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Neutrosophic semi Volterra spaces

R. Vijayalakshmi[1]* and A. Savitha Mary [2]

Abstract
In this paper, we introduced the concepts of Neutrosophic Semi Volterra spaces and some relations of the Neutrosophic Baire spaces and Neutrosophic Volterra spaces are also studied.

Keywords
Neutrosophic SG_δ- set, Neutrosophic SF_σ- set, Neutrosophic Volterra spaces, Neutrosophic Semi Volterra spaces

AMS Subject Classification
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[1] Department of Mathematics, Arignar Anna Government Arts College, Namakkal-637002, Tamil Nadu, India.
[2] Department of Mathematics, St. Joseph's College of Arts and Science(Autonomous), Manjakuppam, Cuddalore-607001, Tamil Nadu, India.
*Corresponding author: [1] viji_lakshmi80@rediffmail.com; [2] savitha.mary139@gmai1.com,
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Contents

1. Introduction and Preliminaries

The concept of fuzzy sets and fuzzy set operations were first introduced by L.A. Zadeh in his classical paper [15] in the year 1965. Thereafter the paper of C.L. Chang [1] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. The concepts of Volterra spaces have been studied extensively in classical topology in [4–7] and [8]. The concept of Volterra spaces in fuzzy setting was introduced and studied by the authors in [13]. The concept of Intuitionstic fuzzy Volterra spaces was introduced and studied by Soundararajan, Rizwan and Syed Tahir Hussainy [12]. The concepts of Neutrosophy and Neutrosophic set were introduced by F. Smarandache [10, 11]. Afterwords, the works of Smarandache inspired A. A. Salama and S. A. Alblowi [9] to introduce and study the concepts of Neutrosophic crisp set and Neutrosophic crisp topological spaces. The Basic definitions and Proposition related to Neutrosophic topological spaces was introduced and discussed by Dhavaseelan et al. [2]. The concepts of Neutrosophic Baire spaces are introduced by R. Dhavaseelan, S. Jafari, R. Narmada Devi, Md. Hanif Page [3].

Definition 1.1 ([15]). *A neutrosophic topology (NT) on a nonempty set N^X is a family N^T of neutrosophic sets in N^X satisfying the following axioms:*

1. $0_N, 1_N \in N^T$,
2. $G_1 \cap G_2 \in N^T$ *for any* $G_1, G_2 \in N^T$.
3. $\cup G_i$ *for arbitrary family* $\{G_i | i \in \Lambda\}$.

In this case the ordered pair (N^X, N^T) or simply N^X is called a neutrosophic Topological Space (briefly NTS) and each Neutrosophic set in N^T is called a neutrosophic open set (briefly NOS). The complement $M_{P_1^*}$ of a NOS $M_{P_1^*}$ in N^X is called a neutrosophic closed set (briefly NCS) in N^X.

Definition 1.2 ([15]). *Let $M_{P_1^*}$ be a neutrosophic set in a neutrosophic topological space N^X. Then*

$$Nint(M_{P_1^*}) = \cup\{G | G \text{ vis Neutrosophic open set in } N^X \text{ and } G \subseteq M_{P_1^*}\}$$

is called the Neutrosophic interior of $M_{P_1^*}$.

$$Ncl(M_{P_1^*}) = \cap\{G | G \text{ is Neutrosophic closed set in } N^X \text{ and } G \supseteq M_{P_1^*}\}$$

is called the Neutrosophic closure of $M_{P_1^*}$. For any Neutrosophic set $M_{P_1^*}$ in a Neutrosophic topological space (N^X, N^T), it is easy to see that

$$1 - Ncl(M_{P_1^*}) = Nint(l - M_{P_1^*}) \text{ and } 1 - Nint(M_{P_1^*}) = Ncl(l - M_{P_1^*}).$$

Definition 1.3 ([15]). *A Neutrosophic set $M_{P_1^*}$ in Neutrosophic topological space (N^X, N^T) is called Neutrosophic dense if there exists no Neutrosophic closed set $M_{P_2^*}$ in (N^X, N^T) such that $M_{P_1^*} \subset M_{P_2^*} \subset 1_N$. That is $Ncl(M_{P_1^*}) = 1_N$.*

