\mathfrak{R}_N^1})$  is an  $N^\beta OS\mathfrak{R}_N^2$ . Thus  $\lambda_{-1}^N$  is Neutrosophic pre- $\beta$ -irresolute open mapping. And also  $\lambda_{-1}^N(C_{\mathfrak{R}_N^1})$  is not  $N^\alpha OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is not Neutrosophic Pre- $\alpha$ -irresolute open mapping.

**Proposition 3.15.** Every Neutrosophic semi- $\alpha$ -irresolute open mapping is Neutrosophic irresolute open mapping.

*Proof.* Take  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  from an Neutrosophic topological space.  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  to another Neutrosophic topological space  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  is Neutrosophic semi-  $\alpha$  -irresolute open mapping. Let  $C_{\mathfrak{R}_N^1}$  be  $N^\beta OS\mathfrak{R}_N^1$ . As  $\lambda_{-1}^N$  is an Neutrosophic semi- $\alpha$ - irresolute open  $\lambda_{-1}^N(C_{\mathfrak{R}_N^1})$  is an  $N^\alpha OS\mathfrak{R}_N^2$ . Every  $N^\alpha OS\mathfrak{R}_N^2$  is also in  $N^\beta OS\mathfrak{R}_N^2$ . So  $\lambda_{-1}^N(C_{\mathfrak{R}_N^1})$  is  $N^\beta OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is Neutrosophic irresolute open mapping.  $\square$

**Remark 3.16.** The above converse of the Proposition necessity not be true as shown by the following below examples.

**Example 3.17.** Let  $\mathfrak{R}_N^1 = \{a_n, b_n, c_n\} = \mathfrak{R}_N^2$ , then  $\mathfrak{S}_N^1 = \{0_N, A_{\mathfrak{R}_N^1}, 1_N\}$ ,  $\mathfrak{S}_N^2 = \{0_N, B_{\mathfrak{R}_N^2}, C_{\mathfrak{R}_N^2}, B_{\mathfrak{R}_N^2} \cup C_{\mathfrak{R}_N^2}, B_{\mathfrak{R}_N^2} \cap C_{\mathfrak{R}_N^2}, 1_N\}$  are Neutrosophic Ts on  $\mathfrak{R}_N^1$  and  $\mathfrak{R}_N^2$ , where

$$\begin{aligned} A_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.5, 0.5, 0.5), (b_n, 0.4, 0.5, 0.6), \\ &\quad (c_n, 0.6, 0.5, 0.4) \rangle; x \in \mathfrak{R}_N^1 \} \\ B_{\mathfrak{R}_N^2} &= \{ \langle y, (a_n, 0.5, 0.5, 0.5), (b_n, 0.3, 0.5, 0.7), \\ &\quad (c_n, 0.6, 0.5, 0.4) \rangle; y \in \mathfrak{R}_N^1 \} \\ C_{\mathfrak{R}_N^2} &= \{ \langle y, (a_n, 0.2, 0.5, 0.7), (b_n, 0.4, 0.5, 0.6), \\ &\quad (c_n, 0.3, 0.5, 0.7) \rangle; y \in \mathfrak{R}_N^1 \} \\ D_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.5, 0.5, 0.4), (b_n, 0.4, 0.5, 0.5), \\ &\quad (c_n, 0.6, 0.5, 0.4) \rangle; x \in \mathfrak{R}_N^1 \}. \end{aligned}$$



Define an Neutrosophic mapping  $\lambda_{-}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  by  $\lambda_{-}^N(a_n) = a_n, \lambda_{-}^N(b_n) = b_n, \lambda_{-}^N(c_n) = c_n$ .

Then  $A_{\mathfrak{R}_N^1}$  and  $D_{\mathfrak{R}_N^1}$  are in  $N^S OS\mathfrak{R}_N^1$  and  $\lambda_{-}^N(A_{\mathfrak{R}_N^1})$  and  $\lambda_{-}^N(D_{\mathfrak{R}_N^1})$  are  $N^S OS$  in  $\mathfrak{R}_N^2$ . So  $\lambda_{-}^N$  is Neutrosophic irresolute open mapping. But  $\lambda_{-}^N(D_{\mathfrak{R}_N^1})$  is not  $N^\alpha OS$  in  $\mathfrak{R}_N^2$ . Therefore  $\lambda_{-}^N$  is not Neutrosophic semi- $\alpha$ -irresolute open mapping.

**Proposition 3.18.** Every Neutrosophic pre- $\beta$ -irresolute open mapping is Neutrosophic  $\beta$ -open mapping.

*Proof.* Let  $\lambda_{-}^N$  be a map from an Neutrosophic topological space  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  to another Neutrosophic topological space  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  and Neutrosophic pre- $\beta$ -irresolute open mapping. Let  $C_{\mathfrak{R}_N^1}$  be  $NOS\mathfrak{R}_N^1$ . Since every  $NOS\mathfrak{R}_N^1$  is an  $N^P OS\mathfrak{R}_N^1$ , hence  $C_{\mathfrak{R}_N^1}$  is an  $N^P OS\mathfrak{R}_N^1$ . As  $\lambda_{-}^N$  is an Neutrosophic pre- $\beta$ -irresolute open. we get  $\lambda_{-}^N(C_{\mathfrak{R}_N^1})$  is an  $N^\beta OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-}^N$  is Neutrosophic  $\beta$ -open mapping.  $\square$

**Remark 3.19.** The above converse of the Proposition necessity not be true as shown by the following below examples.

**Example 3.20.** Let  $\mathfrak{R}_N^1 = \{a_n, b_n, c_n\} = \mathfrak{R}_N^2$ , then  $\mathfrak{S}_N^1 = \{0_N, A_{\mathfrak{R}_N^1}, 1_N\}$ ,  $\mathfrak{S}_N^2 = \{0_N, B_{\mathfrak{R}_N^2}, C_{\mathfrak{R}_N^2}, B_{\mathfrak{R}_N^2} \cup C_{\mathfrak{R}_N^2}, B_{\mathfrak{R}_N^2} \cap C_{\mathfrak{R}_N^2}, 1_N\}$  are Neutrosophic Ts on  $\mathfrak{R}_N^1$  and  $\mathfrak{R}_N^2$ , where

$$\begin{aligned} A_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.5, 0.5, 0.5), (b_n, 0.4, 0.5, 0.6), \\ &\quad (c_n, 0.6, 0.5, 0.4) \rangle; x \in \mathfrak{R}_N^1 \} \\ B_{\mathfrak{R}_N^2} &= \{ \langle y, (a_n, 0.5, 0.5, 0.5), (b_n, 0.3, 0.5, 0.7), \\ &\quad (c_n, 0.6, 0.5, 0.4) \rangle; y \in \mathfrak{R}_N^1 \}. \\ C_{\mathfrak{R}_N^2} &= \{ \langle y, (a_n, 0.2, 0.5, 0.7), (b_n, 0.4, 0.5, 0.6), \\ &\quad (c_n, 0.3, 0.5, 0.7) \rangle; y \in \mathfrak{R}_N^1 \} \\ D_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.5, 0.5, 0.4), (b_n, 0.4, 0.5, 0.5), \\ &\quad (c_n, 0.6, 0.5, 0.4) \rangle; x \in \mathfrak{R}_N^1 \} \end{aligned}$$

Define an Neutrosophic mapping  $\lambda_{-}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  by  $\lambda_{-}^N(a_n) = a_n, \lambda_{-}^N(b_n) = b_n, \lambda_{-}^N(c_n) = c_n$ . Here  $A_{\mathfrak{R}_N^1}$  is an  $NOS\mathfrak{R}_N^1$ . We get  $\lambda_{-}^N$  is an  $N^\beta OS\mathfrak{R}_N^2$  which implies  $\lambda_{-}^N$  is  $\beta$ -open mapping. But  $D_{\mathfrak{R}_N^1}$  is  $N^P OS$  in  $\mathfrak{R}_N^1$  and  $\lambda_{-}^N(D_{\mathfrak{R}_N^1})$  is not  $N^\beta OS\mathfrak{R}_N^2$ . So,  $\lambda_{-}^N$  is not Neutrosophic pre- $\beta$  irresolute open mapping.

**Proposition 3.21.** Every Neutrosophic pre- $\alpha$ -irresolute open mapping is Neutrosophic pre irresolute open mapping.

