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Fuzzy α-ψ^* operator on fuzzy hyperconnected, fuzzy door and fuzzy Urysohn spaces

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Abstract
In this paper, based on the concept of ψ^* operator on a family of fuzzy α-open sets in a fuzzy topological space, the notion of fuzzy α-ψ^* semi-open sets is introduced and fuzzy α-ψ^* hyperconnected spaces is established via fuzzy α-ψ^* semi-open sets. Likely, the ideas of fuzzy α-ψ^*-submaximal spaces, fuzzy α-ψ^*-door spaces and fuzzy α-ψ^*-Urysohn spaces are established and a few of their captivating properties are examined.

Keywords
Fuzzy α-ψ^*-semi-open sets, fuzzy α-ψ^*-dense sets, fuzzy α-ψ^*-hyperconnected spaces, fuzzy α-ψ^*-nowhere dense sets, fuzzy α-ψ^*-submaximal spaces, fuzzy α-ψ^*-door spaces and fuzzy α-ψ^*-Urysohn spaces.

AMS Subject Classification
54A40, 03E72.

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Contents

1. Introduction

Zadeh [9] initiated fuzzy set in 1965. Chang [2] characterized fuzzy topological space. α-open sets were instigated and explored by Njastad [5] in ordinary perspective. Bin Shahna [1], proposed fuzzy α-open sets and fuzzy α-closed sets. Thompson inquested the notion of irreducible or hyperconnected topological spaces. In this paper, based on the concept of ψ^* operator on a family of fuzzy α-open sets in a fuzzy topological space [7], the notion of fuzzy α-ψ^* semi-open sets is propounded and fuzzy α-ψ^* hyperconnected spaces is established via fuzzy α-ψ^* semi-open sets. Likely, the ideas of fuzzy α-ψ^*-submaximal spaces, fuzzy α-ψ^*-door spaces and fuzzy α-ψ^*-Urysohn spaces are established and a few of their captivating properties are examined.

2. Preliminaries

In this section some basic definitions and preliminary results are presented.

Definition 2.1. [4] A non-empty subset F of a topological space is irreducible if whenever $F \subseteq A \cup B$ for closed sets A and B then $F \subseteq A$ or $F \subseteq B$.

Definition 2.2. [7] Let (X,τ) be a fuzzy topological space. A function

$$\psi^* : F\alpha O(X,\tau) \to I^X$$

is called a fuzzy operator on $F\alpha O(X,\tau)$, if for each $\mu \in F\alpha O(X,\tau)$ with $\mu \neq 0_X$, $Fint(\mu) \leq \psi^*(\mu)$ and $\psi^*(0_X) = 0_X$.

Remark 2.1. [7] It is easy to check that some examples of fuzzy operators on $F\alpha O(X,\tau)$ are the well known fuzzy operators viz. $Fint, Fint(Fcl), Fcl(Fint), Fint(Fcl(Fint))$ and $Fcl(Fint(Fcl))$.

Definition 2.3. [7] Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Then any fuzzy α-open set $\mu \in I^X$ is called fuzzy α-ψ^*-open if $\mu \leq \psi^*(\mu)$. The complement of a fuzzy α-ψ^*-open set is said to be a fuzzy α-ψ^*-closed set.

Notation 2.1. [7] Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. The family of all fuzzy α-ψ^*-open (resp. fuzzy α-ψ^*-closed) sets in (X,τ) is denoted by $F\alpha$-ψ^*-$O(X,\tau)$ (resp. $F\alpha$-ψ^*-$C(X,\tau)$).

Definition 2.4. [7] Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. For any $\mu \in I^X$, the fuzzy α-ψ^*-interior of μ (briefly, $F\alpha$-ψ^*-$int(\mu)$) is defined by

$$F\alpha\text{-}\psi^*\text{-}int(\mu) = \vee\{\sigma : \sigma \leq \mu \text{ and each } \sigma \in I^X \text{ is a fuzzy } \alpha\text{-}\psi^*\text{-open set }\}.$$

Definition 2.5. [7] Let (X,τ) be a fuzzy topological space. Let ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. For any $\mu \in I^X$, the fuzzy α-ψ^*-closure of μ (briefly, $F\alpha$-ψ^*-$cl(\mu)$) is defined by

$$F\alpha\text{-}\psi^*\text{-}cl(\mu) = \wedge\{\sigma : \sigma \geq \mu \text{ and each } \sigma \in I^X \text{ is a fuzzy } \alpha\text{-}\psi^*\text{-closed set }\}.$$

Definition 2.6. [7] Let (X,τ) be a fuzzy topological space. Let ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Then (X,τ) is said to be a fuzzy α-ψ^*-irreducible space, if for any $\mu_1,\mu_2 \in F\alpha$-ψ^*-$O(X,\tau)$ where $\mu_1 \neq 0_X, \mu_2 \neq 0_X$, $\mu_1\ q\ \mu_2$.

