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Properties of neutrosophic nano semi open sets

R. Vijayalakshmi[1]* and A. P. Mookambika [2]

Abstract

Smarandache [2] introduced and developed the new concept of Neutrosophic set from the Intuitionistic fuzzy sets. A.A. Salama [1] introduced Neutrosophic topological spaces by using the Neutrosophic crisp sets. M.L. Thivagar et al. [3,4] developed Nano topological spaces and Neutrosophic nano topological spaces. Aim of this paper is we introduce and study the properties of Neutrosophic nano semi closed sets in Neutrosophic nano topological spaces and its characterization are discussed details.

Keywords

Neutrosophic Nano semi open set, Neutrosophic Nano semi closed set, Neutrosophic Nano semi closure, Neutrosophic Nano semi interior, Neutrosophic nano topology.

AMS Subject Classification

32C18.

[1] *Department of Mathematics, Arignar Anna Government Arts College, Namakkal-637002, Tamil Nadu, India.*

[2] *Department of Mathematics, Mahatma Gandhi Government Arts College, New Mahe(PO), Mahe-673311, Puducherry, India.*

***Corresponding author**: [1] viji_lakshmi80@rediffmail.com; [2] mookiratna@gmail.com

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1. Introduction

Nano topology investigated by M.L. Thivagar et.al [3] can be expressed as a collection of nano approximations, Neutrosophic sets established by F. Smarandache [2]. Neutrosophic set is illustrate by three functions: a membership, indeterminacy and a nonmembership functions that are independently related neutrosophic set have wide range of applications in real life. M.L. Thivagar et al. [4] developed Neutrosophic nano topological spaces.Neutrosophic nano semi closed, Neutrosophic nano α closed, Neutrosophic nano pre closed, Neutrosophic nano semi pre closed and Neutrosophic nano regular closed are introduced by M. Parimala et al. [5]. Aim of the present paper is we studied about properties of Nano semi closure, Neutrosophic Nano semi interior in Neutrosophic nano topological spaces.

2. Preliminaries

Definition 2.1. *Let U be a non-empty set and R be an equivalence relation on U. Let M be a neutrosophic set in U with the membership function μ_M, the indeterminancy function σM and the non- membership function ν_M. The neutrosophic nano lower, neutrosophic nano upper approximation and neutrosophic nano boundary of M in the approximation (U,R) denoted by $\underline{N(M)}$, $\overline{N(M)}$ and $B_N(M)$ are respectively defined as follows:*

1. $\underline{N(M)} = \{< u,\ \mu_{\underline{R}}(M_{P_1^*})(u),\ \sigma_{\underline{R}}(M_{P_1^*})(u), \nu_{\underline{R}}(M_{P_1^*})(u) > /y \in [u]_R, u \in U\}.$

2. $\overline{N(M)} = \{< u,\ \mu_{\overline{R}}(M_{P_1^*})(u),\ \sigma_{\overline{R}}(M_{P_1^*})(u), \nu_{\overline{R}}(M_{P_1^*})(u) > /y \in [u]_R,\ u \in U\}.$

3. $B_N(M) = \overline{N(M)} - \underline{N(M)}.$

Definition 2.2. *Let U be an universe, R be an equivalence relation on U and M be a neutrosophic set in U and if the* *collection* $N_N(\tau) = \{0_{N_N},\ 1_{N_N},\ \underline{N(M)},\ \overline{N(M)}, B_N(M)\}$ *forms a topology then it is said to be a neutrosophic nano topology. We call* $(U, N_N(\tau))$ *as the neutrosophic nano topological space. The elements of* $N_N(\tau)$ *are called neutrosophic nano open sets.*

Definition 2.3. *Let* U *be a nonempty set and the neutrosophic sets* $M_{P_1^*}$ *and* $M_{P_2^*}$ *in the form*

$$M_{P_1^*} = \{< u : \mu_{M_{P_1^*}}(u), \sigma_{M_{P_1^*}}(u), \nu_{M_{P_1^*}}(u >, u \in U\},$$
$$M_{P_2^*} = \{< u : \mu_{M_{P_2^*}}(u),\ \sigma_{M_{P_2^*}}(u),\ \nu_{M_{P_2^*}}(u) >,\ u \in U\}$$

Then the following statements hold:

1. $0_{N_N} = \{< u, 0, 0, 1 >: u \in U\}$ *and* $1_{N_N} = \{< u, 1, 1, 0 > : u \in U\}$.
2. $M_{P_1^*} \subseteq M_{P_2^*}$ *iff* $\{\mu_{M_{P_1^*}}(u) \leq \mu_{M_{P_2^*}}(u), \sigma_{M_{P_1^*}}(u) \leq \sigma_{M_{P_2^*}}(u), \nu_{M_{P_1^*}}(u) \geq \nu_{M_{P_2^*}}(u)$ *for all* $u \in U\}$.
3. $M_{P_1^*} = M_{P_2^*}$ *iff* $M_{P_1^*} \subseteq M_{P_2^*}$ *and* $M_{P_2^*} \subseteq M_{P_1^*}$.
4. $M^c_{P_1^*} = \{< u, \nu_{M_{P_1^*}}(u), 1 - \sigma_{M_{P_1^*}}(u),\ \mu_{M_{P_1^*}}(u) >: u \in U\}$.
5. $M_{P_1^*} \cap M_{P_2^*} = \{u, \mu_{M_{P_1^*}}(u) \wedge \mu_{M_{P_2^*}}(u), \sigma_{M_{P_1^*}}(u) \wedge \sigma_{M_{P_2^*}}(u), \nu_{M_{P_1^*}}(u) \vee \nu_{M_{P_2^*}}(u)$ *for all* $u \in U\}$.
6. $M_{P_1^*} \cup M_{P_2^*} = \{u, \mu_{M_{P_1^*}}(u) \vee \mu_{M_{P_2^*}}(u), \sigma_{M_{P_1^*}}(u) \vee \sigma_{M_{P_2^*}}(u), \nu_{M_{P_1^*}}(u) \wedge \nu_{M_{P_2^*}}(u)$ *for all* $u \in U\}$.
7. $\cup M_{P_1^*\,j} = < u, \vee, \vee, \wedge >$.
8. $\cap M_{P_1^*\,j} = < u, \wedge, \wedge, \vee >$.