*Proof.* Let  $\lambda_{-}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  from an Neutrosophic topological space to another Neutrosophic topological space and Neutrosophic pre- $\alpha$ -irresolute open mapping. Let  $A_{\mathfrak{R}_N^1}$  be in  $N^P OS\mathfrak{R}_N^1$ . As  $\lambda_{-}^N$  is Neutrosophic pre- $\alpha$ -irresolute open. We get  $\lambda_{-}^N(A_{\mathfrak{R}_N^1})$  is an  $N^\alpha OS\mathfrak{R}_N^2$ . As every  $N^\alpha OS\mathfrak{R}_N^2$  is  $N^P OS\mathfrak{R}_N^2$ , finally  $\lambda_{-}^N(A_{\mathfrak{R}_N^1})$  is an  $N^P OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-}^N$  is Neutrosophic pre irresolute open mapping.  $\square$

**Remark 3.22.** The above converse of the Proposition necessity not be true as shown by the following below examples.

**Remark 3.23.** Let  $\mathfrak{R}_N^1 = \{a_n, b_n, c_n\} = \mathfrak{R}_N^2$  and take,  $\mathfrak{S}_N^1 = \{0_N, A_{\mathfrak{R}_N^1}, 1_N\}$ ,  $\mathfrak{S}_N^2 = \{0_N, B_{\mathfrak{R}_N^2}, 1_N\}$  are a two Neutrosophic Topological spaces on  $\mathfrak{R}_N^1$  and  $\mathfrak{R}_N^2$ , where

$$\begin{aligned} A_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.5, 0.5, 0.3), (b_n, 0.3, 0.5, 0.5), \\ &\quad (c_n, 0.4, 0.5, 0.6) \rangle; x \in \mathfrak{R}_N^1 \}, \\ B_{\mathfrak{R}_N^2} &= \{ \langle y, (a_n, 0.4, 0.5, 0.6), (b_n, 0.4, 0.5, 0.3), \\ &\quad (c_n, 0.2, 0.5, 0.6) \rangle; y \in \mathfrak{R}_N^1 \}, \\ C_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.4, 0.5, 0.5), (b_n, 0.6, 0.5, 0.3), \\ &\quad (c_n, 0.7, 0.5, 0.3) \rangle; x \in \mathfrak{R}_N^1 \} \end{aligned}$$

Define an Neutrosophic mapping  $\lambda_{-}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$   $\lambda_{-}^N(a_n) = b_n, \lambda_{-}^N(c_n) = b_n, \lambda_{-}^N(b_n) = a_n$ . Then  $A_{\mathfrak{R}_N^1}$  and  $C_{\mathfrak{R}_N^1}$  are  $N^P OS\mathfrak{R}_N^1$  and  $\lambda_{-}^N(A_{\mathfrak{R}_N^1})$  and  $\lambda_{-}^N(C_{\mathfrak{R}_N^1})$  are in  $N^P OS\mathfrak{R}_N^2$ . Therefore  $\lambda_{-}^N$  is Neutrosophic pre irresolute open mapping. But  $\lambda_{-}^N(C_{\mathfrak{R}_N^1})$  is not  $N^\alpha OS\mathfrak{R}_N^2$ . Thus  $\lambda_{-}^N$  is not Neutrosophic pre- $\alpha$ -irresolute open mapping. Hence the converse of the above Proposition need not be true.

**Proposition 3.24.** Every Neutrosophic pre irresolute open mapping is Neutrosophic pre- $\beta$ -irresolute open mapping.

*Proof.* Take,  $\lambda_{-}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  from an Neutrosophic topological space  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  to another Neutrosophic topological space  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  is Neutrosophic pre irresolute open mapping. Let  $A_{\mathfrak{R}_N^1}$  be in  $N^P OS\mathfrak{R}_N^1$ . As  $\lambda_{-}^N$  is Neutrosophic pre irresolute open  $\lambda_{-}^N(A_{\mathfrak{R}_N^1})$  is an  $N^P OS\mathfrak{R}_N^2$ . As every  $N^P OS\mathfrak{R}_N^2$  is  $N^\beta OS\mathfrak{R}_N^2$ . Finally we get  $\lambda_{-}^N(A_{\mathfrak{R}_N^1})$  is an  $N^\beta OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-}^N$  is Neutrosophic pre- $\beta$ -irresolute open mapping.  $\square$

**Remark 3.25.** Let  $\mathfrak{R}_N^1 = \{a_n, b_n, c_n\} = \mathfrak{R}_N^2$  then  $\mathfrak{S}_N^1 = \{0_N, A_{\mathfrak{R}_N^1}, 1_N\}$ ,  $\mathfrak{S}_N^2 = \{0_N, B_{\mathfrak{R}_N^2}, C_{\mathfrak{R}_N^2}, B_{\mathfrak{R}_N^2} \cup C_{\mathfrak{R}_N^2}, B_{\mathfrak{R}_N^2} \cap C_{\mathfrak{R}_N^2}, 1_N\}$  are Neutrosophic Ts on  $\mathfrak{R}_N^1$  and  $\mathfrak{R}_N^2$ , where

$$\begin{aligned} A_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.5, 0.5, 0.5), (b_n, 0.4, 0.5, 0.6), \\ &\quad (c_n, 0.6, 0.5, 0.4) \rangle; x \in \mathfrak{R}_N^1 \} \\ B_{\mathfrak{R}_N^2} &= \{ \langle y, (a_n, 0.5, 0.5, 0.5), (b_n, 0.3, 0.5, 0.7), \\ &\quad (c_n, 0.6, 0.5, 0.4) \rangle; y \in \mathfrak{R}_N^1 \}. \\ C_{\mathfrak{R}_N^2} &= \{ \langle y, (a_n, 0.2, 0.5, 0.7), (b_n, 0.4, 0.5, 0.6), \\ &\quad (c_n, 0.3, 0.5, 0.7) \rangle; y \in \mathfrak{R}_N^1 \} \\ D_{\mathfrak{R}_N^1} &= \{ \langle x, (a_n, 0.5, 0.5, 0.4), (b_n, 0.4, 0.5, 0.5), \\ &\quad (c_n, 0.6, 0.5, 0.4) \rangle; x \in \mathfrak{R}_N^1 \}. \end{aligned}$$

Define an Neutrosophic mapping  $\lambda_{-}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  by  $\lambda_{-}^N(a_n) = a_n, \lambda_{-}^N(b_n) = b_n,$



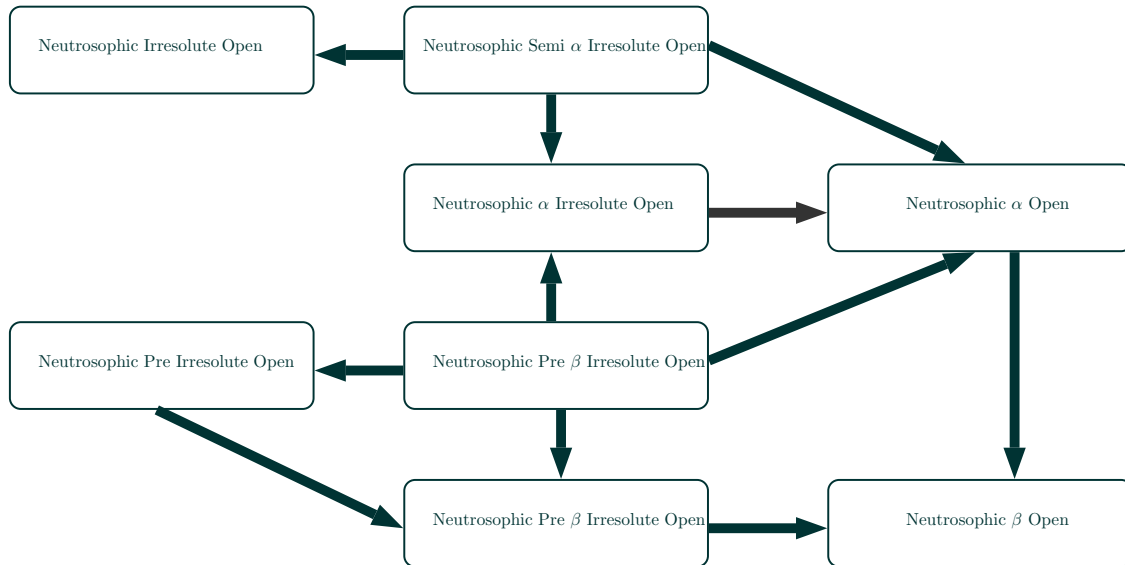


Figure 1

$\lambda_{-1}^N(c_n) = c_n$ . Here  $A_{\mathfrak{R}_N^1}$ . And  $D_{\mathfrak{R}_N^1}$  are in  $N^P OS\mathfrak{R}_N^1$  and  $\lambda_{-1}^N(A_{\mathfrak{R}_N^1})$  and  $\lambda_{-1}^N(D_{\mathfrak{R}_N^1})$  are in  $N^\beta OS\mathfrak{R}_N^2$ . But  $\lambda_{-1}^N(D_{\mathfrak{R}_N^1})$  is not in  $N^P OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is Neutrosophic pre- $\beta$ -irresolute open mapping and not Neutrosophic pre irresolute open mapping. Thus the converse of the above Proposition need not be true.

#### Diagram I

Interrelationships between Neutrosophicpre- $\alpha$  (Neutrosophic alpha, Neutrosophic semi- $\alpha$  and Neutrosophic pre- $\beta$ , resp.)-irresolute open mappings with existing mappings in Neutrosophic topological spaces.

### 4. Properties and Characterizations

**Theorem 4.1.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$ ,  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  and  $(\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  be Neutrosophic TSs. Let  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  and  $\mu_{-1}^N : (\mathfrak{R}_N^2, \mathfrak{S}_N^2) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  be any two maps. If  $\mu_{-1}^N \circ \lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  is Neutrosophic pre- $\alpha$ -irresolute (Neutrosophic semi- $\alpha$ -irresolute) open and  $\lambda_{-1}^N$  is Onto, Neutrosophic pre- $\alpha$ -irresolute (Neutrosophic semi- $\alpha$ -irresolute, resp.) function then  $\mu_{-1}^N$  is Neutrosophic  $\alpha$ -irresolute open mapping.

