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Equitable edge domination in vague graphs

M. Kaliraja[1], P. Kanibose[2]* and A. Ibrahim[3]

Abstract
In this paper, we introduce the notions of equitable edge dominating set, minimal equitable edge domination vague set and equitable edge domination number of vague graph. Moreover, we investigate some related properties in these concepts with illustrations.

Keywords
Vague graph; Equitable edge dominating set; Equitable edge domination number; Equitable edge independent set.

AMS Subject Classification
03E72, 03B52, 05C69.

[1,2,3] *P.G. and Research Department of Mathematics, H.H. The Rajah's College, Pudukkottai-622001, Tamil Nadu, India.*
[1,2,3] *Affiliated to Bharathidasan University, Tiruchirappalli-620024, Tamil Nadu, India.*
***Corresponding author**: [1]mkr.maths009@gmail.com; [2]kanibose77@gmail.com; [3]ibrahimaadhil@yahoo.com
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Contents

1. Introduction

Initially the basic concept of degree of equitable domination on graphs was introduced by V.S. Swaminathan and K. M. Dharmalingam [10]. They [10] have discoursed briefly about the degree of equitable domination graphs to determine the complexity of the "equitable domination." The equitable domination in graphs was introduced by A. Alwardi and N. D. Soner[1]. They have defined equitable dominating set of a graph G as $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leqslant 1$. K. M. Dharmalingam and M. Rani [4] have stated in detail fuzzy equitable minimal fuzzy equitable dominating set, dominating set, fuzzy equitable independent set of fuzzy graphs, strong or weak fuzzy equitable dominating set. A. Nagoor Gani and K. Prasanna Devi [8] introduced the notions of edge domination and fuzzy graph's independent edge dominating set. G. Thamizhendhi and R. Parvathi [11] proposed a another set of domination concepts which can be used with Intuitionistic Fuzzy Graphs (IFGs). J. John Stephan, A. Muthaiyn and N. Vinoth Kumar [6] introduced the idea of equitable domination in an IFGs. Also, they [6] have used the concepts of intuitionistic, minimal intuitionistic fuzzy equitable dominating set and Intuitionistic fuzzy equitable independent set to obtain results in IFGs.
W. L. Gau and D. J. Buehrer [5] had put up the concept of vague set. N. Ramakrishna [9] also had deliberated about the same concept. R.A. Borzooeiy, E.Darabianz, and H. Rashmanlou [2,3] had studied the notion of domination in vague graphs and did obtain the presence of strong domination numbers through applications. Yahya Talebi and Hossein Rashmanlou [12] suggested the concept of application of dominating sets in vague graphs. The authors [7] had used the notion of edge domination vague graph, independent edge domination, and edge domination numbers of vague graphs and investigated a few related properties with illustrations.
In this paper, we have introduced the notions of equitable edge domination set, minimal equitable and equitable edge domination number of vague graphs. Moreover, we have investigated some of the related properties in these concepts with illustrations.

2. Preliminaries

In this section, we review some basic definitions and properties which are used to develop of main results.

Definition 2.1 ([5]). *A vague set P in the universe of discourse X is characterized by two membership functions given by*

(i) Membership function's true value $t_P : X \to [0,1]$

(ii) Membership function's false value $f_P : X \to [0,1]$.

Where $t_p(x)$ is the lower bound value of x derived from the 'evidence for x', and $f_P(x)$ is the lower bound of the negation of x derived from the 'evidence against x' which comes to $t_P(x) + f_P(x) \leq 1$. Thus the grade membership of x in the vague set P is bounded by a subinterval $[t_P(x), 1 - f_P(x)]$ of $[0,1]$. The vague set A is formulated as $P = \{(x, [t_P(x), f_P(x)]) / x \in X\}$. Here the interval $[t_P(x), 1 - f_P(x)]$ is called the value of x for the vague set P and is denoted by $V_P(x)$.