Proposition 2.4. *For any neutrosophic Nano set* $M_{P_1^*}$ *in* $(U,\ N_N(\tau))$ *we have*

1. $N^N Cl((M_{P_1^*})^c) = (N^N Int(M_{P_1^*}))^c$,
2. $N^N Int((M_{P_1^*})^c) = (N^N Cl(M_{P_1^*}))^c$,
3. $M_{P_1^*} \subseteq M_{P_2^*} \Rightarrow N^N Int(M_{P_1^*}) \subseteq N^N Int(M_{P_2^*})$,
4. $M_{P_1^*} \subseteq M_{P_2^*} \Rightarrow N^N Cl(M_{P_1^*}) \subseteq N^N Cl(M_{P_2^*})$,
5. $N^N Int(N^N Int(M_{P_1^*})) = N^N Int(M_{P_1^*})$,
6. $N^N Cl(N^N Cl(M_{P_1^*})) = N^N Cl(M_{P_1^*})$,
7. $N^N Int(M_{P_1^*} \cap M_{P_2^*})) = N^N Int(M_{P_1^*}) \cap N^N Int(M_{P_2^*})$,
8. $N^N Cl(M_{P_1^*} \cup M_{P_2^*}) = N^N Cl(M_{P_1^*}) \cup N^N Cl(M_{P_2^*})$,
9. $N^N Int(0_{N_N}) = 0_{N_N}$,
10. $N^N Int(1_{N_N}) = 1_{N_N}$,
11. $N^N Cl(0_{N_N}) = 0_{N_N}$,
12. $N^N Cl(1_{N_N}) = 1_{N_N}$,
13. $M_{P_1^*} \subseteq M_{P_2^*} \Rightarrow (M_{P_2^*})^c \subseteq (M_{P_1^*})^c$,
14. $N^N Cl(M_{P_1^*} \cap M_{P_2^*}) \subseteq N^N Cl(M_{P_1^*}) \cap N^N Cl(M_{P_2^*})$,
15. $N^N Int(M_{P_1^*} \cup M_{P_2^*}) \supseteq N^N Int(M_{P_1^*}) \cup N^N Int(M_{P_2^*})$.

3. Neutrosophic Nano Semi-Open Sets in Neutrosophic Topological Spaces

In this section, the concepts of the Neutrosophic Nano semi-open set is introduced and also discussed their characterizations.

Definition 3.1. *Let* $M_{P_1^*}$ *be* $N^N S$ *of a* $N^N TSU$. *Then* $M_{P_1^*}$ *is said to be Neutrosophic Nano semi-open [written* $N^N(SO)$*] set of* U *if there exists a Neutrosophic Nano open set* $N^N O$ *such that* $N^N O \subseteq M_{P_1^*} \subseteq N^N Cl(N^N O)$.

Theorem 3.2. *A subset* $M_{P_1^*}$ *in a* $N^N TSU$ *is* $N^N(SO)$ *set if and only if* $M_{P_1^*} \subseteq N^N Cl(N^N Int(M_{P_1^*}))$.

Proof. **Sufficiency:** Let $M_{P_1^*} \subseteq N^N Cl(Int(M_{P_1^*}))$. Then for $N^N O = N^N Int(M_{P_1^*})$, we have $N^N O \subseteq M_{P_1^*} \subseteq N^N Cl(N^N O)$.
Necessity: Let $M_{P_1^*}$ be $N^N(SO)$ set in U. Then $N^N O \subseteq M_{P_1^*} \subseteq N^N Cl(O)$ for some Neutrosophic Nano open set $N^N O$. $M_{P_2^*} ut$ $N^N O \subseteq N^N Int(M_{P_1^*})$ and thus $N^N Cl(N^N O) \subseteq N^N Cl(Int(M_{P_1^*}))$. Hence $M_{P_1^*} \subseteq N^N Cl(N^N O) \subseteq N^N Cl(Int(M_{P_1^*}))$. ☐

Theorem 3.3. *Let* $M_{P_1^*}$ *be* $N^N(SO)$ *set in the* $N^N TSU$ *and suppose* $M_{P_1^*} \subseteq M_{P_2^*} \subseteq N^N Cl(M_{P_1^*})$. *Then* $M_{P_2^*}$ *is* $N^N(SO)$ *set in* U.

Proof. There exists a Neutrosophic Nano open set $N^N O$ such that $N^N O \subseteq M_{P_1^*} \subseteq N^N Cl(N^N O)$. Then $N^N O \subseteq M_{P_2^*}$. But $N^N Cl(M_{P_1^*}) \subseteq N^N Cl(O)$ and thus $M_{P_2^*} \subseteq N^N Cl(N^N O)$. Hence $N^N O \subseteq M_{P_2^*} \subseteq N^N Cl(O)$ and $M_{P_2^*}$ is $N^N(SO)$ set in U. ☐

Theorem 3.4. *Every Neutrosophic Nano open set in the* $N^N TSU$ *is* $N^N(SO)$ *set in* U.

Proof. Let $M_{P_1^*}$ be Neutrosophic Nano open set in $N^N TSU$. Then $M_{P_1^*} = N^N Int(M_{P_1^*})$. Also $N^N Int(M_{P_1^*}) \subseteq N^N Cl(Int(M_{P_1^*}))$. This implies that $M_{P_1^*} \subseteq N^N Cl(Int(M_{P_1^*}))$. Hence by Theorem 3.2, $M_{P_1^*}$ is $N^N(SO)$ set in U. ☐

Remark 3.5. *The converse of the above theorem need not be true as shown by the following example.*

Example 3.6. *Let* U *and* $M_{P_1^*}$ *be two non-empty finite sets, where* U *is the universe and* $M_{P_1^*}$ *the set of attributes* $U = \{F_1, F_2, F_3,\ F\}$ *are Fruits.*

Let $U/R = \{\{F_1, F_2, F_3\}, \{F\}\}$ *be an equivalence relation.*

$M_{P_1^*}$ = {Proteins, Minerals, Vitamins} are three attributes, its Neutrosphic values are given below

$$F_1 = \left\langle \left(\frac{4}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{3}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$
$$F_2 = \left\langle \left(\frac{2}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{1}{10},\frac{1}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$
$$F_3 = \left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{4}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$
$$F_4 = \left\langle \left(\frac{4}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{3}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$

$$N_N(\tau) = \left\{0_{N_N},\ 1_{N_N},\ N(M),\ N(M), M_{P_2^*N}(M)\right\}$$
$$\underline{N(F)} = \left\langle \left(\frac{2}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{1}{10},\frac{1}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$
$$\overline{N(F)} = \left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{4}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$
$$M_{P_2^*N}(F) = \left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{2}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$

$$N_N(\tau) = \Big\{0_{N_N}, 1_{N_N},$$
$$\left\langle \left(\frac{2}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{1}{10},\frac{1}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle,$$
$$\left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{4}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle,$$
$$\left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{2}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle,$$
$$\left\langle \left(\frac{4}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{3}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle\Big\},$$
$$F_5 = \left\langle \left(\frac{3}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{2}{10},\frac{7}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$

Here F_5 is $N^N(SO)$ sets but are not Neutrosophic Nano open sets.

Theorem 3.7. *Let $(U,\ N_N(\tau)$ be a N^NTS. Then union of two $N^N(SO)$ sets is a $N^N(SO)$ set in the N^NTSU.*

Proof. Let $M_{P_1^*}$ and $M_{P_2^*}$ are $N^N(SO)$ sets in U. Then $M_{P_1^*} \subseteq N^NCl(Int(M_{P_1^*}))$ and $M_{P_2^*} \subseteq N^NCl(Int(M_{P_2^*}))$.
Therefore $M_{P_1^*} \cup M_{P_2^*} \subseteq N^NCl(Int(M_{P_1^*})) \cup N^NCl(Int(M_{P_2^*})) = N^NCl(Int(M_{P_1^*}) \cup N^NInt(M_{P_2^*})) \subseteq N^NCl(Int(M_{P_1^*} \cup M_{P_2^*}))$ [By Proposition 2.4]. Hence $M_{P_1^*} \cup M_{P_2^*}$ is $N^N(SO)$ set in U. □

Example 3.8. *Let U and $M_{P_1^*}$ be two non-empty finite sets, where U is the universe and $M_{P_1^*}$ the set of attributes. The members of $U = \{P_1,\ P2,\ P_3, P\}$ are pressure patient.*

Let $U/R = \{\{P_1, P2, P_3\}, \{P\}\}$ be an equivalence relation.