*Proof.* Let  $C_N$  be any in  $N^\alpha OS\mathfrak{R}_N^2$ . Since  $\lambda_{-1}^N$  is an Neutrosophic pre- $\alpha$ -irresolute (Neutrosophic semi- $\alpha$ -irresolute.) function,  $\lambda_{-1}^{N-1}(C_N)$  is  $N^P OS\mathfrak{R}_N^1$  ( $N^\alpha OS\mathfrak{R}_N^1$ ). Also  $\mu_{-1}^N \circ \lambda_{-1}^N$  is Neutrosophic pre- $\alpha$  -irresolute (Neutrosophic semi-  $\alpha$  -irresolute.) open. Therefore  $(\mu_{-1}^N \circ \lambda_{-1}^N)\lambda_{-1}^{N-1}(C_N) = \mu_{-1}^N(C_N)$  is an  $N^\alpha OS\mathfrak{R}_N^3$  in  $(\mathfrak{R}_N^3, \mathfrak{S}_N^3)$ . Hence  $\mu_{-1}^N$  is an Neutrosophic- $\alpha$ -irresolute open mapping.  $\square$

**Theorem 4.2.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$ ,  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  and  $(\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  be Neutrosophic TSs. Let  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  and  $\mu_{-1}^N : (\mathfrak{R}_N^2, \mathfrak{S}_N^2) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  be any two maps. If

$\mu_{-1}^N \circ \lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  is an Neutrosophic  $\alpha$ -irresolute open and  $\lambda_{-1}^N$  is surjective, Neutrosophic  $\alpha$ -continuous function then  $\mu_{-1}^N$  is Neutrosophic  $\alpha$ -open mapping.

*Proof.* Let  $B_N$  be any in  $NOS\mathfrak{R}_N^2$ . Since  $\lambda_{-1}^N$  is an Neutrosophic  $\alpha$ -continuous function,  $\lambda_{-1}^{N-1}(B_N)$  is  $N^\alpha OS\mathfrak{R}_N^1$ . As  $\mu_{-1}^N \circ \lambda_{-1}^N$  is Neutrosophic  $\alpha$ -irresolute open, then  $((\mu_{-1}^N \circ \lambda_{-1}^N)(\lambda_{-1}^{N-1}(B_N))) = \mu_{-1}^N(B_N)$  is an  $N^\alpha OS\mathfrak{R}_N^3$  in  $(\mathfrak{R}_N^3, \mathfrak{S}_N^3)$ . Hence  $\mu_{-1}^N(B_N)$  is an Neutrosophic- $\alpha$  open mapping.  $\square$

**Theorem 4.3.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$ ,  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  and  $(\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  be Neutrosophic TSs. Let  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  and  $\mu_{-1}^N : (\mathfrak{R}_N^2, \mathfrak{S}_N^2) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  be any two maps. If  $\mu_{-1}^N \circ \lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  is Neutrosophic pre- $\beta$ -irresolute open and  $\lambda_{-1}^N$  is surjective, Neutrosophic pre irresolute function then  $\mu_{-1}^N$  is Neutrosophic pre- $\beta$  -irresolute open mapping.

*Proof.* Let  $B_N$  be any in  $NOS\mathfrak{R}_N^2$ . Since  $\lambda_{-1}^N(B_N)$  is Neutrosophic pre-irresolute function,  $\lambda_{-1}^{N-1}(B_N)$  is  $N^P OS\mathfrak{R}_N^1$ . As  $\mu_{-1}^N \circ \lambda_{-1}^N$  is Neutrosophic pre- $\beta$ -irresolute open,  $(\mu_{-1}^N \circ \lambda_{-1}^N)(\lambda_{-1}^{N-1}(B_N)) = \mu_{-1}^N(B_N)$  is an  $N^\beta OS\mathfrak{R}_N^3$  in  $(\mathfrak{R}_N^3, \mathfrak{S}_N^3)$ . Hence  $\mu_{-1}^N$  is an Neutrosophic pre- $\beta$  -irresolute open mapping.  $\square$

**Theorem 4.4.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$ ,  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  and  $(\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  be Neutrosophic TSs. Let  $\lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  and  $\mu_{-1}^N : (\mathfrak{R}_N^2, \mathfrak{S}_N^2) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  be any two maps. If  $\mu_{-1}^N \circ \lambda_{-1}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$ . Then the following statements hold:

1. If  $\lambda_{-1}^N$  is Neutrosophic pre- $\alpha$ -irresolute (Neutrosophic  $\alpha$ -irresolute and Neutrosophic semi- $\alpha$ - irresolute) open and  $\mu_{-1}^N$  is Neutrosophic  $\alpha$ -irresolute open



mappings, then  $\mu_{\perp}^N \circ \lambda_{\perp}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  is Neutrosophic pre- $\alpha$ -irresolute (Neutrosophic  $\alpha$ -irresolute and Neutrosophic semi- $\alpha$ -irresolute, resp.) open mapping.

2. If  $\lambda_{\perp}^N$  is Neutrosophic pre open (Neutrosophic  $\alpha$ -open and Neutrosophic semi open, resp.) mapping and  $\mu_{\perp}^N$  is an Neutrosophic pre- $\alpha$ -irresolute (Neutrosophic  $\alpha$ -irresolute and Neutrosophic semi- $\alpha$ -irresolute, resp.) open mapping then  $\mu_{\perp}^N \circ \lambda_{\perp}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  is an Neutrosophic-open mapping.
3. If  $\lambda_{\perp}^N$  is Neutrosophic pre irresolute open and  $\mu_{\perp}^N$  is Neutrosophic pre- $\beta$ -irresolute open then  $\mu_{\perp}^N \circ \lambda_{\perp}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  is Neutrosophic pre- $\beta$ -irresolute open mapping.
4. If  $\lambda_{\perp}^N$  is Neutrosophic pre open mapping and  $\mu_{\perp}^N$  is Neutrosophic pre- $\beta$ -irresolute open mapping then  $\mu_{\perp}^N \circ \lambda_{\perp}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^3, \mathfrak{S}_N^3)$  is an Neutrosophic- $\beta$ -open mapping.

*Proof.* 1. Let  $B_N$  be an  $N^P OS \mathfrak{R}_N^1$  ( $N^\alpha OS \mathfrak{R}_N^1$  and  $N^S OS \mathfrak{R}_N^1$ , resp.) in  $\mathfrak{R}_N^1$ . Since  $\lambda_{\perp}^N$  is Neutrosophic pre- $\alpha$ -irresolute (Neutrosophic  $\alpha$ -irresolute and Neutrosophic semi- $\alpha$ -irresolute, resp.) open,  $\lambda_{\perp}^N(B_N)$  is an  $N^\alpha OS$  in  $\mathfrak{R}_N^2$ . Now  $(\mu_{\perp}^N \circ \lambda_{\perp}^N)(B_N) = \mu_{\perp}^N(\lambda_{\perp}^N(B_N))$ . Also  $\mu_{\perp}^N$  is Neutrosophic- $\alpha$ -irresolute open,  $\mu_{\perp}^N(\lambda_{\perp}^N(B_N))$  is  $N^\alpha OS \mathfrak{R}_N^3$  in  $\mathfrak{R}_N^3$ . Hence  $\mu_{\perp}^N \circ \lambda_{\perp}^N$  is Neutrosophic pre- $\alpha$ -irresolute (Neutrosophic-irresolute and Neutrosophic semi- $\alpha$ -irresolute, resp.) open mapping.

2. Let  $B_N$  be an in  $NOS \mathfrak{R}_N^1$ . Since  $\lambda_{\perp}^N$  is Neutrosophic pre open (Neutrosophic  $\alpha$ -open and Neutrosophic semi open, resp.),  $\lambda_{\perp}^N(B_N)$  is  $N^P OS \mathfrak{R}_N^2$  ( $N^\alpha OS \mathfrak{R}_N^2$  and  $N^S OS \mathfrak{R}_N^2$ , resp.) in  $\mathfrak{R}_N^2$ . Now  $(\mu_{\perp}^N \circ \lambda_{\perp}^N)(B_N) = \mu_{\perp}^N(\lambda_{\perp}^N(B_N))$ . As  $\mu_{\perp}^N$  is Neutrosophic pre- $\alpha$  (Neutrosophic  $\alpha$  and Neutrosophic semi- $\alpha$  resp.)-irresolute open,  $\mu_{\perp}^N(\lambda_{\perp}^N(B_N))$  is  $N^\alpha OS \mathfrak{R}_N^3$  in  $\mathfrak{R}_N^3$ . Hence  $\mu_{\perp}^N \circ \lambda_{\perp}^N$  is Neutrosophic  $\alpha$ -open mapping.
3. Let  $A_N$  be an  $N^P OS \mathfrak{R}_N^1$  in  $\mathfrak{R}_N^1$ . Since  $\lambda_{\perp}^N$  is Neutrosophic pre irresolute open  $\lambda_{\perp}^N(A_N)$  is an  $N^P OS \mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Now  $(\mu_{\perp}^N \circ \lambda_{\perp}^N)(A_N) = \mu_{\perp}^N(\lambda_{\perp}^N(A_N))$ . But  $\mu_{\perp}^N$  is Neutrosophic pre- $\beta$ -irresolute open,  $\mu_{\perp}^N(\lambda_{\perp}^N(A_N))$  is  $N^\beta OS \mathfrak{R}_N^3$  in  $\mathfrak{R}_N^3$ . Hence  $\mu_{\perp}^N \circ \lambda_{\perp}^N$  is Neutrosophic pre- $\beta$ -irresolute open mapping.
4. Let  $B_N$  be an in  $NOS \mathfrak{R}_N^1$ . Since  $\lambda_{\perp}^N$  is Neutrosophic preopen,  $\lambda_{\perp}^N(B_N)$  is an  $N^P OS \mathfrak{R}_N^2$ . Now  $(\mu_{\perp}^N \circ \lambda_{\perp}^N)(B_N) = \mu_{\perp}^N(\lambda_{\perp}^N(B_N))$ . But  $\mu_{\perp}^N$  is Neutrosophic pre- $\beta$ -irresolute open,  $\mu_{\perp}^N(\lambda_{\perp}^N(B_N))$  is  $N^\beta OS \mathfrak{R}_N^3$  in  $\mathfrak{R}_N^3$ . Hence  $\mu_{\perp}^N \circ \lambda_{\perp}^N$  is Neutrosophic  $\beta$  open mapping. □