Definition 2.2 ([9]). *A vague relation is created from the vague subset $X \times Y$ That is,*

$$R = \{\{(x,y), t_R(x,y), f_R(x,y)\} / x \in X, y \in Y\},$$

where $t_R : X \times Y \to [0,1]$ and $f_R : X \times Y \to [0,1]$ satisfies the condition $0 \leq t_R(x,y) + f_R(x,y) \leq 1$ for all $(x,y) \in X \times Y$.

Definition 2.3 ([9]). *Let $G = (P,Q)$ be a vague graph, where $P = (t_P, f_P)$ and $Q = (t_Q, f_Q)$ are vague sets on V and $Q \subseteq P \times P$ respectively. They defined as $t_Q(uv) \leq \min\{t_P(u), f_P(v)\}$ and $f_Q(uv) \geq \max\{t_P(u), f_P(v)\}$ for all $uv \in Q$.*

Definition 2.4 ([9]). *A vague graph $G = (P,Q)$ is strong, if: $t_Q(v_i v_j) = \min\{t_P(v_i), f_P(v_j)\}$ and $f_Q(v_i v_j) = \max\{t_P(v_i), f_P(v_j)\}$ for all $v_i v_j \in Q$.*

Definition 2.5 ([3]). *Let u be a node the vague graph $G = (P,Q)$, then the neighborhood of u is represented by*

$$N(u) = \{v \in V / (u,v) \text{ which is a strong path }\}.$$

Definition 2.6 ([12]). *Let e_i be a path in vague graph $G = (P,Q)$. Then,*

$$N_s(e_i) = \left\{e_i, e_j \in Q(G) : (e_i, e_j) \text{ is a stron arcs in } G\right\}.$$

The cardinality of $N_s(e_i)$ is the edge vague degree of e_i and is denoted by $d_e(f)$.

Definition 2.7 ([12]). *An edge (u,v) is said to be strong in vague graph $G = (P,Q)$, if, $t_Q(uv) \geq (t_Q)^\infty(uv)$ and $f_Q(uv) \leq (f_Q)^\infty(uv)$. Here*

$$(t_Q)^\infty(uv) = \max\left\{(t_Q)^k(uv) : k = 1,2,\ldots,n\right\}$$

and

$$(f_Q)^\infty(uv) = \min\left\{(f_Q)^k(uv) : k = 1,2,\ldots,n\right\}.$$

Definition 2.8 ([2]). *Let $G = (P,Q)$ be a vague graph. Let $u, v \in P$, then we can say that u dominates v in G if there exists a strong edge between them.*

Definition 2.9 ([2]). *An edge in vague graph $G = (P,Q)$ is an isolated edge, if it is not adjacent to any strong edge in G*

Definition 2.10 ([3]). *Let G be a vague graph. If each edge in G has a similar degree (m,n), then G is said to be an edge regular vague graph.*

Definition 2.11 ([2]). *The strong degree (Q) in vague graph is:*

$$d_s(Q) = \left(\sum_{u \in N_s(v)} t_Q(uv), \sum_{u \in N_s(v)} f_Q(uv)\right).$$

Definition 2.12 ([2]). *The strong degree cardinality of Q in vague graph is*

$$|d_s(Q)| = \sum_{u \in N_s(V)} \frac{t_Q(uv) + (1 - f_Q(uv))}{2}.$$

Definition 2.13 ([9]). *The strong degree of edges in vague graph is*

$$\begin{aligned}
d_\mu(e) &= d_\mu(v_j) + d_\mu(v_i) - 2\mu_2(v_i v_j) \quad \text{or} \\
d_\mu(e) &= \sum_{\substack{v_i v_j \in E \\ r \neq j}} \mu_2(v_i v_r) + \sum_{\substack{v_i v_j \in E \\ r \neq i}} \mu_2(v_r v_j) \\
d_\sigma(e_{ij}) &= d_\sigma(v_j) + d_\sigma(v_i) - 2\sigma_2(v_i v_j) \quad \text{or} \\
d_\sigma(e_{ij}) &= \sum_{\substack{v_i v_j \in E \\ r \neq j}} \sigma_2(v_i v_k) + \sum_{\substack{v_i v_j \in E \\ r \neq i}} \sigma(v_k v_j)
\end{aligned}$$