$M_{P_1^*}$ = {Salt food , cholesterol food} are two attributes.

$$P_1 = \left\langle \left(\frac{3}{10},\frac{5}{10},\frac{4}{10}\right),\left(\frac{6}{10},\frac{2}{10},\frac{5}{10}\right)\right\rangle,$$
$$P_2 = \left\langle \left(\frac{2}{10},\frac{6}{10},\frac{7}{10}\right),\left(\frac{5}{10},\frac{3}{10},\frac{1}{10}\right)\right\rangle,$$
$$P_3 = \left\langle \left(\frac{3}{10},\frac{6}{10},\frac{4}{10}\right),\left(\frac{6}{10},\frac{3}{10},\frac{1}{10}\right)\right\rangle,$$
$$P_4 = \left\langle \left(\frac{2}{10},\frac{5}{10},\frac{7}{10}\right),\left(\frac{5}{10},\frac{2}{10},\frac{5}{10}\right)\right\rangle,$$
$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N(M)}, \overline{N(M)}, M_{P_2^*N}(M)\}$$
$$\underline{N(F)} = \left\langle \left(\frac{2}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{1}{10},\frac{1}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle,$$
$$\overline{N(F)} = \left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{4}{10},\frac{4}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle,$$
$$M_{P_2^*N}(F) = \left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{2}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle,$$

$$N_N(\tau) = \Big\{0_{N_N}, 1_{N_N}$$
$$\left\langle \left(\frac{2}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{1}{10},\frac{1}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle,$$
$$\left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{4}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle,$$
$$\left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{2}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle,$$
$$\left\langle \left(\frac{4}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{3}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle\Big\}$$
$$F_5 = \left\langle \left(\frac{4}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{2}{10},\frac{7}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle,$$

Here F_5 is $N^N(SO)$ sets but are not Neutrosophic Nano open sets.

Theorem 3.9. *Let $(U,\ N_N$ be a N^NTS. Then union of two $N^N(SO)$ sets is a $N^N(SO)$ set in the N^NTSU.*

Proof. Let $M_{P_1^*}$ and $M_{P_2^*}$ are $N^N(SO)$ sets in U. Then $M_{P_1^*} \subseteq N^NCl(Int(M_{P_1^*}))$ and $M_{P_2^*} \subseteq N^NCl(Int(M_{P_2^*}))$. Therefore
$M_{P_1^*} \cup M_{P_2^*} \subseteq N^NCl(Int(M_{P_1^*})) \cup N^NCl(Int(M_{P_2^*})) = N^NCl(Int(M_{P_1^*}) \cup N^NInt(M_{P_2^*})) \subseteq N^NCl(Int(M_{P_1^*} \cup M_{P_2^*}))$ [By Proposition 2.4]. Hence $M_{P_1^*} \cup M_{P_2^*}$ is $N^N(SO)$ set in U. □

Example 3.10. *Let U and $M_{P_1^*}$ be two non-empty finite sets, where U is the universe and $M_{P_1^*}$ the set of attributes.*

The members of $U = \{P_1,\ P2,\ P_3, P_4\}$ are pressure patient. Let $U/R = \{\{P_1,\ P2,\ P_{3}\}, \{P\}\}$ be an equivalence relation. $M_{P_1^*}$ = {Salt food , Colostrol food} are two attributes

$$P_1 = \left\langle \left(\frac{3}{10},\frac{5}{10},\frac{4}{10}\right),\left(\frac{6}{10},\frac{2}{10},\frac{5}{10}\right)\right\rangle,$$

$$P_2 = \left\langle \left(\frac{2}{10}, \frac{6}{10}, \frac{7}{10}\right), \left(\frac{5}{10}, \frac{3}{10}, \frac{1}{10}\right) \right\rangle,$$
$$P_3 = \left\langle \left(\frac{3}{10}, \frac{6}{10}, \frac{4}{10}\right), \left(\frac{6}{10}, \frac{3}{10}, \frac{1}{10}\right) \right\rangle,$$
$$P_4 = \left\langle \left(\frac{2}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{5}{10}, \frac{2}{10}, \frac{5}{10}\right) \right\rangle,$$

$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N(F)}, \overline{N(F)}, M_{P_2^*N}(F)\}$$
$$\underline{N(F)} = \left\langle \left(\frac{2}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{5}{10}, \frac{2}{10}, \frac{5}{10}\right) \right\rangle,$$
$$\overline{N(F)} = \left\langle \left(\frac{3}{10}, \frac{6}{10}, \frac{4}{10}\right), \left(\frac{6}{10}, \frac{3}{10}, \frac{1}{10}\right) \right\rangle,$$
$$M_{P_2^*N}(F) = \left\langle \left(\frac{3}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{5}{10}, \frac{3}{10}, \frac{5}{10}\right) \right\rangle,$$

$$\begin{aligned} & N_N(\tau) \\ &= \Bigg\{ 0_{N_N}, 1_{N_N}, \left\langle \left(\frac{2}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{5}{10}, \frac{2}{10}, \frac{5}{10}\right) \right\rangle, \\ &\qquad \left\langle \left(\frac{3}{10}, \frac{6}{10}, \frac{4}{10}\right), \left(\frac{6}{10}, \frac{3}{10}, \frac{1}{10}\right) \right\rangle, \\ &\qquad \left\langle \left(\frac{3}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{5}{10}, \frac{3}{10}, \frac{5}{10}\right) \right\rangle, \\ &\qquad \left\langle \left(\frac{2}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{5}{10}, \frac{2}{10}, \frac{5}{10}\right) \right\rangle \Bigg\} \end{aligned}$$

is $N^N TS$ on U. Now, we define the two $N^N(SO)$ sets as follows:

$$P_5 = \left\langle \left(\frac{4}{10}, \frac{6}{10}, \frac{4}{10}\right), \left(\frac{8}{10}, \frac{3}{10}, \frac{4}{10}\right) \right\rangle \text{ and}$$
$$P_6 = \left\langle \left(\frac{10}{10}, \frac{9}{10}, \frac{2}{10}\right), \left(\frac{5}{10}, \frac{7}{10}, \frac{1}{10}\right) \right\rangle.$$

But

$$P_5 \cup P_6 = \left\langle \left(\frac{10}{10}, \frac{9}{10}, \frac{2}{10},\right), \left(\frac{8}{10}, \frac{7}{10}, \frac{4}{10},\right) \right\rangle,$$

is a $N^N(SO)$ set in U.

Theorem 3.11. *Let $M_{P_1^*}$ be $N^N(SO)$ set in the $N^N TSU$ and suppose $M_{P_1^*} \subseteq M_{P_2^*} \subseteq N^N Cl(M_{P_1^*})$. Then $M_{P_2^*}$ is $N^N(SO)$ set in U.*

Proof. There exists a Neutrosophic Nano openset $N^N O$ such that $N^N O \subseteq M_{P_1^*} \subseteq N^N Cl(N^N O)$. Then $N^N O \subseteq M_{P_2^*}$. But $N^N Cl(M_{P_1^*}) \subseteq N^N Cl(O)$ and thus $M_{P_2^*} \subseteq N^N Cl(N^N O)$.