Definition 2.7. [8] A fuzzy point p in X is a fuzzy set with membership function

$$\mu_p(x) = \begin{cases} y, & \text{for x = x}_0 \\ 0, & \text{otherwise} \end{cases}$$

where $0 < y < 1$. p is said to have support x_0 and value y. The collection of all fuzzy points in (X,τ) is denoted by $\mathscr{F}\mathscr{P}(X)$

Definition 2.8. [6] A fuzzy set μ_A is quasi-coincident with the fuzzy set μ_B iff $\exists x \in X$ such that $\mu_A(x) + \mu_B(x) > 1$.

Definition 2.9. [3] A prelilter $\mathfrak{F}$ on X is a nonempty collection of subsets of I^X with the properties: (i) If $F_1, F_2 \in \mathfrak{F}$ then $F_1 \wedge F_2 \in \mathfrak{F}$ (ii) If $F \in \mathfrak{F}$ and $F' \geq \mathfrak{F}$, then $F' \in \mathfrak{F}$ (iii) $0 \notin \mathfrak{F}$.

3. Fuzzy α-ψ^*-hyperconnected spaces, Fuzzy α-ψ^*-door spaces and fuzzy α-ψ^*-Urysohn spaces

In this section the characterisation of fuzzy α-ψ^*-Hyper connected Spaces using the concept of fuzzy α-ψ^*-semi-open sets is studied. In addition, α-ψ^*-submaximal spaces, fuzzy α-ψ^*-door spaces and fuzzy α-ψ^*-Urysohn spaces are discussed.

Definition 3.1. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Any $\lambda \in I^X$ is said to be a fuzzy α-ψ^*-semi-open, if there exists a $\gamma \in F\alpha$-ψ^*-O (X,τ) such that $\gamma \leq \lambda \leq F\alpha\psi^* cl(\gamma)$. Then the complement of fuzzy α-ψ^*-semi-open is said to be fuzzy α-ψ^*-semi-closed.

Notation 3.1. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Then the collection of all fuzzy α-ψ^*-semi-open sets in (X,τ) is denoted by $F\alpha$-ψ^*-$SO(X,\tau)$ and the collection of all fuzzy α-ψ^*-semi-closed sets in (X,τ) is denoted by $F\alpha$-ψ^*-$SC(X,\tau)$.

Definition 3.2. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Any $\lambda \in I^X$ is said to be a fuzzy α-ψ^*-dense set in (X,τ) if there exists no $\mu \in F\alpha$-ψ^*-$C(X,\tau)$ in (X,τ) such that $\lambda < \mu < 1_X$. That is, $F\alpha$-ψ^*-$cl(\lambda) = 1_X$

Definition 3.3. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Then (X,τ) is said to be a fuzzy α-ψ^*-hyperconnected space if every $\lambda \in F\alpha$-ψ^*-O (X,τ) is fuzzy α-ψ^*-dense in (X,τ).

Remark 3.1. Let (X,τ) be a fuzzy α-ψ^*-hyperconnected space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Any $\lambda \in F\alpha$-ψ^*-$SO(X,\tau)$ if and only if there exists a $\mu \in F\alpha$-ψ^*-O (X,τ) such that $\mu \leq \lambda$.

Proof. Let $\lambda \in F\alpha$-ψ^*-$SO(X,\tau)$. By the Definition 3.1, there exists $\gamma \in F\alpha$-ψ^*-$O(X,\tau)$ such that $\gamma \leq \lambda$.