Definition 2.14 ([7]). *Let $G = (P,Q)$ be a vague graphs and $e_{i,} e_j \in Q$. Then, we state that $e_{i,}$ dominates e_j, if e_i is a strong arc in adjacent to e_j.*

Definition 2.15 ([7]). *Let D be a minimum dominating set of a vague graph G. If $e_j \in Q - D$, then there exist $e_{i,} \in D$. such that e_i dominates e_j edge. Thus, D is called an edge dominating set of D.*

Definition 2.16 ([7]). *The smallest vague cardinality of each edge domination number of vague graph G is denoted by $d_e(G)$.*

3. Properties of Equitable Edge Dominatiing Set

In this segment, we are introducing the equitable edge dominating set of a vague graph for obtaining a few properties with illustration.

Definition 3.1. *Let $G = (P,Q)$ be a vague graph. Let e_i and e_j be any two edges of G. A subset D of Q is an equitable edge dominating set, if every edge $e_j \in Q(G) - D$. Then there exists an edge $e_i \in D$ such that $e_i e_j \in Q(G)$ and $\left|\deg(e_i) - \deg(e_j)\right| \leq 1$, where $\min\left\{t_P(v_i), f_P(v_j)\right\} \geq t_Q(e)$ and $\max\left\{t_P(v_i), f_P(v_j)\right\} \leq f_Q(e)$ for all $e \in Q$.*

Note 3.2. *The least cardinality of an equitable edge domination vague set is signified by $\gamma_e(G)$.*

Definition 3.3. *A equitable edge dominating set D is said to be a minimal vague equitable dominating set, if it is not an appropriate subset of D.*

Definition 3.4. *(i) Let $G=(P,Q)$ be a vague graph. Then $D \subseteq Q(G)$ is said to be a strong or weak equitable edge dominating set of G, if every edge $e_j \in Q(G)-D$ is strongly (weakly) dominated by some edge e_i in D.*
(ii) The smallest and largest value of degrees in the strong edge G is defined respectively $\Delta_s(G) = {}_{e_i \in Q(G)} \max \mid N_s(e_i) \mid$ and $\delta_s(G) = {}_{e_i \in Q(G)} \min |N_s(e_i)|$.

Example 3.5. *Let $G=(P,Q)$ be a vague graph as shown in the figure 3.1. From the edge set $Q=\{e_1, e_2, e_3, e_4, e_5,\}$, we have $\{e_1,e_5\}, \{e_4,e_6\}, \{e_1,e_2\}, \{e_1,e_6,e_4\}$ which are equitable edge dominating sets where $\{e_4,e_6\}$ is minimal equitable edge dominating set of G and $\gamma_e(G)=0.50$ From the*

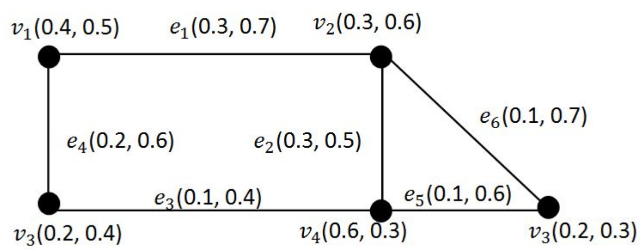

Figure 1

Figure 1, we have the degrees of edges are defined as follows:

$$\begin{aligned}
\deg(e_1) =& (0.5, 1.1), \deg(e_2) = (0.4, 1.2),\\
\deg(e_3) =& (0.6, 1.7), \deg(e_4) = (0.4, 1.0)\\
\deg(e_5) =& (0.3, 0.5), \deg(e_6) = (0.7, 1.7).
\end{aligned}$$

Also, the degrees of edges of strong arcs as follows: $|d(e_1)| = 0.7, |d(e_2)| = 0.7, |d(e_3)| = 0.95, |d(e_4)| = 0.35, |d(e_5)| = 0.8, |d(e_6)| = 1.00$. Then, we have the least and greatest degrees of arcs in G which are $\Delta_s(G) = 0.35, \quad \delta_s(G) = 1.00$.