Definition 1.4 ([15]). *A Neutrosophic set $M_{P_1^*}$ in Neutrosophic topological space (N^X, N^T) is called Neutrosophic nowhere dense if there exists no non-zero Neutrosophic open set $M_{P_2^*}$ in (N^X, N^T) such that $M_{P_2^*} \subset Ncl()$. That is $Nint(Ncl(M_{P_1^*})) = 0_N$.*

Definition 1.5 ([15]). *Let (N^X, N^T) be a Neutrosophic topological space. A Neutrosophic set $M_{P_1^*}$ in (N^X, N^T) is called Neutrosophic first category if $M_{P_1^*} = \bigcup_{i=1}^{\infty} M_{P_1^*\,i}$ where $M_{P_1^*\,i}'s$ are Neutrosophic nowhere dense sets in (N^X, N^T). Any other Neutrosophic set in (N^X, N^T) is said to be of Neutrosophic second category.*

Definition 1.6 ([13]). *A Neutrosophic set $M_{P_1^*}$ in a Neutrosophic topological space (N^X, N^T) is called a Neutrosophic G_δ-set in (N^X, N^T) if $M_{P_1^*} = \bigcap_{i=1}^{\infty} M_{P_1^*\,i}$ where $M_{P_1^*\,i} \in N^T$, for $i \in I$.*

Definition 1.7 ([13]). *A Neutrosophic set $M_{P_1^*}$ in a Neutrosophic topological space $(N^X; N^T)$ is called a Neutrosophic F_σ-set in $(N^X; N^T)$ if $M_{P_1^*} = \bigcup_{i=1}^{\infty} M_{P_1^*\,i}$ where $1 - M_{P_1^*\,i} \in N^T$, for $i \in I$.*

Definition 1.8 ([14]). *Let $M_{P_1^*}$ be a Neutrosophic set in a Neutrosophic topological space N^X. Then*

$$NSint(M_{P_1^*}) = \cup\{G \mid G \text{ is Neutrosophic semi open set in } N^X \text{ and } G \subseteq M_{P_1^*}\}$$

is called the Neutrosophic interior of $M_{P_1^*}$.

$$NScl(M_{P_1^*}) = \cap\{G \mid G \text{ is Neutrosophic semi closed set in } N^X \text{ and } G \supseteq M_{P_1^*}\}$$

is called the Neutrosophic closure of $M_{P_1^*}$.

Result 1.9. *Let $M_{P_1^*}$ be a Neutrosophic set in a Neutrosophic topological space N^X. Then*

$$NScl(M_{P_1^*}) = M_{P_1^*} \cup Nint(Ncl(M_{P_1^*}))$$
$$NSint(M_{P_1^*}) = M_{P_1^*} \cap Ncl(Nint(M_{P_1^*}))$$

Definition 1.10 ([14]). *A neutrosophic set $M_{P_1^*}$ in Neutrosophic topological space $(N^X; N^T)$ is called Neutrosophic semi dense if there exists no Neutrosophic semi closed set $M_{P_2^*}$ in $(N^X; N^T)$ such that $M_{P_1^*} \subset M_{P_2^*} \subset 1_N$. That is $NScl(M_{P_1^*}) = 1_N$.*

Definition 1.11 ([14]). *A Neutrosophic set $M_{P_1^*}$ in Neutrosophic topological space $(N^X; N^T)$ is called Neutrosophic semi nowhere dense if there exists no non-zero Neutrosophic semi open set $M_{P_2^*}$ in $(N^X; N^T)$ such that $M_{P_2^*} \subset NScl(M_{P_1^*})$. That is $NSint(NScl(M_{P_1^*})) = 0_N$.*

Definition 1.12 ([14]). *Let (N^X, N^T) be a Neutrosophic topological space. A Neutrosophic set $M_{P_1^*}$ in (N^X, N^T) is called Neutrosophic semi first category if $M_{P_1^*} = \bigcup_{i=1}^{\infty} M_{P_1^*\,i}$ where $M_{P_1^*\,i}$'s are Neutrosophic semi nowhere dense sets in (N^X, N^T). Any other Neutrosophic set in (N^X, N^T) is said to be of Neutrosophic semi second category.*