Conversely assume that, $\gamma \leq \lambda$, $\gamma \in F\alpha$-ψ^*-$O(X,\tau)$. Since (X,τ) is a fuzzy α-ψ^*-hyperconnected space, $F\alpha$-ψ^*-$cl(\gamma) = 1_X$. Thus $\gamma \leq \lambda \leq F\alpha$-$\psi^*$-$cl(\gamma)$. Hence $\lambda \in F\alpha$-ψ^*-$SO(X,\tau)$. □

Definition 3.4. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Any fuzzy set $\lambda \in I^X$ in (X,τ) is said to be a fuzzy α-ψ^*-nowhere dense if there exists no $\mu \in \alpha$-ψ^*-$O(X,\tau)$ with $\mu \neq 0_X$ such that $\mu < F\alpha$-ψ^*-$cl(\lambda)$. That is., $F\alpha$-ψ^*-$int(F\alpha$-ψ^*-$cl(\lambda)) = 0_X$ or $F\alpha$-ψ^*-$cl(F\alpha$-ψ^*-$int(1_X - \lambda)) = 1_X$.

Proposition 3.1. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Then the following statements are equivalent:

(a) (X,τ) is fuzzy α-ψ^*-hyperconnected;

(b) λ is fuzzy α-ψ^*-dense or fuzzy α-ψ^*-nowhere dense for every $\lambda \in I^X$;

(c) (X,τ) is fuzzy α-ψ^*-irreducible

Proof. **(a) ⇒ (b)**
Let (X,τ) be fuzzy α-ψ^*-hyperconnected and $\lambda \in I^X$. Suppose λ is not a fuzzy α-ψ^*-nowhere dense. Then
$F\alpha$-ψ^*-$cl(F\alpha$-ψ^*-$int(1_X - \lambda)) \neq 1_X$;
Thus $F\alpha$-ψ^*-$cl(1_X - F\alpha$-ψ^*-$cl(\lambda)) \neq 1_X$;
this implies that $1_X - F\alpha$-ψ^*-$int(F\alpha$-ψ^*-$cl(\lambda)) \neq 1_X$;
and so $F\alpha$-ψ^*-$int(F\alpha$-ψ^*-$cl(\lambda)) \neq 0_X$.
Since (X,τ) is fuzzy α-ψ^*-hyperconnected,
$F\alpha$-ψ^*-$cl(F\alpha$-$\psi^* int(F\alpha$-ψ^*-$cl(\lambda))) = 1_X$.
Since $F\alpha$-ψ^*-$cl(F\alpha$-$\psi^* int(F\alpha$-ψ^*-$cl(\lambda)))$
$\leq F\alpha$-ψ^*-$cl(\lambda)$,
it follows that $1_X \leq F\alpha$-ψ^*-$cl(\lambda)$.
Also, $F\alpha$-ψ^*-$cl(\lambda) \leq 1_X$. Thus $F\alpha$-ψ^*-$cl(\lambda) = 1_X$.
Hence λ is fuzzy α--ψ^*-dense.
(b) ⇒ (c)
Suppose that, for some $0_X \neq \lambda_1, \lambda_2 \in F\alpha$-$\psi^*$-$O(X,\tau)$, $\lambda_1\ \not q$

λ_2. Then $F\alpha$-ψ^*-$cl(\lambda_1)\not q\,\lambda_2$. Then λ_1 is not fuzzy α-ψ^*-dense. Since $\lambda_1 \in F\alpha$-ψ^*-$O(X,\tau)$,

$$\lambda_1 = F\alpha\text{-}\psi^*\text{-}int(\lambda_1)$$
$$\text{and hence } \lambda_1 \le F\alpha\text{-}\psi^*\text{-}int(F\alpha\text{-}\psi^*\text{-}cl(\lambda_1)).$$

Since $\lambda_1 \neq 0_X$, $F\alpha$-$\psi^* int(F\alpha$-ψ^*-$cl(\lambda_1)) \neq 0_X$. Thus λ_1 is not fuzzy α-ψ^*-nowhere dense. This contradicts (b). Hence $\lambda_1\, q\, \lambda_2$, for every $\lambda_1, \lambda_2 \in F\alpha$-$\psi^*$-$O(X,\tau)$ where $\lambda_1 \neq 0_X, \lambda_2 \neq 0_X$.