**Theorem 4.5.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  and  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  be two Neutrosophic Topological spaces and let  $\lambda_{\perp}^N : (\mathfrak{R}_N^1, \mathfrak{S}_N^1) \rightarrow (\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  be a mapping. Then the following conditions are equivalent:

1.  $\lambda_{\perp}^N$  is Neutrosophic pre- $\alpha$ -irresolute open mapping.
2.  $\mu_{\perp}^N(Pre_{Neu} int A_N) \subseteq \overset{\alpha}{Neu} int \mu_{\perp}^N(A_N)$  for each Neutrosophic set  $A_N$  in  $\mathfrak{R}_N^1$ .
3.  $Pre_{Neu} int(\lambda_{\perp}^{N-1}(B_N)) \subseteq \lambda_{\perp}^{N-1}(\overset{\alpha}{Neu} int B_N)$  for each Neutrosophic set  $B_N$  in  $\mathfrak{R}_N^2$ .
4. For any  $NS, A_N$  in  $\mathfrak{R}_N^1$  and  $NSB_N$  in  $\mathfrak{R}_N^2$  and let  $A_N$  be  $N^P CSC \mathfrak{R}_N^1$  such that  $\lambda_{\perp}^{N-1}(B_N) \subseteq A_N$ . Then there exists an  $C_N \in N^\alpha CSC \mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$  and  $B_N \subseteq C_N$  such that  $\lambda_{\perp}^{N-1}(C_N) \subseteq A_N$ .

*Proof.* (i)  $\Rightarrow$  (ii):

Here  $Pre_{Neu} int A_N \subseteq A_N \Rightarrow \lambda_{\perp}^N(Pre_{Neu} int A_N) \subseteq \lambda_{\perp}^N(A_N)$ . But,  $Pre_{Neu} int A_N$  is an  $N^P OS \mathfrak{R}_N^1$ . And  $\lambda_{\perp}^N(Pre_{Neu} int A_N)$  is an  $N^\alpha OS \mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Hence

$$\lambda_{\perp}^N(Pre_{Neu} int A_N) = \overset{\alpha}{Neu} int \lambda_{\perp}^N(Pre_{Neu} int A_N) \subseteq \overset{\alpha}{Neu} int \lambda_{\perp}^N(A_N).$$

(ii)  $\Rightarrow$  (iii):

Let  $A_N = \lambda_{\perp}^{N-1}(B_N)$  By (ii),

$$\lambda_{\perp}^N(Pre_{Neu} int(\lambda_{\perp}^{N-1}(B_N))) \subseteq \overset{\alpha}{Neu} int \lambda_{\perp}^N(\lambda_{\perp}^{N-1}(B_N)) \subseteq \overset{\alpha}{Neu} int(B_N)$$

which gives

$$\begin{aligned} Pre_{Neu} int(\lambda_{\perp}^{N-1}(B_N)) &\subseteq \lambda_{\perp}^{N-1}(\lambda_{\perp}^N(Pre_{Neu} int(\lambda_{\perp}^{N-1}(B_N)))) \\ &\subseteq \lambda_{\perp}^{N-1}(\overset{\alpha}{Neu} int(B_N)). \end{aligned}$$

Thus  $Pre_{Neu} int(\lambda_{\perp}^{N-1}(B_N)) \subseteq \lambda_{\perp}^{N-1}(\overset{\alpha}{Neu} int(B_N))$ .

(iii)  $\Rightarrow$  (iv) :

Let  $A_N$  be  $N^P CS$  in  $\mathfrak{R}_N^1$  and  $B_N$  be an Neutrosophic set in  $\mathfrak{R}_N^2$ .

Such that  $\lambda_{\perp}^{N-1}(B_N) \subseteq A_N$ . Then

$$\overline{\lambda_{\perp}^{N-1}(B_N)} \supseteq \overline{A_N} \Rightarrow \overline{A_N} \subseteq \overline{\lambda_{\perp}^{N-1}(B_N)} = \lambda_{\perp}^{N-1}(B_N).$$

But  $\overline{A_N}$  is an  $N^P OS \mathfrak{R}_N^1$ . Thus

$$\overline{A_N} = Pre_{Neu} int(A_N) \subseteq Pre_{Neu} int(\lambda_{\perp}^{N-1}(B_N)) \subseteq (\lambda_{\perp}^N)^{-1}(\overset{\alpha}{Neu} int(B_N)).$$

Hence  $(\lambda_{\perp}^N)^{-1}(\overset{\alpha}{Neu} int(\overline{B_N})) = \lambda_{\perp}^{N-1}(\alpha cl(B_N))$ . Take

$\overset{\alpha}{Neu} cl(B_N) = C_N$ . Therefore  $\lambda_{\perp}^{N-1}(C_N) \subseteq A_N$ .

(iv)  $\Rightarrow$  (i) :

Let  $D$  be an  $N^P OS \mathfrak{R}_N^1$ . And  $B_N = \overline{\lambda_{\perp}^N(D)}$  and  $A_N = \overline{D}$ . Then  $A$  is an  $N^P CSC \mathfrak{R}_N^1$ . Hence

$$\lambda_{\perp}^{N-1}(B_N) = \lambda_{\perp}^{N-1}(\overline{\lambda_{\perp}^N(D)}) = \overline{\lambda_{\perp}^{N-1} \lambda_{\perp}^N(D)} \subseteq \overline{D} = A_N.$$

Then there exists an  $N^\alpha CSC_N$  and  $B_N \subseteq C_N$  such that  $\lambda_{\perp}^{N-1}(C_N) \subseteq A_N = \overline{D}$ . Thus  $D \subseteq \lambda_{\perp}^{N-1}(C_N)$ , which implies  $\lambda_{\perp}^N(D) \subseteq \lambda_{\perp}^N(\lambda_{\perp}^{N-1}(\overline{C_N})) \subseteq \overline{C_N}$ . On the other hand, by  $\overline{B_N} \subseteq C_N, \lambda_{\perp}^N(D) = \overline{B_N} \supseteq \overline{C_N}$ . Hence  $\lambda_{\perp}^N(D) = \overline{C_N}$ . Since  $\overline{C_N}$  is an  $N^\alpha OS$ , then  $\lambda_{\perp}^N(D)$  is an  $N^\alpha OS \mathfrak{R}_N^2$ . Therefore  $\lambda_{\perp}^N$  is Neutrosophic pre- $\alpha$ -irresolute open mapping. □





**Theorem 4.6.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  and  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  be two NTSSs and let  $\lambda_{-1}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  be a mapping. Then the following conditions are equivalent:

1.  $\lambda_{-1}^N$  is Neutrosophic  $\alpha$ -irresolute open mapping.
2.  $\lambda_{-1}^N(\alpha_{Neu} int A_N) \subseteq \alpha_{Neu} int \lambda_{-1}^N(A_N)$  for each Neutrosophic set in  $\mathfrak{R}_N^1$ .
3.  $\alpha_{Neu} int((\lambda_{-1}^N)^{-1}(B_N)) \subseteq \alpha_{Neu} int(\lambda_{-1}^N)^{-1}(B_N)$  for each Neutrosophic set  $B$  in  $\mathfrak{R}_N^2$ .
4. For any Neutrosophic set  $A_N$  in  $\mathfrak{R}_N^1$ , Neutrosophic set  $B_N$  in  $\mathfrak{R}_N^2$  and let  $A_N$  be  $N^\alpha CS\mathfrak{R}_N^1$  such that  $\lambda_{-1}^N(A_N) \subseteq B_N$ . Then there exists an  $N^\alpha CS\mathfrak{R}_N^2$ ,  $C_N$  in  $\mathfrak{R}_N^2$  and  $B_N \subseteq C_N$  such that  $\lambda_{-1}^N(C_N) \subseteq A_N$ .

*Proof.* (i)  $\Rightarrow$  (ii):

$\alpha_{Neu} int A_N \subseteq A_N \Rightarrow \lambda_{-1}^N(\alpha_{Neu} int A_N) \subseteq \lambda_{-1}^N(A_N)$ . But  $\alpha_{Neu} int A_N$  is an  $N^\alpha OS\mathfrak{R}_N^1$ ,  $\lambda_{-1}^N(\alpha_{Neu} int A_N)$  is an  $N^\alpha OS$  in  $\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N(\alpha_{Neu} int A_N) = \alpha_{Neu} int \lambda_{-1}^N(\alpha_{Neu} int A_N) \subseteq \alpha_{Neu} int \lambda_{-1}^N(A_N)$ .