Proposition 1.13 ([14]). *If $M_{P_1^*}$ is a Neutrosophic semi closed set in (N^X, N^T) with $NSint(M_{P_1^*}) = 0_N$, then $M_{P_1^*}$ is a Neutrosophic semi nowhere dense set in $(N^X; N^T)$.*

Definition 1.14 ([14]). *Let $M_{P_1^*}$ be a Neutrosophic semi first category set in (N^X, N^T). Then $\overline{M_{P_1^*}}$ is called a Neutrosophic semi residual set in (N^X, N^T).*

2. Neutrosophic Volterra Spaces

Definition 2.1 ([3]). *Let (N^X, N^T) be a Neutrosophic topological space. Then (N^X, N^T) is called a Neutrosophic Baire space if $Nint(\bigcup_{i=1}^{\infty}(M_{P_1^*\,i})) = 0_N$, where $M_{P_1^*\,i}$'s are Neutrosophic nowhere dense sets in (N^X, N^T).*

Definition 2.2. *A Neutrosophic topological space (N^X, N^T) is called a Neutrosophic Volterra space if $Ncl(\bigcap_{i=1}^{\infty}(M_{P_1^*\,i})) = 1_N$, where $M_{P_1^*\,i}$'s are Neutrosophic dense and Neutrosophic G_δ sets in (N^X, N^T).*

Example 2.3. *Let $N^X = \{a, b\}$. Define the Neutrosophic set $M_{P_1^*}, M_{P_2^*}, M_{P_3^*}, M_{P_4^*}$ and K on N^X as follows:*

$$M_{P_1^*} = \{x, \langle 0.4, 0.8, 0.9\rangle, \langle 0.7, 0.5, 0.3\rangle\},$$
$$M_{P_2^*} = \{x, \langle 0.5, 0.8, 0.6\rangle, \langle 0.8, 0.4, 0.3\rangle\},$$
$$M_{P_3^*} = \{x, \langle 0.4, 0.7, 0.9\rangle, \langle 0.6, 0.4, 0.4\rangle\},$$
$$M_{P_4^*} = \{x, \langle 0.5, 0.7, 0.5\rangle, \langle 0.8, 0.4, 0.6\rangle\},$$

and

$$K = \{x, \langle 1, 1, 0.3\rangle, \langle 0.7, 0.3, 0.6\rangle\}$$

Then the family

$$N^T = \{0_N, 1_N, M_{P_1^*}, M_{P_2^*}, M_{P_3^*}, M_{P_4^*}, M_{P_1^*} \cup M_{P_2^*}, M_{P_1^*} \cap M_{P_2^*}\}$$

is Neutrosophic topology on N^X. Thus (N^X, N^T) is a Neutrosophic topological space. Now Neutrosophic Open

Sets $= \{M_{P_1^*}, M_{P_2^*}, M_{P_3^*}, M_{P_4^*}, M_{P_1^*} \cup M_{P_2^*}, M_{P_1^*} \cap M_{P_2^*}\}$ *and Neutrosophic Closed Sets*

$$= \{\overline{M_{P_1^*}}, \overline{M_{P_2^*}}, \overline{M_{P_3^*}}, \overline{M_{P_4^*}}, \overline{M_{P_1^*} \cup M_{P_2^*}}, \overline{M_{P_1^*} \cap M_{P_2^*}}\}.$$

Then Neutrosophic G_δ-sets are $\{M_{P_1^*}, M_{P_2^*}, M_{P_3^*}, M_{P_1^*} \cap M_{P_2^*}\}$. *Here* $M_{P_1^*} \cap M_{P_2^*} \cap M_{P_3^*} \cap (M_{P_1^*} \cap M_{P_2^*}) = M_{P_3^*}$ *and* $Ncl(M_{P_3^*}) = 1_N$. *So Neutrosophic topological space* (N^X, N^T) *is Neutrosophic Volterra space.*

Lemma 2.4. *$M_{P_1^*}$ is a Neutrosophic G_δ Neutrosophic topological space (N^X, N^T) if and only if $1 - M_{P_1^*}$ is Neutrosophic F_σ set in (N^X, N^T).*