(c) ⇒ (a)
Let (X,τ) be a fuzzy α-ψ^*-irreducible space. Then for every $\lambda_1,\ \lambda_2 \in F\alpha$-$\psi^*$-$O(X,\tau)$, $\lambda_1+\lambda_2 \ge 1_X$. Suppose that (X,τ) is not a fuzzy α-ψ^*-hyperconnected space. Then there is $\mu \in F\alpha$-ψ^*-$O(X,\tau)$ such that μ is not fuzzy α-ψ^*-dense in (X,τ). Therefore $F\alpha$-ψ^*-$cl(\mu) \neq 1_X$. Thus $(1_X - F\alpha$-$\psi^* cl(\mu))$ and μ are fuzzy α-ψ^*-open sets in (X,τ) such that $([1_X - F\alpha\psi^*$-$cl(\mu)] + \mu) \le 1_X$. This contradicts (c). Hence (X,τ) is a fuzzy α-ψ^*-hyperconnected space. □

Proposition 3.2. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. If (X,τ) is fuzzy α-ψ^*-hyperconnected then $F\alpha$-ψ^*-$SO(X,\tau) - \{0_X\}$ $\big(0_X$ is excluded from $F\alpha$-ψ^*-$SO(X,\tau)\big)$ is a fuzzy fillter on (X,τ).

Proof. Let (X,τ) be a fuzzy α-ψ^*-hyperconnected space. Then (X,τ) is a fuzzy α-ψ^*-irreducible space. If $\lambda_1,\lambda_2 \in F\alpha$-$\psi^*$-$SO(X,\tau)-\{0_X\}$, then there exists $\mu_1,\mu_2 \in F\alpha$-ψ^*-SO $(X,\tau)-\{0_X\}$ such that $\mu_1 \le \lambda_1$ and $\mu_2 \le \lambda_2$. Since (X,τ) is α-ψ^*-irreducible, $(\mu_1 \wedge \mu_2) \neq 0_X, (\lambda_1 \wedge \lambda_2) \neq 0_X$. Hence $\lambda_1 \wedge \lambda_2 \in F\alpha$-$\psi^*$-$SO(X,\tau)-\{0_X\}$. Let $\lambda_1 \in F\alpha$-ψ^*-$SO(X,\tau)-\{0_X\}$ and $\lambda_1 \le \lambda_2$. Then there exists a $\mu_1 \in F\alpha$-ψ^*-$O(X,\tau)-\{0_X\}$ such that $\mu_1 \le \lambda_1$. Thus $\mu_1 \le \lambda_1 \le \lambda_2$. Therefore $\lambda_2 \in F\alpha$-ψ^*-$SO(X,\tau)-\{0_X\}$. Hence $F\alpha$-ψ^*-$SO(X,\tau)-\{0_X\}$ is a fuzzy filter on (X,τ). □

Definition 3.5. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Then (X,τ) is said to be a fuzzy α-ψ^*-submaximal space if and only if every fuzzy α-ψ^*-dense set $\lambda \in I^X$ is fuzzy α-ψ^*-open in (X,τ).

Definition 3.6. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Then (X,τ) is said to be a fuzzy α-ψ^*-door space if every fuzzy set $\lambda \in I^X$ is either fuzzy α-ψ^*-open or fuzzy α-ψ^*-closed in (X,τ).

Proposition 3.3. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Every fuzzy α-ψ^*-door space (X,τ) is a fuzzy α-ψ^*-submaximal space.

Proof. Let $\lambda \in I^X$ be a fuzzy α-ψ^*-dense set. If $\lambda \notin F\alpha$-ψ^*-$O(X,\tau)$, then $\lambda \in F\alpha$-ψ^*-$C(X,\tau)$, since (X,τ) is a fuzzy α-ψ^*-door space. Then $\lambda = F\alpha$-ψ^*-$cl(\lambda) = 1_X$. That is, $\lambda = 1_X$ which shows that λ is not a proper fuzzy set. This is a contradiction to the fact that λ is a fuzzy α-ψ^*-dense set. Therefore $\lambda \in F\alpha$-ψ^*-$O(X,\tau)$. Hence (X,τ) is a fuzzy α-ψ^*-submaximal space. □

Proposition 3.4. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Every fuzzy α-ψ^*-irreducible submaximal space (X,τ) is a fuzzy α-ψ^*-door space.