Proposition 3.6. *Let $G=(P,Q)$ be a vague graph. Then D is an equitable edge dominating set of G, which is minimal, if and only if, for all the edges in $e_i \in D$ satisfies the following conditions.*

(i) Either $N(e_i) \cap D = \emptyset$ or $\left|\deg(e_i) - \deg(e_j)\right| \geq 2$.

(ii) There exists an edge $e_j \in Q(G)-D$ such that $N(e_j) \cap D = \{e_i\}$ and $\left|\deg(e_i) - \deg(e_j)\right| \leq 1$.

Proof. Let G be a vague graph and D be the least equitable edge dominating set of G. If the above conditions (i) and (ii) are not satisfied, then for some edge $e_i \in D$, there exists an edge $e_j \in N(e_i) \cap D$ such that, $\left|\deg(e_i) - \deg(e_j)\right| \leq 1$, and for some edge $e_j \in Q(G)-D$, either $N(e_j) \cap D$ is not equal to $\{e_i\}$ or $\left|\deg(e_i) - \deg(e_j)\right| \geq 2$ or together. Therefore, $D - \{e_i\}$ is an equitable edge dominating set, which is inconsistent to the minimality of D. Hence, (i) and (ii) are true.

Then again, for every edge in a vague graph G there is a strong edge $e_i \in Q(G)$ such that one of the statements gets satisfied. Assume D is not a minimal one, then there exists $e_i \in D$, such that $D - \{e_i\}$ is an equitable dominating set. Therefore there exist $e_j \in D - e_i$ such that, e_j is an equitable dominating set of e_i. Then the edge $e_j \in N(e_i) \cap D$, $\left|\deg(e_i) - \deg(e_j)\right| \leq 1$ and e_i are not equal to e_j, which is a disagreement to $N(e_j) \cap D = \{e_i\}$. Thus D is a least equitable edge dominating set. □

Proposition 3.7. *If $G=(P,Q)$ is a regular vague graph, then $\gamma(G) = \gamma_e(G)$.*

Proof. Let G be a regular vague graph where the edges deg $e_i = \deg e_j$ in G are strong. We can state that every edge is denoted by b. Let D be an edge dominating set G, then $\gamma_e(G) = |D|$.

Let us consider $e_j \in Q(G)-D$ Here D must be an edge dominating set, where strong edge e_ie_j that exists is adjacent, and also $\deg e_i = \deg e_j = b$. Therefore, $\left|\deg(e_i) - \deg(e_j)\right| = 0 < 1$ is an equitable edge dominating set D in the vague graph G. So that, $\gamma_e(G) \leq |D| = \gamma(G)$ But $\gamma_e(G) \geq \gamma(G)$ which implies, $\gamma(G) = \gamma_e(G)$. □

Proposition 3.8. *Let $G=(P,Q)$ be a vague graph of order r. Then*

(i) $\gamma_e(G) \leq \gamma_{se}(G) \leq r - \Delta_e(G)$

(ii) $\gamma_e(G) \leq \gamma_{we}(G) \leq r - \delta_e(G)$.

Proof. Let G be a vague graph of order r. In G, every strong equitable edge dominating vague set is an equitable edge dominating vague graph. Hence $\gamma_e(G) \leq \gamma_{se}(G)$. similarly we know that all the weak equitable edge dominating set is also a part of vague graph G. Let $e_ie_j \in E$. If $d_e(e) = \Delta_e(G)$ and $d_e(e) = \delta_e(G)$, clearly $Q - N_e(e)$ is a strong vague equitable edge dominating set G. Therefore, we have

$$\begin{aligned}
\delta_{se}(G) &\leq |Q - N_e(e)|\\
\delta_{we}(G) &\leq |Q - N_e(e)|\\
\gamma_{se}(G) &\leq r - \Delta_e(G)\\
\gamma_{we}(G) &\leq r - \delta_e(G).
\end{aligned}$$