Lemma 2.5. *If $M_{P_1^*}$ is a Neutrosophic dense and Neutrosophic G_δ-set in a Neutrosophic topological space (N^X, N^T), then $1 - M_{P_1^*}$ is a Neutrosophic first category set in (N^X, N^T).*

Proposition 2.6. *If $M_{P_1^*}$ is a Neutrosophic nowhere dense set and Neutrosophic F_σ-sets in the Neutrosophic topological space $(N^X; N^T)$, then $1 - M_{P_1^*}$ is a Neutrosophic second category set in $(N^X; N^T)$.*

Proof. Let $M_{P_1^*}$ be a Neutrosophic nowhere dense and Neutrosophic F_σ-set in (N^X, N^T). Then $M_{P_1^*} = \bigcup_{i=1}^{\infty} M_{P_1^*\,i}$, where $1 - M_{P_1^*\,i} \in N^T$ and since $M_{P_1^*}$ is a Neutrosophic nowhere dense set in (N^X, N^T). Then

$$Nint(Ncl(\bigcup_{i=1}^{\infty}(M_{P_1^*\,i})) = 0_N.$$

But $Nint(Ncl(\bigcup_{i=1}^{\infty}(M_{P_1^*\,i})) \subseteq \bigcup_{i=1}^{\infty}(Nint(Ncl(M_{P_1^*\,i}))$. Here

$$0_N \subseteq \bigcup_{i=1}^{\infty}(Nint(Ncl(M_{P_1^*\,i})).$$

That is $\bigcup_{i=1}^{\infty}(Nint(Ncf(M_{P_1^*\,i})) = 0_N$. Then we have $Nint(Ncl(M_{P_1^*\,i})) = 0_N$ for each where $1 - M_{P_1^*\,i} \in N^T$ so $Ncl(M_{P_1^*\,i}) = M_{P_1^*\,i}$ which implies that $Nint(Ncl(M_{P_1^*\,i}))$ $= Nint(M_{P_1^*\,i}) = 0_N$ and hence $1 - Nint(M_{P_1^*\,i}) = 1_N = Ncl(1 - M_{P_1^*\,i}) = 1_N$. Therefore $1 - M_{P_1^*\,i}$ is a Neutrosophic dense set in (N^X, N^T). Here

$$1 - M_{P_1^*} = 1 - \bigcup_{i=1}^{\infty} M_{P_1^*\,i} = \bigcap_{i=1}^{\infty} 1 - M_{P_1^*\,i}.$$

Therefore $1 - M_{P_1^*} = \bigcap_{i=1}^{\infty} 1 - M_{P_1^*\,i}$ where are Neutrosophic dense sets in (N^X, N^T). Hence $1 - M_{P_1^*\,i}$'s a Neutrosophic second category set in (N^X, N^T). □

Proposition 2.7. *If the Neutrosophic second category sets $M_{P_2^*\,i}$ are formed from the Neutrosophic nowhere dense and Neutrosophic F_σ-sets $M_{P_1^*\,i}$ in an Neutrosophic Volterra space (N^X, N^T), then $Ncl(Nint(\bigcap_{i=1}^{N} M_{P_2^*\,i})) = 1_N$.*

Proof. Let $M_{P_1^*\,i}$'s ($i = 1$ to N) be Neutrosophic nowhere dense and Neutrosophic F_σ-sets in a Neutrosophic Volterra space (N^X, N^T). Then $Nint(Ncl(\bigcup_{i=1}^{N}(M_{P_1^*\,i})) = 0_N$. Now

$$1 - Nint(Ncl(\bigcup_{i=1}^{N}(M_{P_1^*\,i})) = 1_N$$

implies that $Ncl(Nint(\bigcap_{i=1}^{N} 1 - M_{P_1^*\,i})) = 1_N$. Since $M_{P_1^*\,i}$'s ($i =$ 1 to N) be Neutrosophic nowhere dense and Neutrosophic F_σ-sets in (N^X, N^T) by Proposition 2.6 , $1 - M_{P_1^*\,i}$'s a Neutrosophic second category set in (N^X, N^T). Let $M_{P_2^*\,i} = 1 - M_{P_1^*\,i}$. Hence $Ncl(Nint(\bigcap_{i=1}^{N} M_{P_2^*\,i})) = 1_N$, where $M_{P_2^*\,i}$'s are Neutrosophic second category sets in (N^X, N^T). □