Hence $N^N O \subseteq M_{P_2^*} \subseteq N^N Cl(O)$ and $M_{P_2^*}$ is $N^N(SO)$ set in U. ☐

4. Neutrosophic Nano Semi-Closed Sets in Neutrosophic Topological Spaces

In this section, the Neutrosophic Nano semi-closed set is introduced and studied their properties.

Definition 4.1. *Let $M_{P_1^*}$ be $N^N S$ of a $N^N TSU$. Then $M_{P_1^*}$ is said to be Neutrosophic Nano semi-closed [written as $N^N(SC)$] set of U if there exists a neutrosophic nano closed set $N^N c$ such that $N^N Int(N^N C) \subseteq M_{P_1^*} \subseteq N^N C$.*

Theorem 4.2. *A subset $M_{P_1^*}$ in a $N^N TSU$ is $N^N CS$ set if and only if $N^N Int(N^N Cl(M_{P_1^*})) \subseteq M_{P_1^*}$.*

Proof. **Sufficiency:**
Let $N^N Int(N^N Cl(M_{P_1^*})) \subseteq M_{P_1^*}$. Then for $N^N c = N^N Cl(M_{P_1^*})$, we have $N^N Int(N^N C) \subseteq M_{P_1^*} \subseteq N^N C$.
Necessity:
Let $M_{P_1^*}$ be $N^N(SC)$ set in U. Then $N^N Int(N^N C) \subseteq M_{P_1^*} \subseteq N^N C$ for some Neutrosophic nano closed set $N^N C$. But $N^N Cl(M_{P_1^*}) \subseteq N^N C$ and thus $N^N Int(N^N Cl(M_{P_1^*})) \subseteq N^N Int(N^N C))$. Hence $N^N Int(Cl(M_{P_1^*})) \subseteq N^N Int(N^N C) \subseteq M_{P_1^*}$. ☐

Theorem 4.3. *Every Neutrosophic nano closed set in the $N^N TSU$ is $N^N(SC)$ set in U.*

Proof. Let $M_{P_1^*}$ be Neutrosophic nano closed set in $N^N TSU$. Then $M_{P_1^*} = N^N Cl(M_{P_1^*})$. Also $N^N Int(N^N Cl(M_{P_1^*})) \subseteq N^N Cl(M_{P_1^*})$. This implies that $N^N Int(N^N Cl(M_{P_1^*})) \subseteq M_{P_1^*}$. Hence, $M_{P_1^*}$ is $N^N(SC)$ set in U. ☐

Remark 4.4. *The converse of the above theorem need not be true as shown by the following example.*

Example 4.5. *Let U and $M_{P_1^*}$ be two non-empty finite sets, where U is the universe and $M_{P_1^*}$ the set of attributes $U = \{P_1, P_2, P_3, P_4\}$ are Patients. Let $U/R = \{\{P_1, P_2, P_3\}, \{P_4\}\}$ be an equivalence relation. $M_{P_1^*} = \{$Head ache, Temperature, Cold$\}$ are three attributes, its Neutrosphic values are given below*

$$P_1 = \left\langle \left(\frac{3}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{2}{10}, \frac{8}{10}, \frac{3}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{9}{10}\right) \right\rangle$$
$$P_2 = \left\langle \left(\frac{5}{10}, \frac{6}{10}, \frac{2}{10}\right), \left(\frac{2}{10}, \frac{8}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{6}{10}\right) \right\rangle$$
$$P_3 = \left\langle \left(\frac{3}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{2}{10}, \frac{8}{10}, \frac{4}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{9}{10}\right) \right\rangle$$
$$P_4 = \left\langle \left(\frac{5}{10}, \frac{6}{10}, \frac{4}{10}\right), \left(\frac{2}{10}, \frac{8}{10}, \frac{3}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{5}{10}\right) \right\rangle$$

$$N_N(\tau) = \{0_{N_N},\ 1_{N_N}, \underline{N(M)},\ \overline{N(M)}, M_{P_2^*N}(M)\}$$

$$\underline{N(F)} = \left\langle \left(\frac{3}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{2}{10},\frac{8}{10},\frac{4}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{9}{10}\right)\right\rangle$$

$$\overline{N(F)} = \left\langle \left(\frac{5}{10},\frac{6}{10},\frac{2}{10}\right),\left(\frac{2}{10},\frac{8}{10},\frac{1}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{5}{10}\right)\right\rangle$$

$$M_{P_2^*N}(F) = \left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{2}{10},\frac{8}{10},\frac{2}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$

$$N_N(\tau) = \Bigg\{0_{N_{N'}} 1_{N_N}, \left\langle \left(\frac{3}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{2}{10},\frac{8}{10},\frac{4}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{9}{10}\right)\right\rangle, \left\langle \left(\frac{5}{10},\frac{6}{10},\frac{2}{10}\right),\left(\frac{2}{10},\frac{8}{10},\frac{1}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{5}{10}\right)\right\rangle \left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{2}{10},\frac{8}{10},\frac{2}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle \left\langle \left(\frac{5}{10},\frac{6}{10},\frac{4}{10}\right),\left(\frac{2}{10},\frac{8}{10},\frac{3}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{5}{10}\right)\right\rangle\Bigg\}$$

$$P_5 = \left\langle \left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{4}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{8}{10}\right)\right\rangle$$

Here P_5 is $N^N(SO)$ sets but are not Neutrosophic Nano open sets.

Also E is $N^N(SC)$ set but is not Neutrosophic nano closed set.

Theorem 4.6. *Let $(U, N_N$ be a N^NTS. Then intersection of two $N^N(SC)$ sets is a $N^N(SC)$ set in the N^NTSU.*

Proof. Let $M_{P_1^*}$ and $M_{P_2^*}$ are $N^N(SC)$ sets in U. Then $N^NInt(N^NCl(M_{P_1^*})) \subseteq M_{P_1^*}$ and $N^NInt(N^NCl(M_{P_2^*})) \subseteq M_{P_2^*}$. Therefore $M_{P_1^*} \cap M_{P_2^*} \supseteq N^NInt(N^NCl(M_{P_1^*})) \cap N^NInt(N^NCl(M_{P_2^*})) = N^NInt(N^NCl(M_{P_1^*}) \cap N^NCl(M_{P_2^*})) \supseteq N^NInt\,(N^NCl(M_{P_1^*} \cap M_{P_2^*}))$. Hence $M_{P_1^*} \cap M_{P_2^*}$ is $N^N(SC)$ set in U. □

Example 4.7. *Let U and $M_{P_1^*}$ be two non-empty finite sets, where U is the universe and $M_{P_1^*}$ the set of attributes.*
$U = \{P_1, P_2, P_3,\ P_4\}$ are Patients.
Let $U/R = \{\{P_1, P_2, P_3\},\ \{P_4\}\}$ be an equivalence relation.
$M_{P_1^*} = \{Temperature\}$ are one attributes

$$U/R = \{P_1\}\{P_2, P_3,\ P_4\}$$

$$P_1 = \left\langle \left(\frac{10}{10},\frac{5}{10},\frac{7}{10}\right)\right\rangle$$

$$P_2 = \left\langle \left(\frac{0}{10},\frac{9}{10},\frac{2}{10}\right)\right\rangle$$

$$P_3 = \left\langle \left(\frac{10}{10},\frac{9}{10},\frac{2}{10}\right)\right\rangle$$

$$P_4 = \left\langle \left(\frac{10}{10},\frac{6}{10},\frac{7}{10}\right)\right\rangle.$$