□

Proposition 3.9. *Let $G=(P,Q)$ be a vague graph and D be a minimal edge dominating set of connected graph G. Then $Q(G)-D$ is an edge dominating set of G.*

Proof. Let D be a minimal edge dominating set of G and $Q(G)-D$ is not an edge dominating set. Then there is an edge $e_i \in D$ such that, e_i is not dominated by any edge in $Q(G)-D$ and G is connected. since e_i is a strong neighbor of at least one edge in $D - \{e_i\}, D - \{e_i\}$ is an edge dominating set, which is a denial to the minimality of D. This is true for all edges e_i in D. Then, at least one edge e_j in $Q(G)-D$ is present, such that $t_Q(e) = t_P(u_j) \wedge t_P(v_i)$ and $f_Q(e) = f_P(v_j) \wedge f_P(v_i)$. Thus, $Q(G)-D$ is a dominating set. □

Definition 3.10. *The edges e_i and e_j are said to be edge independent of a vague graph $=(P,Q)$, if $e_i \notin N_s(e_j)$ and $e_j \notin N_s(e_i)$.*

Example 3.11. *Let $G = (P,Q)$ be a vague graph, as shown in the Figure 2 Here $\{e_1, e_3, e_4, e_2\}$ all edges in the are no strong arc between them. These are all edges of independent vague graph.*

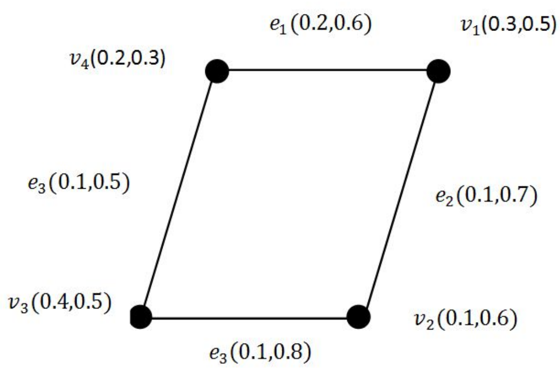

Figure 2

Proposition 3.12. *Let $G = (P,Q)$ be a vague graph, then an equitable dominating set of G having only strong edge is a minimal equitable set, if and only if, it is an edge independent set.*

Proof. Let D be an edge independent set which are equitable of a vague graph G having only strong arc. If the equitable edge independent set D is maximal, then for every edge $e_i \in Q(G) - D$, the set $D \cup \{e_i\}$ is not an equitable independent set of a vague graph.

That is, for every $e_i \in Q(G) - D$, there is an edge $e_j \in D$, such that e_j is strong neighbor to e_i, So D is an equitable edge dominating set, and also an edge independent set of G.

Conversely, D is each equitable edge independent and edge dominating set in G.

To prove: D is maximal equitable edge independent set of G having strong edges:

If D is not a maximal equitable independent set, there exist an edge $e_i \notin D$, such that $D \cup \{e_i\}$ is a equitable edge dominating set. Therefore there is no equitable edge in D which is a strong neighbor to e_i Thus, D cannot be a equitable edge dominating set which is a contradiction. Therefore D is a maximal equitable edge independent set of G having only strong edges. □

Proposition 3.13. *Let $G = (P,Q)$ be a vague graph having edge independent set that has only strong arcs. It tends be a maximal vague independent set, if and only if, it is both a edge independent set and also an equitable edge dominating set.*

Proof. Let G be a vague graph and M be a maximal edge independent set of G having only a strong arc. Then for every $e_i \in Q - M$, the set $M \cup \{e_i\}$ will not be a vague edge independent set because in $e_i \in Q - M$ the edge $e_j \in N_s(e_i)$. Hence, M is an edge dominating set of G and hence it is vague edge independent for set G.

Conversely, the same M is mutually vague edge independent as well as an edge dominating set of G.