Proposition 2.8. *If $M_{P_1^*\,i}$'s are the Neutrosophic nowhere dense sets in a Neutrosophic Volterra space (N^X, N^T), then it a Neutrosophic Baire Space.*

Proof. The sets $M_{P_1^*\,i}$'s are Neutrosophic nowhere dense sets in (N^X, N^T). This gives $1 - M_{P_1^*\,i}$'s are Neutrosophic dense and Neutrosophic G_δ sets in (N^X, N^T). Since (N^X, N^T) is Neutrosophic Volterra space, therefore $Ncl(\bigcap_{j=1}^{\infty} 1 - M_{P_1^*\,i}) = 1_N$. That is $1 - Ncl(\bigcap_{i=1}^{\infty} 1 - M_{P_1^*\,i}) = 0_N$ implies that $Nint(\bigcup_{i=1}^{\infty} M_{P_1^*\,i}) = 0_N$, where $M_{P_1^*\,i}$'s are Neutrosophic nowhere dense sets in (N^X, N^T). Hence (N^X, N^T) is a Neutrosophic Baire Space. □

Proposition 2.9. *If $M_{P_1^*\,i}'s$ ($i = 1$ to N) are the Neutrosophic nowhere dense and Neutrosophic F_σ sets in a Neutrosophic Volterra space (N^X, N^T) if and only if (N^X, N^T) is a Neutrosophic Baire Space.*

Proof. The sets $M_{P_1^*\,i}'s$ are Neutrosophic nowhere dense and Neutrosophic F_σ sets in (N^X, N^T). Therefore, $Nint(Ncl(M_{P_1^*\,i})) = 0_N$. But $Nint(M_{P_1^*}) \subseteq Nint(Ncl(M_{P_1^*}))$ $= 0_N$. Therefore $Nint(M_{P_1^*}) = 0_N$. Since $M_{P_1^*\,i}$'s are F_σ sets in (N^X, N^T), so $M_{P_1^*} = \bigcup_{i=1}^{\infty} M_{P_1^*\,i} \supset Nint(M_{P_1^*}) = Nint$ $(\bigcup_{i=1}^{\infty} M_{P_1^*\,i}) = 0_N$. Hence (N^X, N^T) is a Neutrosophic Baire Space.

Conversely, let (N^X, N^T) is a Neutrosophic Baire Space then $Nint(\bigcup_{i=1}^{\infty} M_{P^*_1 i}) = 0_N$ where $M_{P^*_1 i}$'s are Neutrosophic nowhere dense set in (N^X, N^T). Since $M_{P^*_1 i}$'s Neutrosophic F_σ sets, therefore $M_{P^*_1} = \bigcup_{i=1}^{\infty} M_{P^*_1 i} \Rightarrow 1 - M_{P^*_1} = 1 - \bigcup_{i=1}^{\infty} M_{P^*_1 i} = \bigcap_{i=1}^{\infty} 1 - M_{P^*_1 i}$ where $1 - M_{P^*_1 i}$'s are Neutrosophic dense and Neutrosophic G_δ-set in (N^X, N^T). That is $Ncl(\bigcap_{i=1}^{\infty} 1 - M_{P^*_1 i}) = 1_N$. Hence (N^X, N^T) is a Neutrosophic Volterra space. □

Proposition 2.10. *Every Neutrosophic Baire space (N^X, N^T) need not to be Neutrosophic Volterra space.*

Example 2.11. *Let $N^X = \{a, b\}$. Define the Neutrosophic set $M_{P^*_1}, M_{P^*_2}$ and $M_{P^*_3}$ on N^X as follows:*

$$M_{P^*_1} = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.6}\right), \left(\frac{a}{0.6}, \frac{b}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right) \right\rangle$$
$$M_{P^*_2} = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.5}\right), \left(\frac{a}{0.7}, \frac{b}{0.3}\right), \left(\frac{a}{0.8}, \frac{b}{0.1}\right) \right\rangle$$