Proof. Let $\lambda \in I^X$ and λ be a fuzzy α-ψ^*-dense in (X,τ). Since (X,τ) is fuzzy α-ψ^*-submaximal, $\lambda \in F\alpha$-ψ^*-$O(X,\tau)$. Suppose that λ is not fuzzy α-ψ^*-dense in (X,τ). Then there exists $\mu \in F\alpha$-ψ^*-$O(X,\tau)$ such that $\mu < (1_X - \lambda)$. Since (X,τ) is fuzzy α-ψ^*-irreducible, μ is fuzzy α-ψ^*-dense in (X,τ). Then $(1_X-\lambda)$ is fuzzy α-ψ^*-dense in (X,τ). Again by submaximality of (X,τ), $(1_X-\lambda) \in F\alpha$-$\psi^*$-$O(X,\tau)$.
Therefore $\lambda \in F\alpha$-ψ^*-$C(X,\tau)$. Thus in any case $\lambda \in F\alpha$-ψ^*-$O(X,\tau)$ or $\lambda \in F\alpha$-ψ^*-$C(X,\tau)$. Hence (X,τ) is a fuzzy α-ψ^*-door space. □

Definition 3.7. Let (X,τ) be a fuzzy topological space and ψ^* be a fuzzy operator on $F\alpha O(X,\tau)$. Then (X,τ) is said to be a fuzzy α-ψ^*-Urysohn space if for every pair of fuzzy points $x_{t_1}, y_{t_2} \in \mathscr{F}\mathscr{P}(X)$ and $x_{t_1} \not q\, y_{t_2}$ there exist $\lambda,\mu \in F\alpha$-ψ^*-$SO(X,\tau)$ with $\lambda \neq 1_X, \mu \neq 1_X \in I^X$ such that $x_t \le \lambda$, $y_t \le \mu$ and $F\alpha$-ψ^*-$cl(\lambda)\not q\, F\alpha$-$\psi^*$-$cl(\mu)$.

Definition 3.8. Let (X,τ) and (Y,σ) be any two fuzzy topological spaces. Let ψ_1^* and ψ_2^* be fuzzy operators on $F\alpha O(X,\tau)$ and $F\alpha O(Y,\sigma)$ respectively. Any function $f:(X,\tau)\to(Y,\sigma)$ is said to be a fuzzy α-ψ_{12}^*-irresolute function if $f^{-1}(\lambda) \in F\alpha$-$\psi_1^*$-$O(X,\tau)$ for every $\lambda \in F\alpha$-ψ_2^*-$O(Y,\sigma)$.

Proposition 3.5. Let (X,τ) and (Y,σ) be any two fuzzy topological spaces. Let ψ_1^* and ψ_2^* be fuzzy operators on $F\alpha O(X,\tau)$ and $F\alpha O(Y,\sigma)$ respectively. Then for any function $f:(X,\tau)\to(Y,\sigma)$, the following statements are equivalent:

(a) f is a fuzzy α-ψ_{12}^*-irresolute function;

(b) $f(F\alpha$-ψ_1^*-$cl(\lambda)) \le F\alpha$-ψ_2^*-$cl(f(\lambda))$ for every $\lambda \in I^X$;

(c) $F\alpha$-ψ_1^*-$cl(f^{-1}(\mu)) \le f^{-1}(F\alpha$-$\psi_2^*$-$cl(\mu))$ for every $\mu \in I^Y$.

Proof. **(a) ⇒ (b)**
Let f be a fuzzy α-ψ_{12}^*-irresolute function. Let $\lambda \in I^X$. Then $F\alpha$-ψ_2^*-$cl(f(\lambda)) \in F\alpha$-$\psi_2^*$-$C(Y,\sigma)$. By (a), $f^{-1}(F\alpha$-ψ_2^*-$cl(f(\lambda))) \in F\alpha$-$\psi_1^*$-$C(X,\tau)$. Now,
$\lambda \le f^{-1}(f(\lambda))$
Thus $F\alpha$-ψ_1^*-$cl(\lambda) \le F\alpha$-ψ_1^*-$cl(f^{-1}(f(\lambda)))$
$\le F\alpha$-ψ_1^*-$cl(f^{-1}(F\alpha$-$\psi_2^* cl(f(\lambda))))$
$= f^{-1}(F\alpha$-ψ_2^*-$cl(f(\lambda)))$.
Hence, $f(F\alpha$-$\psi_1^* cl(\lambda)) \le F\alpha$-$\psi_2^* cl(f(\lambda))$.