Then,

$$N_N(\tau) = \{0_{N_N},\ 1_{N_N}, \underline{N(M)},\ \overline{N(M)}, M_{P_2^*N}(M)\}$$

$$\underline{N(F)} = \left\langle \left(\frac{0}{10},\frac{5}{10},\frac{7}{10}\right)\right\rangle$$

$$\overline{N(F)} = \left\langle \left(\frac{10}{10},\frac{9}{10},\frac{2}{10}\right)\right\rangle$$

$$M_{P_2^*N}(F) = \left\langle \left(\frac{7}{10},\frac{5}{10},\frac{2}{10}\right)\right\rangle$$

$$N_N(\tau) = \Bigg\{0_{N_N},\ 1_{N_N}, \left(\frac{0}{10},\frac{5}{10},\frac{7}{10}\right),\ \left(\frac{1}{10},\frac{9}{10},\frac{2}{10}\right), \left(\frac{7}{10},\frac{5}{10},\frac{2}{10}\right),\ \left(\frac{10}{10},\frac{5}{10},\frac{7}{10}\right)\Bigg\}$$

is N^NTS on U. Now, we define the two $N^N(SC)$ sets as follows :

$$P_5 = \left\langle \left(\frac{7}{10},\frac{5}{10},\ \frac{2}{10}\right)\right\rangle \quad and$$

$$P_6 = \left\langle \left(\frac{2}{10},\frac{0}{10},\frac{8}{10}\right)\right\rangle.$$

But $P_5 \cap P_6 = \langle(\frac{2}{10},\frac{0}{10},\frac{8}{10})\rangle$ is a $N^N(SC)$ set in U.

Theorem 4.8. *Let $M_{P_1^*}$ be $N^N(SC)$ set in the N^NTSU and suppose $N^NInt(M_{P_1^*}) \subseteq M_{P_2^*} \subseteq M_{P_1^*}$. Then $M_{P_2^*}$ is $N^N(SC)$ set in U.*

Proof. There exists a neutrosophic nano closed set N^NC such that $N^NInt(N^NC) \subseteq M_{P_1^*} \subseteq N^NC$. Then $M_{P_2^*} \subseteq N^NC$. But $N^NInt(N^NC) \subseteq N^NInt(M_{P_1^*})$ and thus $N^NInt(N^NC) \subseteq M_{P_2^*}$. Hence $N^NInt(N^NC) \subseteq M_{P_2^*} \subseteq N^NC$ and $M_{P_2^*}$ is $N^N(SC)$ set in U. □

5. Neutrosophic Nano Semi-Interior in Neutrosophic Topological Spaces

In this section, we introduce the Neutrosophic Nano semi-interior operator and their properties in neutrosophic topological space.

Definition 5.1. *Let $(U, N_N(\tau)$ be a N^NTS. Then for a neutrosophic subset $M_{P_1^*}$ of U, the Neutrosophic Nano semi-interior of $M_{P_1^*}$[$N^NSInt(M_{P_1^*})$ for short] is the union of all Neutrosophic Nano semi-open sets of U contained in $M_{P_1^*}$.*

That is, $N^NSInt(M_{P_1^*}) = \cup\{G : G$ is a $N^N(SO)$ set in U and $G \subseteq M_{P_1^*}\}$.

Proposition 5.2. *Let $(U, N_N\tau)$ be a N^NTS. Then for any neutrosophic subsets $M_{P_1^*}$ and $M_{P_2^*}$ of a N^NTSU we have*

1. *$N^NSInt(M_{P_1^*}) \subseteq M_{P_1^*}$.*
2. *$M_{P_1^*}$ is $N^N(SO)$ set in $U \Leftrightarrow N^NSIntM_{P_1^*} = M_{P_1^*}$.*
3. *$N^NSInt(N^NSInt(M_{P_1^*})) = N^NSInt(M_{P_1^*})$.*

4. *If $M_{P_1^*} \subseteq M_{P_2^*}$ then $N^N SInt(M_{P_1^*}) \subseteq N^N SInt(M_{P_2^*})$.*

Proof. Let $M_{P_1^*}$ be $N^N(SO)$ set in U. Then $M_{P_1^*} \subseteq N^N SInt(M_{P_1^*})$. (1) $\Rightarrow M_{P_1^*} = N^N SInt(M_{P_1^*})$.

Conversely, assume $M_{P_1^*} = N^N SInt(M_{P_1^*})$. Hence, $M_{P_1^*}$ is $N^N(SO)$ set in U. Thus (2) is proved.

(2) $\Rightarrow N^N SInt(N^N SInt(M_{P_1^*})) = N^N SInt(M_{P_1^*})$. Thus (3) is proved.

Since $M_{P_1^*} \subseteq M_{P_2^*}$, by using (1), $N^N Int(M_{P_1^*}) \subseteq M_{P_1^*} \subseteq M_{P_2^*}$. That is $N^N SInt(M_{P_1^*}) \subseteq M_{P_2^*}$. By (3), $N^N SInt(N^N SInt(M_{P_1^*})) \subseteq N^N SInt(M_{P_2^*})$. Thus $N^N SInt(M_{P_1^*}) \subseteq N^N SInt(M_{P_2^*})$. Thus (4) is proved. □

Theorem 5.3. *Let $(U, N_N(\tau)$ be a $N^N TS$. Then for any neutrosophic subset $M_{P_1^*}$ and $M_{P_2^*}$ of a $N^N TS$, we have*

1. $N^N SInt(M_{P_1^*} \cap M_{P_2^*}) = N^N SInt(M_{P_1^*}) \cap N^N SInt(M_{P_2^*})$.
2. $N^N SInt(M_{P_1^*} \cup M_{P_2^*}) \supseteq N^N SInt(M_{P_1^*}) \cup N^N SInt(M_{P_2^*})$.

Proof. $M_{P_1^*} \cap M_{P_2^*} \subseteq M_{P_1^*}$ and $M_{P_1^*} \cap M_{P_2^*} \subseteq M_{P_2^*} \Rightarrow N^N SInt(M_{P_1^*} \cap M_{P_2^*}) \subseteq N^N SInt(M_{P_1^*})$ and $N^N SInt (M_{P_1^*} \cap M_{P_2^*}) \subseteq N^N SInt(M_{P_2^*})$. Hence, $N^N SInt\ (M_{P_1^*} \cap M_{P_2^*}) \subseteq N^N SInt(M_{P_1^*}) \cap N^N SInt(M_{P_2^*})$.