Then the family

$$N^T = \{0_N, 1_N, M_{P^*_1}, M_{P^*_2}, M_{P^*_1} \cup M_{P^*_2}, M_{P^*_1} \cap M_{P^*_2}\}$$

is Neutrosophic topology on N^X. Thus (N^X, N^T) is a Neutrosophic topological space. Now the Neutrosophic open sets $= \{M_{P^*_1}, M_{P^*_2}, M_{P^*_1} \cup M_{P^*_2}, M_{P^*_1} \cap M_{P^*_2}\}$ and the sets $\overline{M_{P^*_1}}, \overline{M_{P^*_2}}, \overline{M_{P^*_1} \cup M_{P^*_2}}$ are Neutrosophic nowhere dense set.

Here $\overline{M_{P^*_1}} \cup \overline{M_{P^*_2}} \cup \overline{M_{P^*_1} \cup M_{P^*_2}} = \overline{M_{P^*_2}}$ and $Nint(\overline{M_{P^*_2}}) = 0_N$. So Neutrosophic topological space (N^X, N^T) is a Neutrosophic Baire Space. But $Nscl(M_{P^*_1} \cap M_{P^*_2}) \neq 1_N$, where $M_{P^*_1} \cap M_{P^*_2}$ is G_δ-set in (N^X, N^T). Hence Neutrosophic topological space (N^X, N^T) is not a Neutrosophic Volterra space.

3. Neutrosophic Semi Volterra Spaces

Definition 3.1. *A Neutrosophic set $M_{P^*_1}$ in a Neutrosophic topological space (N^X, N^T) is called a Neutrosophic SG_δ-set in (N^X, N^T) if $M_{P^*_1} = \bigcap_{i=1}^{\infty} M_{P^*_1 i}$ where $M_{P^*_1 i}$'s are Neutrosophic semi open for $i \in I$.*

Definition 3.2. *A Neutrosophic set $M_{P^*_1}$ in a Neutrosophic topological space (N^X, N^T) is called a Neutrosophic SF_σ-set in (N^X, N^T) if $M_{P^*_1} = \bigcup_{i=1}^{\infty} M_{P^*_1 i}$ where $M_{P^*_1 i}$'s are Neutrosophic semi closed for $i \in I$.*

Definition 3.3. *Let (N^X, N^T) be a Neutrosophic topological space. Then (N^X, N^T) is called a Neutrosophic Semi- Baire space if $N\sin t(\bigcup_{i=1}^{\infty} M_{P^*_1 i}) = 0_N$, where $M_{P^*_1 i}$'s are Neutrosophic semi-nowhere dense sets in (N^X, N^T).*

Definition 3.4. *A Neutrosophic topological space (N^X, N^T) is called a Neutrosophic Semi -Volterra space if $Nscl(\bigcap_{i=1}^{\infty}(M_{P^*_1 i})) = 1_N$, where $M_{P^*_1 i}$'s are Neutrosophic semi-dense and Neutrosophic SG_δ sets in (N^X, N^T).*

Example 3.5. *Let $N^X = \{a, b\}$. Define the Neutrosophic set $M_{P^*_1}, M_{P^*_2}, M_{P^*_3}$ and $M_{P^*_4}$ on N^X as follows:*

$$M_{P^*_1} = \left\langle x, \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}\right) \right\rangle$$
$$M_{P^*_2} = \left\langle x, \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}\right) \right\rangle$$
$$M_{P^*_3} = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.5}\right) \right\rangle$$
$$M_{P^*_4} = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}\right) \right\rangle$$

Then the families $N^T = \{0_N, 1_N, M_{P^*_1}, M_{P^*_2}\}$ is Neutrosophic topology on N^X. Thus (N^X, N^T) is a Neutrosophic topological space. Now Neutrosophic Semi Open Sets $= \{M_{P^*_1}, M_{P^*_2}\}$ and Neutrosophic Semi Closed Sets $= \{\overline{M_{P^*_1}}, \overline{M_{P^*_2}}, M_{P^*_3}, M_{P^*_4}\}$. Then $M_{P^*_1}$ is SG_δ set in (N^X, N^T) and $Nscl(M_{P^*_1}) = 1_N$. So Neutrosophic topological space (N^X, N^T) is Neutrosophic Semi Volterra space.