(b) ⇒ (c)
Let $\mu \in I^Y$ then $f^{-1}(\mu) \in I^X$. By (b),

$$f(F\alpha\text{-}\psi_1^*\text{-}cl(f^{-1}(\mu))) \le F\alpha\text{-}\psi_2^*\text{-}cl(f(f^{-1}(\mu))) \le F\alpha\text{-}\psi_1^* cl(\mu)$$

Thus $f^{-1}(f(F\alpha$-ψ_1^*-$cl(f^{-1}(\mu)))) \le f^{-1}(F\alpha$-$\psi_2^*$-$cl(\mu))$
Then $F\alpha$-ψ_1^*-$cl(f^{-1}(\mu)) \le f^{-1}(F\alpha$-$\psi_2^*$-$cl(\mu))$. **(c) ⇒ (a)**

Let $\gamma \in F\alpha$-ψ_2^*-$C(Y,\sigma)$. Then, $F\alpha$-ψ_2^*-$cl(\gamma) = \gamma$. By (c), it follows that

$F\alpha$-ψ_1^*-$cl(f^{-1}(\gamma)) \leq f^{-1}(F\alpha$-$\psi_2^*$-$cl(\gamma))$
$= f^{-1}(\gamma)$

But, $f^{-1}(\gamma) \leq F\alpha$-$\psi_1^*$-$cl(f^{-1}(\gamma))$. Therefore, $f^{-1}(\gamma) = F\alpha$-ψ_1^*-$cl(f^{-1}(\gamma))$. Hence $f^{-1}(\gamma) \in F\alpha$-$\psi_1^*$-$C(X,\tau)$. Thus f is a fuzzy α-ψ_{12}^*-irresolute function. □

Proposition 3.6. Let (X,τ) be a fuzzy α-ψ_1^*-irreducible space, (Y,σ) be a fuzzy α-ψ_2^*-Urysohn space and ψ_1^* and ψ_2^* be fuzzy operators on $F\alpha O(X,\tau)$ and $F\alpha O(Y,\sigma)$ respectively. If $x_t \in \mathscr{F}\mathscr{P}(X)$, $y_t \in \mathscr{F}\mathscr{P}(Y)$ and $f:(X,\tau) \to (Y,\sigma)$ is a fuzzy α-ψ_{12}^*-irresolute function, then $f(x_t) = y_t$ for all $x_t \in \mathscr{F}\mathscr{P}(X)$.

Proof. Suppose that there exist $x_{t_1}, x_{t_2} \in \mathscr{F}\mathscr{P}(X)$ with $x_{t_1} \not{q}\, x_{t_2}$ such that $f(x_{t_1}) \neq f(x_{t_2})$. Since (Y,σ) is a fuzzy α-ψ_2^*-Urysohn space, there exist $\lambda, \mu \in F\alpha$-ψ_2^*-$O(Y,\sigma)$ with $\lambda \neq 1_Y$ and $\mu \neq 1_Y$ such that $f(x_{t_1}) \leq \lambda$, $f(x_{t_2}) \leq \mu$ and $F\alpha$-ψ_2^*-$cl(\lambda) \not{q}\, F\alpha$-$\psi_2^*$-$cl(\mu)$.
By the fuzzy α-ψ^*-irresoluteness of f, there exist $\gamma, \delta \in F\alpha$-$\psi_1^*$-$O(X,\tau)$, such that $x_{t_1} \leq \gamma$, $x_{t_2} \leq \delta$ and
$f(F\alpha$-ψ_1^*-$cl(\gamma)) \leq F\alpha$-ψ_2^*-$cl(\lambda)$ and
$f(F\alpha$-ψ_1^*-$cl(\delta)) \leq F\alpha$-ψ_2^*-$cl(\mu)$. Therefore, $F\alpha$-ψ_1^*-$cl(\gamma) \not{q}\, F\alpha$-$\psi_1^*$-$cl(\delta)$. Thus $\gamma \not{q}\, \delta$. This contradicts the assumption that (X,τ) is fuzzy α-ψ^*-irreducible. Therefore $f(x_{t_1}) = f(x_{t_2})$ for all $x_{t_1}, x_{t_2} \in \mathscr{F}\mathscr{P}(X)$ with $x_{t_1} \not{q}\, x_{t_2}$. □

4. Conclusion

In this paper, in view of the idea of ψ^* operator on a family of fuzzy α-open sets in a fuzzy topological spaces, the notion of fuzzy α-ψ^* semi-open sets is introduced and fuzzy α-ψ^* hyperconnected spaces is established by means of fuzzy α-ψ^* semi-open sets. Likely, the ideas of fuzzy α-ψ^*-submaximal spaces, fuzzy α-ψ^*-door spaces and fuzzy α-ψ^*-Urysohn spaces are established and a few of their captivating properties are discussed.

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