Now, $N^N SInt(M_{P_1^*}) \subseteq M_{P_1^*}$ and $N^N SInt(M_{P_2^*}) \subseteq M_{P_2^*}$. we get, $N^N SInt(M_{P_1^*}) \cap N^N SInt(M_{P_2^*}) \subseteq M_{P_1^*} \cap M_{P_2^*}$.
$\Rightarrow N^N SInt((SInt(M_{P_1^*}) \cap N^N SInt(M_{P_2^*})) \subseteq N^N SInt\ (M_{P_1^*} \cap M_{P_2^*})$, which implies, $N^N SInt\ (N^N SInt(M_{P_1^*})) \cap N^N SInt\ (N^N SInt(M_{P_2^*})) \subseteq N^N SInt\ (M_{P_1^*} \cap M_{P_2^*})$.

$\Rightarrow N^N SInt(M_{P_1^*}) \cap N^N SInt(M_{P_2^*}) \subseteq N^N SInt(M_{P_1^*} \cap M_{P_2^*})$. Hence,

$$N^N SInt(M_{P_1^*} \cap M_{P_2^*}) = N^N SInt(M_{P_1^*}) \cap N^N SInt(M_{P_2^*}).$$

This implies (1).
Since $M_{P_1^*} \subseteq M_{P_1^*} \cup M_{P_2^*}$ and $M_{P_2^*} \subseteq M_{P_1^*} \cup M_{P_2^*}$, we get, $N^N SInt(M_{P_1^*}) \subseteq N^N SInt(M_{P_1^*} \cup M_{P_2^*})$ and $N^N SInt(M_{P_2^*}) \subseteq N^N SInt(M_{P_1^*} \cup M_{P_2^*})$. $\Rightarrow N^N SInt(M_{P_1^*}) \cup N^N SInt(M_{P_2^*}) \subseteq N^N SInt\ (M_{P_1^*} \cup M_{P_2^*})$. Hence (2) is proved. □

The following example shows that the equality need not be hold.

Example 5.4. *Let U and $M_{P_1^*}$ be two non-empty finite sets, where U is the universe and $M_{P_1^*}$ the set of attributes. $U = \{S_1, S_2, S_3, S_4\}$ are Higher secondary student for wait for NEET entrance exam*

$$M_{P_1^*} = \{\text{ Physics, Chemistry, Biology }\}$$

are three attributes are Exam subjects its Neutrosphic values are given below. The members of $U = \{S_1, S_2, S_3, S_4\}$. Let $U/R = \{\{S_1\}, \{S2, S_3, S_4\}\}$ be an equivalence relation

$$S_1 = \left\langle \left(\frac{4}{10}, \frac{7}{10}, \frac{1}{10}\right), \left(\frac{5}{10}, \frac{6}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{7}{10}, \frac{3}{10}\right) \right\rangle$$
$$S_2 = \left\langle \left(\frac{4}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{7}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right) \right\rangle$$
$$S_3 = \left\langle \left(\frac{4}{10}, \frac{7}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{7}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{7}{10}, \frac{1}{10}\right) \right\rangle$$
$$S_4 = \left\langle \left(\frac{4}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{5}{10}, \frac{6}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{3}{10}\right) \right\rangle$$
$$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N(M)}, \overline{N(M)}, M_{P_2^*N}(M)\}$$
$$\underline{N(F)} = \left\langle \left(\frac{4}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{5}{10}, \frac{6}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{3}{10}\right) \right\rangle$$
$$\overline{N(F)} = \left\langle \left(\frac{4}{10}, \frac{7}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{7}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{7}{10}, \frac{1}{10}\right) \right\rangle$$
$$M_{P_2^*N}(F) = \left\langle \left(\frac{1}{10}, \frac{4}{10}, \frac{4}{10}\right), \left(\frac{2}{10}, \frac{4}{10}, \frac{5}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{9}{10}\right) \right\rangle$$

$$N_N(\tau) = \Big\{ 0_{N_N}, 1_{N_N}, \left\langle \left(\frac{4}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{5}{10}, \frac{6}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{3}{10}\right) \right\rangle \left\langle \left(\frac{4}{10}, \frac{7}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{7}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{7}{10}, \frac{1}{10}\right) \right\rangle \left\langle \left(\frac{1}{10}, \frac{4}{10}, \frac{4}{10}\right), \left(\frac{2}{10}, \frac{4}{10}, \frac{5}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{9}{10}\right) \right\rangle \left\langle \left(\frac{4}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{5}{10}, \frac{6}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{3}{10}\right) \right\rangle \Big\}$$

$$S_5 = \left\langle \left(\frac{7}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right) \right\rangle$$
$$S_6 = \left\langle \left(\frac{4}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{5}{10}, \frac{7}{10}, \frac{2}{10}\right), \left(\frac{10}{10}, \frac{7}{10}, \frac{1}{10}\right) \right\rangle$$

Then $(U, N_N(\tau)$ is a $N^N TS$. It follows that $N^N SInt(S_5 \cup s_{6)\not\subseteq N^N S}$ $Int\ (S_{5)} \cup N^N S\ Int(S_6)$.

6. Neutrosophic Nano Semi-Closure in Neutrosophic Topological Spaces

In this section, we introduce the concept of Neutrosophic Nano semi-closure operators in a $N^N TS$.

Definition 6.1. *Let $(U, N_N(\tau)$ be a $N^N TS$. Neutrosophic Nano semi-closure of $M_{P_1^*}[N^N(S)Cl(M_{P_1^*})$ for short] is, $N^N(S)Cl(M_{P_1^*}) = \cap\{K : K$ is a $N^N(SC)$ set in U and $K \supseteq M_{P_1^*}\}$.*

Proposition 6.2. *Let $(U, N_N$ be a $N^N TS$. Then for any neutrosophic nano subsets $M_{P_1^*}$ of U,*

1. $(N^N SInt(M_{P_1^*})^c = N^N(S)Cl((M_{P_1^*})^c)$,
2. $(N^N(S)Cl(M_{P_1^*}))^c = N^N SInt((M_{P_1^*})^c)$.

Proof. By Definition, $N^N SInt(M_{P_1^*}) = \cup\{G : G \text{ is a } N^N(SO) \text{ set in } U \text{ and } G \subseteq M_{P_1^*}\}$.

Taking complement on both sides, $(N^N SInt(M_{P_1^*}))^c = (\cup\{G : G$ is a $N^N(SO)$ set in U and $G \subseteq M_{P_1^*}\})^c = \cap\{(G)^c : (G)^c$ is a $N^N(SC)$ set in U and $(M_{P_1^*})^c \subseteq (G)^c\}$.