To prove: We have to show that M is a maximal vague edge independent set, with only strong arcs. since M is an equitable edge dominating set of G, it has only a strong arcs. If M is not a maximal vague edge independent set, then there exist an edge $e_i \notin M$, so that $M \cup e_i$ is an equitable edge independent set of G. since $M \cup \{e_i\}$ is vague edge independent set there happens to be no edge in M that belongs to $N_s(e_i)$, and hence e_i cannot be dominated by M. The statement that M is not an equitable edge dominating set of G is true. Hence, it is confirmed that M should be a maximal vague edge independent set of G having just strong arcs. □

Proposition 3.14. *Let $G = (P,Q)$ be a vague graph without any equitable isolated edges. Here D be a minimal equitable edge domination set and $Q(G) - D$ is an edge dominating set which is equitable.*

Proof. Let G be a vague graph and D be the least value of equitable edge dominating set. $Q(G) - D$ is not an equitable edge dominating set of G. Then there exists an edge e_i such that $e_i \in D$ is not an equitable edge dominating set in $Q(G) - D$. The vague graph G has no isolated edge and there is no edge $e_i \in G$, such that $N_s(e_i) \subseteq D$. Then, e_i is adjacent to at least one strong edge e_j in $Q(G).Q(G) - D$ is not an equitable edge dominating set of G and e_j is also not in $Q(G) - D$, $D - \{e_j\}$ is an equitable edge dominating set which is a denial to $Q(G)$ that, it is the least value of a equitable edge dominating set. Therefore, $Q(G) - D$ is similar to the edges of the dominating set D. □

4. conclusion

In this paper, we have introduced the notions of equitable edge dominating set, equitable edge domination set of minimal vague graphs and equitable edge domination number of vague graphs. Moreover, we have studied some properties with illustrations.

References

[1] A. Alwardi and N. D. Soner, Equitable Edge Domination in Graph, *Bulletin of International Mathematics Virtual Institute*, 3(2013), 7–13.

[2] R.A. Borzooei, E. Darabian and H. Rashmanlou, Strong Domination Numbers of Vague Graphs with Applications, *Applied Mathematics E-Notes*, 16(2016), 251–267.

[3] R. A. Borzooei and H. Rashmanlou, Domination in Vague Graphs and its Applications, *Journal of Intelligent & Fuzzy Systems*, 29(2015), 1933–1940.

[4] K. M. Dharmalingam and M. Rani, Equitable Domination in Fuzzy Graphs, *International Journal of Pure and Applied Mathematics,* 94(3)(2014), 661–667.

[5] W. L. Gau and D. J. Buehrer, Vague Sets, IEEE Transactions on Systems, *Man and Cybernetics*, 23(2), (1993), 610–614.

[6] J. John Stephan, A. Muthaiyn and N. Vinoth Kumar, Equitable Dominating in an Intunionistic Fuzzy Graphs, *Global Journal of Pure and Applied Mathematics*, 12(3)(2016), 2335–2341.

[7] M. Kaliraja, P. Kanibose and A. Ibrahim, Edge Domination in Vague Graph, *International Journal of Advanced Science and Technology,* 29(1)(2020), 1474–1480.

[8] A. Nagoor Gani and K. Prasanna Devi, Edge Domination and Independent in Fuzzy Graph, *Advances in Fuzzy Sets and Systems,* 15(2)(2013), 73–84.

[9] N. Ramakrishna, Vague graphs, *International Journal of Computational Cognition*, 7(2009), 51–58.

[10] V. S. Swaminathan and K. M. Dharmalingam, Degree Equitable Domination on Graphs, *Kragujevac Journal of Mathematics,* 35(1)(2011), 177–183.

[11] G. Thamizhendhi and R. Parvathi, Equitable, Restrained and k-Domination in Intuitionistic Fuzzy Graphs, *Global Journal of Pure and Applied Mathematics*, 12(1), (2015), 125–145.

[12] Yahya Talebiy, Hossein Rashmanlouz, Application Of Dominating Sets In Vague Graphs, *Applied Mathematics, E-Notes*, 17(2017), 251–267.

[13] L. A. Zadeh, Fuzzy sets, *Information and Control*, 8(1965), 338–353.

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