Example 3.6. *Let $N^X = \{a, b\}$. Define the Neutrosophic set $M_{P^*_1}, M_{P^*_2}$ and $M_{P^*_3}$ on N^X as follows:*

$$M_{P^*_1} = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.6}\right), \left(\frac{a}{0.5}, \frac{b}{0.2}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}\right) \right\rangle$$
$$M_{P^*_2} = \left\langle x, \left(\frac{a}{0.2}, \frac{b}{0.5}\right), \left(\frac{a}{0.6}, \frac{b}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.1}\right) \right\rangle.$$

Then the families

$$N^T = \{0_N, 1_N, M_{P^*_1}, M_{P^*_2}, M_{P^*_1} \cup M_{P^*_2}, M_{P^*_1} \cap M_{P^*_2}\}$$

is neutrosophic topology on N^X. Thus (N^X, N^T) is a Neutrosophic topological space. Now the neutrosophic semi open sets $= \{M_{P^*_1}, M_{P^*_2}, M_{P^*_1} \cup M_{P^*_2}, M_{P^*_1} \cap M_{P^*_2}\}$ and the sets $\overline{M_{P^*_1}}, \overline{M_{P^*_2}}, \overline{M_{P^*_1} \cup M_{P^*_2}}$ are neutrosophic semi nowhere dense set. Then $M_{P^*_1} \cap M_{P^*_2}$ is SG_δ-set in (N^X, N^T), but $Nscl(M_{P^*_1} \cap M_{P^*_2}) \neq 1_N$. So Neutrosophic topological space (N^X, N^T) is not a Neutrosophic Semi Volterra space.

Proposition 3.7. *In a Neutrosophic topological space (N^X, N^T), a Neutrosophic set $M_{P^*_1}$ is Neutrosophic semi nowhere dense if and only if $1 - M_{P^*_1}$ is a Neutrosophic semi dense in (N^X, N^T).*

Proof. Let $M_{P_1^*}$ be a Neutrosophic nowhere dense set in (N^X, N^T), then $NSint(NScl(M_{P_1^*})) = 0_N$. That is, $1 - NSint(NScl(M_{P_1^*})) = 1_N$. That is, $NScl(NSint(1 - M_{P_1^*})) = 1_N$. Here $1 - M_{P_1^*}$ is open, since $M_{P_1^*}$ is closed. So that $NScl(1 - M_{P_1^*})) = 1_N$. Hence, $1 - M_{P_1^*}$ is a Neutrosophic dense set in (N^X, N^T). □

Proposition 3.8. *If $M_{P_1^*} = \bigcap_{i=1}^{N} M_{P_1^*{}_i}$ where $M_{P_1^*{}_i}$'s are Neutrosophic semi dense and Neutrosophic SG_δ-sets in a Neutrosophic Semi Volterra space (N^X, N^T), then $M_{P_1^*}$ is not a Neutrosophic semi closed set.*

Proof. Let $M_{P_1^*} = \bigcap_{i=1}^{N} M_{P_1^*{}_i}$ where $M_{P_1^*{}_i}$'s are Neutrosophic semi dense and Neutrosophic SG_δ-sets in (N^X, N^T). Since (N^X, N^T) is a Neutrosophic Semi Volterra space, we have $NScl(\bigcap_{i=1}^{N}(M_{P_1^*{}_i})) = 1_N$. That is, $NScl(M_{P_1^*}) = 1_N$. This implies that $NScl(M_{P_1^*}) \neq M_{P_1^*}$. Hence, $M_{P_1^*}$ is not a Neutrosophic semi closed set in (N^X, N^T). □

Proposition 3.9. *If $M_{P_2^*} = \bigcup_{i=1}^{N} M_{P_2^*{}_i}$ where $M_{P_2^*{}_i}$'s are Neutrosophic semi nowhere dense and Neutrosophic SF_σ-sets in a Neutrosophic Semi Volterra space (N^X, N^T), then $M_{P_2^*}$ is not a Neutrosophic semi open set.*