(ii)  $\Rightarrow$  (iii):

Let  $A_N = \lambda_{-1}^N(B_N)$ . By (ii),  $\lambda_{-1}^N(\alpha_{Neu} int(\lambda_{-1}^N(B_N))) \subseteq \alpha_{Neu} int(B_N)$  which implies  $\alpha_{Neu} int((\lambda_{-1}^N)^{-1}(B_N)) \subseteq \lambda_{-1}^N(\alpha_{Neu} int(\lambda_{-1}^N)^{-1}(B_N)) \subseteq \lambda_{-1}^N(\alpha_{Neu} int B_N)$ . Thus,  $\alpha_{Neu} int(\lambda_{-1}^N)^{-1}(B_N) \subseteq \lambda_{-1}^N(\alpha_{Neu} int B_N)$ .

(iii)  $\Rightarrow$  (iv):

Let  $A_N$  be  $N^\alpha CS\mathfrak{R}_N^1$  and  $B_N$  be an Neutrosophic set in  $\mathfrak{R}_N^2$ . Such that  $\lambda_{-1}^N(A_N) \subseteq B_N$ . Hence  $\overline{\lambda_{-1}^N(A_N)} \supseteq \overline{A_N} \Rightarrow \overline{A_N} \subseteq \lambda_{-1}^N(B_N) = \lambda_{-1}^N(\overline{B_N})$ . But  $\overline{A_N}$  is an  $N^\alpha OS\mathfrak{R}_N^1$ . Thus  $\overline{A_N} = \alpha_{Neu} int(\overline{A_N}) \subseteq \alpha_{Neu} int(\lambda_{-1}^N(B_N)) \subseteq \lambda_{-1}^N(\alpha_{Neu} int(\overline{B_N}))$ . As  $A_N \supseteq \lambda_{-1}^N(\alpha_{Neu} int(\overline{B_N})) = \lambda_{-1}^N(\alpha cl(B_N))$ . Put  $\alpha cl(B_N) = C_N$ . Hence  $\lambda_{-1}^N(C_N) \subseteq A_N$ .

(iv)  $\Rightarrow$  (i):

Let  $D$  be an  $N^\alpha OS\mathfrak{R}_N^1$ ,  $B_N = \lambda_{-1}^N(D)$  and  $A_N = \overline{D}$ . Then  $A_N$  is an  $N^\alpha CS\mathfrak{R}_N^1$ . Hence  $\lambda_{-1}^N(A_N) = \lambda_{-1}^N(\overline{\lambda_{-1}^N(D)}) = \overline{\lambda_{-1}^N(\lambda_{-1}^N(D))} = A_N$ . Then, there exists an  $N^\alpha CS\mathfrak{R}_N^1$ ,  $C_N$  and  $B_N \subseteq C_N$ . Such that  $\lambda_{-1}^N(C_N) \subseteq A_N = \overline{D}$ . Thus,  $D \subseteq (\lambda_{-1}^N)^{-1}(C_N) \Rightarrow \lambda_{-1}^N(D) \subseteq \lambda_{-1}^N((\lambda_{-1}^N)^{-1}(C_N)) \subseteq \overline{C_N}$ . On the other hand by  $B_N \subseteq C_N$ ,  $\lambda_{-1}^N(D) = \overline{B_N} \supseteq \overline{C_N}$ . Therefore  $\lambda_{-1}^N(D) = \overline{C_N}$ . As  $\overline{C_N}$  is an  $N^\alpha OS\mathfrak{R}_N^1$ ,  $\lambda_{-1}^N(D)$  is an  $N^\alpha OS\mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is Neutrosophic  $\alpha$ -irresolute open mapping.  $\square$

**Theorem 4.7.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  and  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  be two NTSSs and let  $\lambda_{-1}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  be a mapping. Then the following conditions are equivalent:

1.  $\lambda_{-1}^N$  is Neutrosophic semi- $\alpha$ -irresolute open mapping.
2.  $\lambda_{-1}^N(S_{Neu} int(A_N)) \subseteq \alpha_{Neu} int \lambda_{-1}^N(A_N)$  for each Neutrosophic set  $A_N$  in  $\mathfrak{R}_N^1$ .
3.  $S_{Neu} int(\lambda_{-1}^N)^{-1}(B_N) \subseteq \lambda_{-1}^N(\alpha_{Neu} int B_N)$  for each Neutrosophic set  $B_N$  in  $\mathfrak{R}_N^2$ .

4. For any Neutrosophic set in  $\mathfrak{R}_N^1$ , Neutrosophic set in  $\mathfrak{R}_N^2$  and let  $A_N$  be  $N^\alpha CS\mathfrak{R}_N^1$  such that  $\lambda_{-1}^N(A_N) \subseteq B_N$ . Then there exists an  $N^\alpha CS\mathfrak{R}_N^2$ ,  $C_N$  in  $\mathfrak{R}_N^2$  and  $B_N \subseteq C_N$  such that  $\lambda_{-1}^N(C_N) \subseteq A_N$ .

*Proof.* (i)  $\Rightarrow$  (ii):

$S_{Neu} int A_N \subseteq A_N \Rightarrow \lambda_{-1}^N(S_{Neu} int A_N) \subseteq \lambda_{-1}^N(A_N)$ . But  $S_{Neu} int A_N$  is an  $N^\alpha OS\mathfrak{R}_N^1$ ,  $\lambda_{-1}^N(S_{Neu} int A_N)$  is an  $N^\alpha OS\mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N(S_{Neu} int A_N) = \alpha_{Neu} int \lambda_{-1}^N(S_{Neu} int A_N) \subseteq \alpha_{Neu} int \lambda_{-1}^N(A_N)$ .

(ii)  $\Rightarrow$  (iii):

Let  $A_N = \lambda_{-1}^N(B_N)$ . By (ii),  $\lambda_{-1}^N(S_{Neu} int(\lambda_{-1}^N(B_N))) \subseteq \alpha_{Neu} int \lambda_{-1}^N(\lambda_{-1}^N(B_N)) \subseteq \alpha_{Neu} int(B_N)$  which implies  $S_{Neu} int(\lambda_{-1}^N)^{-1}(B_N) \subseteq \lambda_{-1}^N(\alpha_{Neu} int(\lambda_{-1}^N)^{-1}(B_N)) \subseteq \lambda_{-1}^N(\alpha_{Neu} int B_N)$ . Thus,  $S_{Neu} int(\lambda_{-1}^N)^{-1}(B_N) \subseteq (\lambda_{-1}^N)^{-1}(\alpha_{Neu} int B_N)$ .

(iii)  $\Rightarrow$  (iv):

Let  $A_N$  be  $N^\alpha CS\mathfrak{R}_N^1$  and  $B_N$  be an Neutrosophic set in  $\mathfrak{R}_N^2$  such that  $\lambda_{-1}^N(A_N) \subseteq B_N$ . Hence  $\overline{\lambda_{-1}^N(A_N)} \supseteq \overline{A_N} \Rightarrow \overline{A_N} \subseteq \lambda_{-1}^N(B_N) = \lambda_{-1}^N(\overline{B_N})$ . But  $\overline{A_N}$  is an  $N^\alpha OS\mathfrak{R}_N^1$ . Thus,  $\overline{A_N} = S_{Neu} int(\overline{A_N}) \subseteq S_{Neu} int(\lambda_{-1}^N(B_N)) \subseteq \lambda_{-1}^N(\alpha_{Neu} int(\overline{B_N}))$ . Hence  $A_N \supseteq \lambda_{-1}^N(\alpha_{Neu} int(\overline{B_N})) = \lambda_{-1}^N(\alpha_{Neu} cl(B_N))$ . Put  $\alpha_{Neu} cl(B_N) = C_N$ , obtain  $\lambda_{-1}^N(C_N) \subseteq A_N$ .