Replacing $(G)^c$ by K, we get $(N^N SInt(M_{P_1^*}))^c = \cap\{K : K$ is a $N^N(SC)$ set in U and $K \supseteq (M_{P_1^*})^c\}$. By Definition, $(N^N S\ Int\ (M_{P_1^*}))^c = N^N(S)Cl((M_{P_1^*})^c)$. This proves (1).
(1) ⇒

$$(N^N SInt((M^c_{P_1^*})))^c = N^N(S)Cl((M^c_{P_1^*}))^c = N^N(S)Cl(M_{P_1^*}).$$

Taking complement on both sides, we get $N^N SInt((M_{P_1^*})^c) = (N^N(S)Cl(M_{P_1^*}))^c$. This proves (2). □

Proposition 6.3. *Let $(U, N_N(\tau)$ be a $N^N TS$. Then for any neutrosophic subsets $M_{P_1^*}$ and $M_{P_2^*}$ of a $N^N TSU$ we have*

1. *$M_{P_1^*} \subseteq N^N(S)Cl(M_{P_1^*})$.*
2. *$M_{P_1^*}$ is $N^N(SC)$ set in $U \Leftrightarrow N^N(S)Cl(M_{P_1^*}) = M_{P_1^*}$.*
3. *$N^N(S)Cl(N^N(S)Cl(M_{P_1^*})) = N^N(S)Cl(M_{P_1^*})$.*
4. *If $M_{P_1^*} \subseteq M_{P_2^*}$ then $N^N(S)Cl(M_{P_1^*}) \subseteq N^N(S)Cl(M_{P_2^*})$.*

Proof. (1) Proof is obvious.
Let $M_{P_1^*}$ be $N^N(SC)$ set in U. By Definition, $(M_{P_1^*})^C$ is $N^N(SO)$ set in U. Now, $N^N SInt((M_{P_1^*})^c) = (M_{P_1^*})^c \Leftrightarrow (N^N(S)Cl(M_{P_1^*}))^c = (M_{P_1^*})^c \Leftrightarrow N^N(S)Cl(M_{P_1^*}) = M_{P_1^*}$. Thus proved (2).
(2) $\Rightarrow N^N(S)Cl(N^N(S)Cl(M_{P_1^*})) = N^N(S)Cl(M_{P_1^*})$. Thus proved (3).
Since $M_{P_1^*} \subseteq M_{P_2^*}, (M_{P_2^*})^c \subseteq (M_{P_1^*})^c$ we get, $N^N SInt\ ((M_{P_2^*})^c) \subseteq N^N SInt((M_{P_1^*})^c)$. Taking complement on both sides, $(N^N SInt((M^c_{P_2^*})^c \supseteq (N^N SInt((M_{P_1^*})^c))^c$. $\Rightarrow N^N(S)Cl\ (M_{P_1^*}) \subseteq N^N(S)Cl(M_{P_2^*})$. This proves (4). □

Proposition 6.4. *Let $M_{P_1^*}$ be a neutrosophic nano set in a $N^N TSU$. Then $N^N Int(M_{P_1^*}) \subseteq N^N SInt(M_{P_1^*}) \subseteq M_{P_1^*} \subseteq N^N(S)Cl\ (M_{P_1^*}) \subseteq N^N Cl(M_{P_1^*})$.*

Proof. It follows from the definitions of corresponding operators. □

Proposition 6.5. *Let $(U, N_N(\tau)$ be a $N^N TS$. Then for a neutrosophic subset $M_{P_1^*}$ and $M_{P_2^*}$ of a $N^N TSU$, we have*

1. *$N^N(S)Cl(M_{P_1^*} \cup M_{P_2^*}) = N^N(S)Cl(M_{P_1^*}) \cup N^N(S)Cl\ (M_{P_2^*})$ and*
2. *$N^N(S)Cl(M_{P_1^*} \cap M_{P_2^*}) \subseteq N^N(S)Cl(M_{P_1^*}) \cap N^N(S)Cl\ (M_{P_2^*})$.*

Proof. Since $N^N(S)Cl(M_{P_1^*} \cup M_{P_2^*}) = N^N(S)Cl(C(C(M_{P_1^*} \cup M_{P_2^*}$ we have, $N^N(S)Cl(M_{P_1^*} \cup M_{P_2^*}) = (N^N SInt((M_{P_1^*} \cup M_{P_2^*})^C))^C = (N^N SInt((M_{P_1^*})^C \cap (M_{P_2^*})^C))^C$. $\Rightarrow N^N(S)Cl(M_{P_1^*} \cup M_{P_2^*}) = (N^N SInt\ ((M_{P_1^*})^C)^C \cap N^N SInt\ (M_{P_2^*})^C)) = (N^N SInt((M_{P_1^*})^C))^C \cup (N^N SInt\ ((M_{P_2^*})^C))^C$. $\Rightarrow N^N(S)Cl\ (M_{P_1^*}\ U M_{P_2^*}) = N^N(S)Cl(((M_{P_1^*})^C))^C \cup N^N(S)Cl\ (((M^C_{P_2^*}))^C) = N^N(S)Cl(M_{P_1^*}) \cup N^N(S)Cl(M_{P_2^*})$. Thus proved (1).

Since $M_{P_1^*} \cap M_{P_2^*} \subseteq M_{P_1^*}$ and $M_{P_1^*} \cap M_{P_2^*} \subseteq M_{P_2^*}$ we have, $N^N(S)Cl(M_{P_1^*} \cap M_{P_2^*}) \subseteq N^N(S)Cl(M_{P_1^*})$ and $N^N(S)Cl\ (M_{P_1^*} \cap M_{P_2^*}) \subseteq N^N(S)Cl(M_{P_2^*})$. $\Rightarrow N^N(S)Cl\ (M_{P_1^*} \cap M_{P_2^*}) \subseteq N^N(S)Cl(M_{P_1^*}) \cap N^N(S)Cl(M_{P_2^*})$. Thus proved (2). □

The following example shows that the equality need not be hold.

Example 6.6. *Let U and $M_{P_1^*}$ be two non-empty finite sets, where U is the universe and $M_{P_1^*}$ the set of attributes $U = \{P_1, P_2, P_3,\ P_4\}$ are sugar patient.*
Let $U/R = \{\{P_1, P_2, P_3\}, \{P_4\}\}$ be an equivalence relation $M_{P_1^*} = \{$Thirsty, Urinal, wait lose$\}$ are three attributes its Neutrosphic values are given below

$$P_1 = \left\langle \left(\frac{5}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{6}{10}, \frac{7}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{2}{10}\right) \right\rangle$$
$$P_2 = \left\langle \left(\frac{4}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{8}{10}, \frac{6}{10}, \frac{3}{10}\right), \left(\frac{9}{10}, \frac{7}{10}, \frac{3}{10}\right) \right\rangle$$
$$P_3 = \left\langle \left(\frac{4}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{6}{10}, \frac{6}{10}, \frac{3}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{3}{10}\right) \right\rangle$$

$$P_4 = \left\langle \left(\frac{5}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{7}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{7}{10}, \frac{2}{10}\right) \right\rangle$$
$$N_N(\tau) = \{0_{N_N},\ 1_{N_N}, \underline{N(M)}, \overline{N(M)}, M_{P^*_2 N}(M)\}$$
$$\underline{N(F)} = \left\langle \left(\frac{4}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{6}{10}, \frac{6}{10}, \frac{3}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{3}{10}\right) \right\rangle$$
$$\overline{N(F)} = \left\langle \left(\frac{5}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{7}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{7}{10}, \frac{2}{10}\right) \right\rangle$$
$$M_{P^*_2 N}(F) = \left\langle \left(\frac{2}{10}, \frac{4}{10}, \frac{4}{10}\right), \left(\frac{3}{10}, \frac{4}{10}, \frac{6}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{9}{10}\right) \right\rangle$$
$$N_N(\tau) = \Big\{ 0_{N_N}, 1_{N_N},$$
$$\left\langle \left(\frac{4}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{6}{10}, \frac{6}{10}, \frac{3}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{3}{10}\right) \right\rangle$$
$$\left\langle \left(\frac{5}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{7}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{7}{10}, \frac{2}{10}\right) \right\rangle$$
$$\left\langle \left(\frac{2}{10}, \frac{4}{10}, \frac{4}{10}\right), \left(\frac{3}{10}, \frac{4}{10}, \frac{6}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{9}{10}\right) \right\rangle$$
$$\left\langle \left(\frac{5}{10}, \frac{6}{10}, \frac{1}{10}\right), \left(\frac{8}{10}, \frac{7}{10}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{7}{10}, \frac{2}{10}\right) \right\rangle \Big\}$$