Proof. Let $M_{P_2^*} = \bigcup_{i=1}^{N} M_{P_2^*{}_i}$ where $M_{P_2^*{}_i}$'s are Neutrosophic semi dense and Neutrosophic SF_σ-sets in (N^X, N^T). Since (N^X, N^T) is a Neutrosophic Semi Volterra space, we have $NScl(\bigcap_{i=1}^{N}(1 - M_{P_2^*{}_i})) = 1_N$. Since, $M_{P_2^*{}_i}$'s are Neutrosophic nowhere dense sets, which implies $1 - M_{P_2^*{}_i}$'s are Neutrosophic dense sets. Since

$$1 - M_{P_2^*} = 1 - \bigcup_{i=1}^{N} M_{P_2^*{}_i} = \bigcap_{i=1}^{N}(1 - M_{P_2^*{}_i})$$

we have, $NScl(1 - M_{P_2^*}) = 1_N$. This implies that $NSint(M_{P_2^*}) = 0_N$. That is $NSint(M_{P_2^*}) \neq M_{P_2^*}$. Hence, $M_{P_1^*}$ is not a Neutrosophic semi open set in (N^X, N^T). □

Proposition 3.10. *Let (N^X, N^T) be a Neutrosophic topological space. A Neutrosophic set $M_{P_1^*}$ is a Neutrosophic semi dense and Neutrosophic semi open set in (N^X, N^T), then $1 - M_{P_1^*}$ is a Neutrosophic semi nowhere dense set in (N^X, N^T).*

Proof. Since, $M_{P_1^*}$ is a Neutrosophic semi dense set in (N^X, N^T). We have $NScl(M_{P_1^*}) = 1_N$. Also, since $M_{P_1^*}$ is a Neutrosophic semi open set, we have $Nsint(M_{P_1^*}) = M_{P_1^*}$. Now,

$$\begin{aligned} NSint(NScl(1 - M_{P_1^*}) &= 1 - NScl((NSint(M_{P_1^*})) \\ &= 1 - Nscl((M_{P_1^*}) \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

Hence, $1 - M_{P_1^*}$ is an Neutrosophic semi nowhere dense set in (N^X, N^T). □

Proposition 3.11. *Every Neutrosophic semi Baire space is a Neutrosophic semi Volterra space only ifthe complement ofNeutrosophic semi dense SG_δ -sets is Neutrosophic semi nowhere dense SF_σ-set in (N^X, N^T).*

Proof. Let $M_{P_1^*{}_i}$'s ($i = 1$ to ∞) be Neutrosophic semi dense and Neutrosophic SG_δ-set in (N^X, N^T). Consider the Neutrosophic set $NScl(\bigcap_{i=1}^{N}(M_{P_1^*{}_i}))$. Now,

$$\begin{aligned} 1 - NScl(\bigcap_{i=1}^{N}(M_{P_1^*{}_i})) &= NSint(1 - \bigcap_{i=1}^{N}(M_{P_1^*{}_i})) \\ &= NSint(\bigcup_{i=1}^{N}(1 - M_{P_1^*{}_i})). \end{aligned}$$

But

$$NSint(\bigcup_{i=1}^{N}(1 - M_{P_1^*{}_i})) \leq NSint(\bigcup_{i=1}^{\infty}(1 - M_{P_1^*{}_i})).$$

By the hypothesis $1 - M_{P_1^*{}_i}$'s are Neutrosophic semi nowhere dense sets in (N^X, N^T). Also since (N^X, N^T) is a Neutrosophic semi Baire space

$NSint(\bigcup_{i=1}^{\infty}(1 - M_{P_1^*{}_i})) = 0_N$ Hence, $NSint(\bigcup_{i=1}^{N}(1 - M_{P_1^*{}_i})) = 0_N$. Then $Nsint(1 - \bigcap_{i=1}^{N}(M_{P_1^*{}_i})) = 0_N$ implies that $1 - NScl(\bigcap_{i=1}^{N}(M_{P_1^*{}_i})) = 0_N$. Then we have $NScl(\bigcap_{i=1}^{N}(M_{P_1^*{}_i})) = 1_N$. Hence $(N^X, \cdot N^T)$ is a Neutrosophic semi Volterra space. □

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