(iv)  $\Rightarrow$  (i):

Let  $D$  be an  $N^\alpha OS\mathfrak{R}_N^1$ ,  $B_N = \overline{\lambda_{-1}^N(D)}$  and  $A_N = \overline{D}$ . Then  $A_N$  is an  $N^\alpha CS\mathfrak{R}_N^1$ . Hence  $\lambda_{-1}^N(A_N) = \lambda_{-1}^N(\overline{\lambda_{-1}^N(D)}) = \overline{\lambda_{-1}^N(\lambda_{-1}^N(D))}$ . Then, there exists an  $N^\alpha CS\mathfrak{R}_N^1$ ,  $C_N$  and  $B_N \subseteq C_N$ . Such that  $\lambda_{-1}^N(C_N) \subseteq A_N = \overline{D}$ , thus,  $D \subseteq \lambda_{-1}^N(C_N) \Rightarrow \lambda_{-1}^N(D) \subseteq \lambda_{-1}^N(\lambda_{-1}^N(C_N)) \subseteq \overline{C_N}$ . On the other hand by  $B_N \subseteq C_N$ ,  $\lambda_{-1}^N(D) = \overline{B_N} \supseteq \overline{C_N}$ . Hence  $\lambda_{-1}^N(D) = \overline{C_N}$ . Since  $\overline{C_N}$  is an  $N^\alpha OS\mathfrak{R}_N^1$ ,  $\lambda_{-1}^N(D)$  is an  $N^\alpha OS\mathfrak{R}_N^2$ . Therefore  $\lambda_{-1}^N$  is Neutrosophic semi- $\alpha$ -irresolute open mapping.  $\square$

**Theorem 4.8.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  and  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  be two NTSSs and let  $\lambda_{-1}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  be a mapping. Then the following conditions are equivalent:

1.  $\lambda_{-1}^N$  is a Neutrosophic pre- $\beta$ -irresolute open mapping.
2.  $\lambda_{-1}^N(Pre_{Neu} int A_N) \subseteq \beta_{Neu} int \lambda_{-1}^N(A_N)$  for each Neutrosophic set in  $\mathfrak{R}_N^1$ .
3.  $Pre_{Neu} int(\lambda_{-1}^N)^{-1}(B_N) \subseteq \lambda_{-1}^N(\beta_{Neu} int B)$  for each Neutrosophic set  $B_N$  in  $\mathfrak{R}_N^2$ .
4. For any Neutrosophic set  $A_N$  in  $\mathfrak{R}_N^1$ , Neutrosophic set  $B_N$  in  $\mathfrak{R}_N^2$  and let  $A_N$  be  $N^\beta CS\mathfrak{R}_N^1$  such that  $\lambda_{-1}^N(A_N) \subseteq B_N$ . Then there exists an  $N^\beta CS\mathfrak{R}_N^2$ ,  $C_N$  in  $\mathfrak{R}_N^2$  and  $B_N \subseteq C_N$  such that  $\lambda_{-1}^N(C_N) \subseteq A_N$ .

*Proof.* (i)  $\Rightarrow$  (ii):

$Pre_{Neu} int A_N \subseteq A_N \Rightarrow \lambda_{-1}^N(Pre_{Neu} int A_N) \subseteq \lambda_{-1}^N(A_N)$ . But  $Pre_{Neu} int A_N$  is an  $N^\beta OS$  in  $\mathfrak{R}_N^1$ ,  $\lambda_{-1}^N(Pre_{Neu} int A_N)$  is an Neutrosophic  $N^\beta OS$  in



$\mathfrak{R}_N^2$ . Hence  $\lambda_{\downarrow}^N(\text{Pre int } A_N) = \beta \text{ int } \lambda_{\downarrow}^N(\text{Pre int } A_N) \subseteq \beta \text{ int } \lambda_{\downarrow}^N(A_N)$ .

(ii)  $\Rightarrow$  (iii):

Let  $A_N = \lambda_{\downarrow}^{N^{-1}}$ . By (ii),  $\lambda_{\downarrow}^N(\text{Pre int } (\lambda_{\downarrow}^{N^{-1}}(B_N))) \subseteq \beta \text{ int } \lambda_{\downarrow}^N(\lambda_{\downarrow}^{N^{-1}}(B_N)) \subseteq \beta \text{ int } (B_N)$  which implies  $\text{Pre int } (\lambda_{\downarrow}^{N^{-1}}(B_N)) \subseteq \lambda_{\downarrow}^{N^{-1}}(\lambda_{\downarrow}^N(\text{Pre int } (\lambda_{\downarrow}^{N^{-1}}(B_N)))) \subseteq \lambda_{\downarrow}^{N^{-1}}(\beta \text{ int } (B_N))$ . Thus  $\text{Pre int } (\lambda_{\downarrow}^{N^{-1}}(B_N)) \subseteq \lambda_{\downarrow}^{N^{-1}}(\beta \text{ int } B)$ .

(iii)  $\Rightarrow$  (iv):

Let  $A_N$  be  $N^P \text{CS} \mathfrak{R}_N^1$  and  $B_N$  be a Neutrosophic set in  $\mathfrak{R}_N^2$ . Such that  $\lambda_{\downarrow}^{N^{-1}}(B_N) \subseteq A_N$ . Therefore  $\overline{\lambda_{\downarrow}^{N^{-1}}(B_N)} \supseteq \overline{A_N}$  which implies  $\overline{A_N} \subseteq \overline{\lambda_{\downarrow}^{N^{-1}}(B_N)} = \lambda_{\downarrow}^{N^{-1}}(\overline{B_N})$ . But  $\overline{A_N}$  is an  $N^P \text{OS} \mathfrak{R}_N^1$ . Thus  $\overline{A_N} = \text{Pre int } (\overline{A_N}) \subseteq \text{Pre int } (\lambda_{\downarrow}^{N^{-1}}(\overline{B_N})) \subseteq \lambda_{\downarrow}^{N^{-1}}(\beta \text{ int } (\overline{B_N}))$ . Hence  $A_N \supseteq \lambda_{\downarrow}^{N^{-1}}(\beta \text{ int } (\overline{B_N})) = \lambda_{\downarrow}^{N^{-1}}(\beta \text{ int } (B_N))$ . Take  $\beta \text{ int } (B_N) = C_N$ . Therefore  $\lambda_{\downarrow}^{N^{-1}}(C_N) \subseteq A_N$ .

(iv)  $\Rightarrow$  (i):

Let  $D$  be an  $N^P \text{OS}$  in  $\mathfrak{R}_N^1$ ,  $B_N = \overline{\lambda_{\downarrow}^N(D)}$  and  $A_N = \overline{D}$ . Then  $A_N$  is an  $N^P \text{CS} \mathfrak{R}_N^1$ . Hence  $\lambda_{\downarrow}^{N^{-1}}(B_N) = \lambda_{\downarrow}^{N^{-1}}(\overline{\lambda_{\downarrow}^N(D)}) = \overline{\lambda_{\downarrow}^{N^{-1}}(\lambda_{\downarrow}^N(D))} \subseteq \overline{D} = A_N$ . Then there exists an  $N^{\beta} \text{CSC}_N$  and  $B_N \subseteq C_N$  such that  $\lambda_{\downarrow}^{N^{-1}}(C_N) \subseteq A_N = \overline{D}$ . Thus  $D \subseteq \lambda_{\downarrow}^{N^{-1}}(C_N) \Rightarrow \lambda_{\downarrow}^N(D) \subseteq \lambda_{\downarrow}^N(\lambda_{\downarrow}^{N^{-1}}(C_N)) \subseteq C_N$ . On the other hand by  $B_N \subseteq C_N$ ,  $\lambda_{\downarrow}^N(D) = \overline{B_N} \supseteq \overline{C_N}$ . Hence  $\lambda_{\downarrow}^N(D) = \overline{C}$ . Since  $\overline{C}$  is an  $N^{\beta} \text{OS}$ ,  $\lambda_{\downarrow}^N(D)$  is an  $N^{\beta} \text{OS}$  in  $\mathfrak{R}_N^2$ . Therefore  $\lambda_{\downarrow}^N$  is Neutrosophic pre- $\beta$ -irresolute open mapping.  $\square$

## 5. Properites of Neutrosophic PRE- $\alpha$ , SEMI- $\alpha$ and PRE- $\beta$ Irresolute closed Mappings

**Theorem 5.1.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  and  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  be two NTSs. A function  $\text{let } \lambda_{\downarrow}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  Neutrosophic pre- $\alpha$ -irresolute closed mapping if and only if  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\text{pre cl } A_N)$  for each  $N^{\mathcal{S}}$ ,  $A_N$  in  $\text{NTS} \mathfrak{R}_N^1$ .

*Proof.* Let  $\lambda_{\downarrow}^N$  be Neutrosophic pre- $\alpha$ -irresolute closed mapping, then  $\lambda_{\downarrow}^N(\text{pre cl } A_N)$  is  $N^{\alpha} \text{CS} \mathfrak{R}_N^2$ . Therefore  $\lambda_{\downarrow}^N(\text{pre cl } A_N) = \alpha \text{ Neu cl } \lambda_{\downarrow}^N(\text{pre cl } A_N)$  and  $\lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$ . Thus  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \alpha \text{ Neu cl } \lambda_{\downarrow}^N(\text{pre cl } A_N) = \lambda_{\downarrow}^N(\text{pre cl } A_N)$ . Hence  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\text{pre cl } A_N)$ .

Conversely, Let  $A_N$  be an  $N^P \text{CS} \mathfrak{R}_N^1$ . Then  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\text{pre cl } A_N) = \lambda_{\downarrow}^N(A_N)$ . Thus  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N$ . But  $\lambda_{\downarrow}^N(A_N) \subseteq \alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N)$ . So  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) = \lambda_{\downarrow}^N(A_N)$ . Therefore  $\lambda_{\downarrow}^N(A_N)$  is an  $N^{\alpha} \text{CS} \mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Hence  $\lambda_{\downarrow}^N$  is Neutrosophic pre- $\alpha$ -irresolute closed mapping.  $\square$

**Theorem 5.2.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  and  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  be two NTSs. A function  $\text{let } \lambda_{\downarrow}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  is Neutrosophic  $\alpha$ -irresolute closed mapping if and only if  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$  for each Neutrosophic set  $A_N$  in  $\text{NTS} \mathfrak{R}_N^1$ .