Then $(U, N_N(\tau))$ is a $N^N TS$. Consider the $N^N Ss$ are

$$P_5 = \left\langle \left(\frac{1}{10}, \frac{2}{10}, \frac{5}{10}\right), \left(\frac{2}{10}, \frac{3}{10}, \frac{7}{10}\right), \left(\frac{3}{10}, \frac{3}{10}, \frac{10}{10}\right) \right\rangle \text{ and}$$

$$P_6 = \left\langle \left(\frac{2}{10}, \frac{4}{10}, \frac{4}{10}\right), \left(\frac{1}{10}, \frac{2}{10}, \frac{8}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{9}{10}\right) \right\rangle.$$

Then $N^N(S)Cl(P_5) \cap N^N(S)Cl(P_6) \nsubseteq N^N(S)Cl(P_5 \cap P_6)$.

Theorem 6.7. *If $M_{P_1^*}$ and $M_{P_2^*}$ are $N^N Ss$ of $N^N TSs$ U and V respectively, then*

1. *$N^N(S)Cl(M_{P_1^*}) \times N^N(S)Cl(M_{P_2^*}) \supseteq N^N(S)Cl(M_{P_1^*} \times M_{P_2^*})$,*
2. *$N^N SInt(M_{P_1^*}) \times N^N SInt(M_{P_2^*}) \subseteq N^N Int(M_{P_1^*} \times M_{P_2^*})$.*

Proof. (1) Since $M_{P_1^*} \subseteq N^N(S)Cl(M_{P_1^*})$ and $M_{P_2^*} \subseteq N^N(S)Cl(M_{P_2^*})$, hence $M_{P_1^*} \times M_{P_2^*} \subseteq N^N(S)Cl\ (M_{P_1^*}) \times N^N(S)Cl\ (M_{P_2^*})$.
$\Rightarrow N^N(S)Cl(M_{P_1^*} \times M_{P_2^*}) \subseteq N^N(S)Cl(N^N(S)Cl(M_{P_1^*}) \times N^N(S)Cl(M_{P_2^*}))$ and $N^N(S)Cl(M_{P_1^*} \times M_{P_2^*}) \subseteq N^N(S)Cl\ (M_{P_1^*}) \times N^N(S)Cl(M_{P_2^*})$.
(2) follows from (1) and the fact that $N^N SInt\ (M_{P_1^*})^C = (N^N(S)Cl(M_{P_1^*}))^C$. □

Lemma 6.8. *For $N^N Ss\, M_{P_1^*\,i}$'s and $M_{P_2^*\,j}$'s of $N^N TSs\, U$ and V respectively, we have*

1. *$\cap\{M_{P_1^*\,i}, M_{P_2^*\,j}\} = \min(\cap M_{P_1^*\,i}, \cap M_{P_2^*\,j})$;*
   $\cup\{M_{P_1^*\,i}, M_{P_2^*\,j}\} = \max(\cup M_{P_1^*\,i}, \cup M_{P_2^*\,j})$.
2. *$\cap\{M_{P_1^*\,i}, 1_{N_N}\} = (\cap M_{P_1^*\,i}) \times 1_{N_N}$;*
   $\cup\{M_{P_1^*\,i}, 1_{N_N}\} = (\cup M_{P_1^*\,i}) \times 1_{N_N}$.
3. *$\cap\{1_{N_N} \times M_{P_2^*\,j}\} = 1_{N_N} \times (\cap M_{P_2^*\,j})$;*
   $\cup\{1_{N_N} \times M_{P_2^*\,j}\} = 1_{N_N} \times (\cup M_{P_2^*\,j})$.

Proof. Obvious. □

Theorem 6.9. *Let $N_N(\tau)$ be a $N^N TS$. Then for a neutrosophic subset $M_{P_1^*}$ and $M_{P_2^*}$ of U we have,*

1. *$N^N(S)Cl(M_{P_1^*}) \supseteq M_{P_1^*} \cup N^N(S)Cl(N^N SInt(M_{P_1^*}))$,*
2. *$N^N SInt(M_{P_1^*}) \subseteq M_{P_1^*} \cap N^N SInt N^N(S)Cl(M_{P_1^*}))$,*
3. *$N^N Int(N^N(S)Cl(M_{P_1^*})) \subseteq N^N Int(N^N Cl(M_{P_1^*}))$,*
4. *$N^N Int(N^N(S)Cl(M_{P_1^*})) \subseteq N^N Int(N^N(S)Cl(N^N SInt\ M_{P_1^*}))$.*

Proof. $M_{P_1^*} \subseteq N^N(S)Cl(M_{P_1^*}) \Rightarrow N^N SInt(M_{P_1^*}) \subseteq M_{P_1^*}$. Then $N^N(S)Cl(N^N SInt(M_{P_1^*})) \subseteq N^N(S)Cl(M_{P_1^*})$ From the above,

$$M_{P_1^*} \cup N^N(S)Cl(N^N SInt(M_{P_1^*})) \subseteq N^N(S)Cl(M_{P_1^*}).$$

Thus proved (1).
Now, $N^N SInt(M_{P_1^*}) \subseteq M_{P_1^*} \Rightarrow M_{P_1^*} \subseteq N^N(S)Cl(M_{P_1^*})$. Then $N^N SInt(M_{P_1^*}) \subseteq N^N SInt(N^N(S)Cl(M_{P_1^*}))$ From the above, we have $N^N SInt(M_{P_1^*}) \subseteq M_{P_1^*} \cap N^N SInt(N^N(S)Cl(M_{P_1^*}))$. Thus proved (2).
Since $N^N(S)Cl(M_{P_1^*}) \subseteq N^N Cl(M_{P_1^*})$ We get,

$$N^N Int(N^N(S)Cl(M_{P_1^*})) \subseteq N^N Int(N^N Cl(M_{P_1^*})).$$

Thus proved (3).

(1) $\Rightarrow N^N(S)Cl(M_{P_1^*}) \supseteq M_{P_1^*} \cup N^N(S)Cl(N^N SInt(M_{P_1^*}))$. We have $N^N Int(N^N(S)Cl(M_{P_1^*}) \supseteq N^N Int(M_{P_1^*} \cup N^N(S)Cl(N^N SInt(M_{P_1^*})))$.

Since, $N^N Int(M_{P_1^*} \cup M_{P_2^*}) \supseteq N^N Int(M_{P_1^*}) \cup N^N Int(M_{P_2^*})$, $N^N Int(N^N(S)Cl(M_{P_1^*}) \supseteq N^N Int(M_{P_1^*} \cup N^N Int(N^N(S) Cl(N^N SInt(M_{P_1^*}))) \supseteq N^N Int(N^N(S)Cl(N^N SInt(M_{P_1^*})))$. Thus proved (4). □

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