*Proof.* Let  $\lambda_{\downarrow}^N$  is Neutrosophic  $\alpha$ -irresolute closed mapping, then  $\lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$  is  $N^{\alpha} \text{CS} \mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Therefore  $\lambda_{\downarrow}^N$

$(\alpha \text{ Neu cl } A_N) = \alpha \text{ Neu cl } \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$  and  $\lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$ . Thus  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$ .

Conversely, Let  $A_N$  be an  $N^{\alpha} \text{CS} \mathfrak{R}_N^1$ . Then  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N) = \lambda_{\downarrow}^N(A_N)$ . Thus  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(A_N)$ . But  $\lambda_{\downarrow}^N(A_N) \subseteq \alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N)$  obtain  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) = \lambda_{\downarrow}^N(A_N)$ . Therefore  $\lambda_{\downarrow}^N(A_N)$  is an  $N^{\alpha} \text{CS} \mathfrak{R}_N^2$ . Hence  $\lambda_{\downarrow}^N$  is Neutrosophic  $\alpha$ -irresolute closed mapping.  $\square$

**Theorem 5.3.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  and  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  be two NTSs. A function  $\lambda_{\downarrow}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  is Neutrosophic semi- $\alpha$ -irresolute closed mapping if and only if  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$  for each Neutrosophic set  $A_N$  in  $\text{NTS} \mathfrak{R}_N^1$ .

*Proof.* Let  $\lambda_{\downarrow}^N$  Neutrosophic semi- $\alpha$ -irresolute closed mapping, then  $\lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$  in  $N^{\alpha} \text{CS} \mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Therefore  $\lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N) = \alpha \text{ Neu cl } \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$  also  $\lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$ . Thus  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \alpha \text{ Neu cl } \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N) = \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$ . Hence  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N)$ .

Conversely, let  $A_N$  be an  $N^{\mathcal{S}} \text{CS} \mathfrak{R}_N^1$ . Then  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\alpha \text{ Neu cl } A_N) = \lambda_{\downarrow}^N(A_N)$ . Thus  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N$ . But  $\lambda_{\downarrow}^N(A_N) \subseteq \alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N)$  obtain  $\alpha \text{ Neu cl } \lambda_{\downarrow}^N(A_N) = \lambda_{\downarrow}^N(A_N)$ . Thus  $\lambda_{\downarrow}^N(A_N)$  is an  $N^{\alpha} \text{CS} \mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Hence  $\lambda_{\downarrow}^N$  is Neutrosophic semi- $\alpha$ -irresolute closed mapping.  $\square$

**Theorem 5.4.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  and  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  be two NTSs. A function  $\text{let } \lambda_{\downarrow}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  is Neutrosophic pre- $\beta$ -irresolute closed mapping if and only if  $\beta \text{ Neu Cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\text{pre cl } A_N)$  for each Neutrosophic set  $A_N$  in  $\text{NTS} \mathfrak{R}_N^1$ .

*Proof.* Let  $\lambda_{\downarrow}^N$  be Neutrosophic pre- $\beta$ -irresolute closed mapping, then  $\lambda_{\downarrow}^N(\text{pre cl } A_N)$  is Neutrosophic  $N^{\beta} \text{CS}$  in  $\mathfrak{R}_N^2$ . Therefore  $\lambda_{\downarrow}^N(\text{pre cl } A_N) = \beta \text{ Neu Cl } \lambda_{\downarrow}^N(\text{pre cl } A_N)$ . Also  $\lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\text{pre cl } A_N)$ . Thus  $\beta \text{ Neu Cl } \lambda_{\downarrow}^N(A_N) \subseteq \beta \text{ Neu Cl } \lambda_{\downarrow}^N(\text{pre cl } A_N) = \lambda_{\downarrow}^N(\text{pre cl } A_N)$ . Hence  $\beta \text{ Neu Cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\text{pre cl } A_N)$ . Conversely, Let  $A_N$  be an  $N^{\beta} \text{CS} \mathfrak{R}_N^1$  in  $\mathfrak{R}_N^1$ . Then  $\beta \text{ Neu Cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(\text{pre cl } A_N) = \lambda_{\downarrow}^N(A_N)$ . Thus  $\beta \text{ Neu Cl } \lambda_{\downarrow}^N(A_N) \subseteq \lambda_{\downarrow}^N(A_N)$ . But  $\lambda_{\downarrow}^N(A_N) \subseteq \beta \text{ Neu Cl } \lambda_{\downarrow}^N(A_N)$ . So,  $\beta \text{ Neu Cl } \lambda_{\downarrow}^N(A_N) = \lambda_{\downarrow}^N(A_N)$ . Therefore  $\lambda_{\downarrow}^N(A_N)$  is an Neutrosophic  $N^{\beta} \text{CS} \mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Hence  $\lambda_{\downarrow}^N$  is Neutrosophic pre- $\beta$ -irresolute closed mapping.  $\square$

**Theorem 5.5.** Let  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  and  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  be two NTSs. A function  $\text{let } \lambda_{\downarrow}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  is Neutrosophic pre- $\alpha$ -irresolute (Neutrosophic  $\alpha$ -irresolute and Neutrosophic semi- $\alpha$ -irresolute, resp.) closed mapping if and only if for each Neutrosophic set  $B_N$  in  $\mathfrak{R}_N^2$  and each  $N^P \text{OS} \mathfrak{R}_N^2$  ( $N^{\alpha} \text{OS} \mathfrak{R}_N^2$  and  $N^{\mathcal{S}} \text{OS} \mathfrak{R}_N^2$ , resp.)  $A_N$  in  $\mathfrak{R}_N^1$  with  $A_N \supseteq (\lambda_{\downarrow}^N)^{-1}(B_N)$  there exists an  $N^{\alpha} \text{OS} \mathfrak{R}_N^2$ ,  $C_N$  in  $\mathfrak{R}_N^2$  with  $C_N \supseteq B_N$  such that  $\lambda_{\downarrow}^{N^{-1}}(C_N) \subseteq A_N$ .

*Proof.* Let  $A_N$  be any arbitrary  $N^P \text{OS} \mathfrak{R}_N^1$  ( $N^{\alpha} \text{OS} \mathfrak{R}_N^1$  and  $N^{\mathcal{S}} \text{OS} \mathfrak{R}_N^1$ , resp.) in  $\mathfrak{R}_N^1$ . With  $A_N \supseteq (\lambda_{\downarrow}^N)^{-1}(B_N)$  where  $B_N$  is an Neutrosophic set in  $\mathfrak{R}_N^2$ . Then  $\overline{A_N}$  is an



$N^P CS\mathfrak{R}_N^1(N^\alpha CS\mathfrak{R}_N^1$  and  $N^\mathcal{S} CS\mathfrak{R}_N^1$ , resp.) in  $\mathfrak{R}_N^1$ . Since  $\lambda_{-1}^N$  is Neutrosophic pre- $\alpha$ -irresolute (Neutrosophic  $\alpha$ -irresolute and Neutrosophic semi- $\alpha$  irresolute, resp.) closed mapping  $\lambda_{-1}^N(\overline{A_N})$  is  $N^\alpha CS\mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Then  $\overline{\lambda_{-1}^N(A_N)} = C_N$  (say) is  $N^\alpha OS\mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Since  $\lambda_{-1}^{N-1}(B_N) \subseteq A_N$ ,  $B_N \subseteq C_N$ . Moreover, obtain  $\lambda_{-1}^{N-1}(C_N) = \lambda_{-1}^{N-1}(\overline{\lambda_{-1}^N(A_N)}) = \lambda_{-1}^{N-1}(\overline{\lambda_{-1}^N(A_N)}) \subseteq A_N$ . Thus  $\lambda_{-1}^{N-1}(C_N) \subseteq A_N$ . Conversely Let  $A_N$  be  $N^P CS\mathfrak{R}_N^1(N^\alpha CS\mathfrak{R}_N^1$  and  $N^\mathcal{S} CS\mathfrak{R}_N^1$ , resp.) in  $\mathfrak{R}_N^1$ . Then  $\overline{\lambda_{-1}^N(A_N)} = B_N$  (say) is a Neutrosophic set in  $\mathfrak{R}_N^2$  and  $\overline{A_N}$  is  $N^P OS\mathfrak{R}_N^1(N^\alpha OS\mathfrak{R}_N^1$  and  $N^\mathcal{S} OS\mathfrak{R}_N^1$ , resp.) in  $\mathfrak{R}_N^1$ . Such that  $\lambda_{-1}^{N-1}(B_N) \subseteq \overline{A_N}$ . By hypothesis, there is an  $N^\alpha OS\mathfrak{R}_N^2, C_N$  of  $\mathfrak{R}_N^2$ . Such that  $B_N \subseteq C_N$  and  $\lambda_{-1}^{N-1}(C_N) \subseteq \overline{A_N}$ . Therefore,  $A_N \subseteq \lambda_{-1}^{N-1}(C_N)$ . Hence  $\overline{C_N} \subseteq \overline{B_N} = \lambda_{-1}^N(A_N) \subseteq \lambda_{-1}^N(\lambda_{-1}^{N-1}(C_N))\lambda_{-1}^N(A_N) = \overline{C_N}$ . Since  $\overline{C_N}$  is  $N^\alpha CS\mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ ,  $\lambda_{-1}^N(A_N)$  is  $N^\alpha CS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is Neutrosophic pre- $\alpha$ -irresolute (Neutrosophic  $\alpha$ -irresolute and Neutrosophic semi- $\alpha$ -irresolute, resp.) closed mapping.  $\square$

**Theorem 5.6.**  $(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  and  $(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$  be two NTSs. A function let  $\lambda_{-1}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  is Neutrosophic pre- $\beta$ -irresolute closed mapping if and only if for each Neutrosophic set  $B_N$  in  $\mathfrak{R}_N^2$  and each  $N^P OS A_N$  in  $\mathfrak{R}_N^1$  with  $A_N \supseteq \lambda_{-1}^{N-1}(B_N)$  there exists an  $N^\beta OS, C_N$  in  $\mathfrak{R}_N^2$  with  $C_N \supseteq B_N$  such that  $\lambda_{-1}^{N-1}(C_N) \subseteq A_N$ .

*Proof.* Let  $A_N$  be any arbitrary  $N^P OS\mathfrak{R}_N^1$  in  $\mathfrak{R}_N^1$  with  $A_N \supseteq \lambda_{-1}^{N-1}(B_N)$  where  $B_N$  is a Neutrosophic set in  $\mathfrak{R}_N^2$ . Then  $\overline{A_N}$  is an  $N^P CS\mathfrak{R}_N^1$  in  $\mathfrak{R}_N^1$ . Since  $\lambda_{-1}^N$  is Neutrosophic pre- $\beta$ -irresolute closed mapping  $\lambda_{-1}^N(\overline{A_N})$  is Neutrosophic  $N^\beta CS\mathfrak{R}_N^2$ . Then  $\overline{\lambda_{-1}^N(A_N)} = C_N$  (say) is Neutrosophic  $N^\beta OS$  in  $\mathfrak{R}_N^2$ . Since  $\lambda_{-1}^{N-1}(B_N) \subseteq A_N$ ,  $B_N \subseteq C_N$ . Moreover we have  $\lambda_{-1}^{N-1}(C_N) = \lambda_{-1}^{N-1}(\overline{\lambda_{-1}^N(A_N)}) = \lambda_{-1}^{N-1}(\overline{\lambda_{-1}^N(A_N)})$ . Thus  $\lambda_{-1}^{N-1}(C_N) \subseteq A_N$ .

Conversely, Let  $B_N$  be  $N^P CS$  in  $\mathfrak{R}_N^1$ . Then  $\lambda_{-1}^N(A_N) = B_N$  (say) is a Neutrosophic set in  $\mathfrak{R}_N^2$  and  $\overline{A_N}$  is  $N^P OS\mathfrak{R}_N^1$  in  $\mathfrak{R}_N^1$ . Such that  $\lambda_{-1}^{N-1}(B_N) \subseteq \overline{A_N}$ . By hypothesis, there is a Neutrosophic OS  $C_N$  of  $\mathfrak{R}_N^2$ . Such that  $B_N \subseteq C_N$  and  $\lambda_{-1}^{N-1}(C_N) \subseteq \overline{A_N}$ . Therefore  $A_N \subseteq \lambda_{-1}^{N-1}(C_N)$ . Hence  $\overline{C_N} \subseteq \overline{B_N} = \lambda_{-1}^N(A_N) \subseteq \lambda_{-1}^N(\lambda_{-1}^{N-1}(C_N)) \subseteq \overline{C_N}$ .  $\lambda_{-1}^N(A_N) = \overline{C_N}$ . Since  $\overline{C_N}$  is Neutrosophic  $N^\beta CS\mathfrak{R}_N^2$ ,  $\lambda_{-1}^N(A_N)$  is  $N^\beta CS$  in  $\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is Neutrosophic pre- $\beta$ -irresolute closed mapping.  $\square$

**Theorem 5.7.** Let  $\lambda_{-1}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  be a bijective mapping from  $NTS(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  to another  $NTS(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$ . Then the following statements are equivalent:

1.  $\lambda_{-1}^N$  is an Neutrosophic  $\alpha$ -irresolute open mapping.
2.  $\lambda_{-1}^N$  is an Neutrosophic  $\alpha$ -irresolute closed mapping.
3.  $\lambda_{-1}^{N-1}$  is an Neutrosophic  $\alpha$ -irresolute function.

*Proof.* (i)  $\Rightarrow$  (ii):

Let  $A_N$  be  $N^\alpha CS$  in  $\mathfrak{R}_N^1$ . Then  $\overline{A_N}$  is an  $N^\alpha OS\mathfrak{R}_N^1$ . By

hypothesis  $\lambda_{-1}^N(\overline{A_N}) = \overline{\lambda_{-1}^N(A_N)}$  is  $N^\alpha OS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N(A_N)$  is  $N^\alpha CS\mathfrak{R}_N^2$ . Thus  $\lambda_{-1}^N$  is Neutrosophic  $\alpha$ -irresolute closed mapping.

(ii)  $\Rightarrow$  (iii):

Let  $A_N$  be  $N^\alpha CS$  in  $\mathfrak{R}_N^1$ . Then,  $\lambda_{-1}^N(A_N)$  is  $N^\alpha CS$  in  $\mathfrak{R}_N^2$ . That is,  $(\lambda_{-1}^{N-1})^{-1}(A_N) = \lambda_{-1}^N(A_N)$  is  $N^\alpha CS$  in  $\mathfrak{R}_N^2$ . Therefore  $\lambda_{-1}^{N-1}$  is Neutrosophic  $\alpha$ -irresolute function.

(iii)  $\Rightarrow$  (i):

Let  $A_N$  be  $NOS$  in  $\mathfrak{R}_N^1$  and  $\lambda_{-1}^{N-1}$  is Neutrosophic-irresolute function. So  $((\lambda_{-1}^{N-1})^{-1}(A_N) = \lambda_{-1}^N(A_N))$  is  $N^\alpha OS$  in  $\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N$  is Neutrosophic  $\alpha$ -irresolute open mapping.  $\square$

**Theorem 5.8.** Let  $\lambda_{-1}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  be a mapping from Neutrosophic  $TS(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  to another Neutrosophic  $TS(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$ . Then the following statements are equivalent:

1.  $\lambda_{-1}^N$  is an Neutrosophic pre- $\alpha$  (Neutrosophic semi- $\alpha$ , resp.)-irresolute open mapping.
2.  $\lambda_{-1}^N$  is an Neutrosophic pre- $\alpha$  (Neutrosophic semi- $\alpha$ , resp.)-irresolute closed mapping.

*Proof.* (i)  $\Rightarrow$  (ii):

Let  $A_N$  be  $N^P CS\mathfrak{R}_N^1(N^\mathcal{S} CS\mathfrak{R}_N^1$ , resp.) in  $\mathfrak{R}_N^1$ . Then  $\overline{A_N}$  is an  $N^P OS\mathfrak{R}_N^1(N^\mathcal{S} OS\mathfrak{R}_N^1$ , resp.) in  $\mathfrak{R}_N^1$ . By hypothesis  $\lambda_{-1}^N(A_N) = \overline{\lambda_{-1}^N(A_N)}$  is  $N^\alpha OS\mathfrak{R}_N^2$  in  $\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N(A_N)$  is  $N^\alpha CS\mathfrak{R}_N^2$ . Thus  $\lambda_{-1}^N$  is Neutrosophic pre- $\alpha$  (Neutrosophic semi- $\alpha$ , resp.)-irresolute closed mapping.

(ii)  $\Rightarrow$  (i):

Let  $A_N$  be  $N^P OS\mathfrak{R}_N^1(N^\mathcal{S} OS\mathfrak{R}_N^1$ , resp.) in  $\mathfrak{R}_N^1$ . Then  $\overline{A_N}$  is an  $N^P CS\mathfrak{R}_N^1(N^\mathcal{S} CS\mathfrak{R}_N^1$ , resp.) in  $\mathfrak{R}_N^1$ . By hypothesis  $\lambda_{-1}^N(A_N) = \overline{\lambda_{-1}^N(A_N)}$  is  $N^\alpha CS\mathfrak{R}_N^2$ . Hence  $\lambda_{-1}^N(A_N)$  is  $N^\alpha OS\mathfrak{R}_N^2$ . Thus  $\lambda_{-1}^N$  is Neutrosophic pre- $\alpha$  (Neutrosophic semi- $\alpha$ , resp.)-irresolute open mapping.  $\square$

**Theorem 5.9.** Let  $\lambda_{-1}^N : \mathfrak{R}_N^1 \rightarrow \mathfrak{R}_N^2$  be a mapping from Neutrosophic  $TS(\mathfrak{R}_N^1, \mathfrak{S}_N^1)$  to another Neutrosophic  $TS(\mathfrak{R}_N^2, \mathfrak{S}_N^2)$ . Then the following statements are equivalent:

1.  $\lambda_{-1}^N$  is an Neutrosophic pre- $\beta$ -irresolute open mapping.
2.  $\lambda_{-1}^N$  is an Neutrosophic pre- $\beta$ -irresolute closed mapping.

*Proof.* Proof is similar  $\square$

## 6. Conclusion

The concepts of Neutrosophic pre- $\alpha$  (Neutrosophic  $\alpha$ , Neutrosophic semi- $\alpha$  and Neutrosophic  $\alpha$  and  $\beta$ , resp.)-irresolute open and closed mappings have been introduced and studied. The relationships between these mappings with other existing mappings in Neutrosophic topological spaces are investigated.



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