Malaya Journal of Matematik, Vol. 8, No. 4, 1894-1897, 2020
https://doi.org/10.26637/MJM0804/0093

Connected $\mathscr{F}$ domination

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Abstract
In this paper we introduce connected $\mathscr{F}$ dominating set and minimum connected $\mathscr{F}$ dominating set. Moreover, we determine some bounds of minimum connected $\mathscr{F}$ dominating set and some basic results.

Keywords
Connected $\mathscr{F}$ domination, Graph Operations.

AMS Subject Classification
57K10, 57M15.

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Article History: Received **14** July **2020**; Accepted **22** October **2020**

Contents

1. Introduction

The present world is fighting against COVID-19 virus. Almost half of the population in the world is in the grip of this killer virus. No panacea or effective vaccine is invented for eradicating the virus. Researchers all over the world is pursuing their studies to tackle the problems of this pandemic. Virus once enter in a body incubates within 14 days. To find out whether one person is affected or not require tideous process of tests. Different modes or models or tools have to be employed to tackle or trace the pandemic without wasting much time. Also it is impossible to test all the people. This situation can be made into a mathematical problem by introducing a new type of [1] dominating set, namely minimum connected $\mathscr{F}$ dominating set. Similar kind of studies are done by [3] Manju Raju, Kiran R. Bhutani, Babak Moazzez, S. Arumugam.

[1] Let $G=(V,E)$ be a graph and a subset D of $V(G)$ is said to be a dominating set if for every $u \in V\setminus D$ there exists a $v \in D$ such that u is adjacent to v. D is a minimal dominating set of G if $D\setminus\{u\}$ is not a dominating set of G for any $u \in D$.

[2] For any subset F of $V(G)$, the induced subgraph $\langle F\rangle$ is the maximal subgraph of G with vertex set F.

[3] Let $G=(V,E)$ be a graph and $\mathscr{F}$ be a family of subsets of V whose union is V. A dominating set D of G is called an $\mathscr{F}$- dominating set if $D\cap F\neq\emptyset$ for all $F\in\mathscr{F}$. The minimum cardinality of an $\mathscr{F}$-dominating set of G is called the $\mathscr{F}$-domination number of G and is denoted by $\gamma_{\mathscr{F}}(G)$.

In this paper we consider only simple graphs. We use the notations G for a graph, $V(G)$ or V for the set of vertices of G and $\mathscr{F}$ a collection of subsets of $V(G)$ whose union is $V(G)$ and each element F of $\mathscr{F}$ induces a connected subgraph of G. For undefined terms we refer [1–4].

2. Connected $\mathscr{F}$ domination in graphs

Definition 2.1. *Let $G=(V,E)$ be a graph and $\mathscr{F}$ be a collection of subsets of $V(G)$ whose union is $V(G)$ and each element F of $\mathscr{F}$ induces a connected subgraph of G. A dominating set D of G is called connected $\mathscr{F}$ dominating set if $D\cap F\neq\emptyset\ \forall\ F\in\mathscr{F}$. The minimum cardinality of a connected $\mathscr{F}$ dominating set of G is called the connected $\mathscr{F}$ domination number of G and is denoted by $\gamma_{c\mathscr{F}}(G)$. Any connected $\mathscr{F}$ dominating set D with $|D|=\gamma_{c\mathscr{F}}(G)$ is called a $\gamma_{c\mathscr{F}}$ set. A connected $\mathscr{F}$ dominating set S is said to be minimal connected $\mathscr{F}$ dominating set if $S\setminus\{v\}$ is not a connected $\mathscr{F}$ dominating set for every $v\in S$.*

From the definition of a connected $\mathscr{F}$ dominating set of a graph G, $\gamma(G)\leq\gamma_{c\mathscr{F}}(G)$.

Remark 2.2. *If $\mathscr{F}$ is a family of vertex disjoint subsets of V then $\gamma_{c\mathscr{F}}$ is greater than or equal to $|\mathscr{F}|$.*

As a consequence of this result we have the following theorem.

Theorem 2.3. *Let $G = (V, E)$ be a graph and $\mathscr{F}$ be a collection of vertex disjoint subsets of $V(G)$ such that each element of $\mathscr{F}$ induces a complete graph in G. Then $\gamma_{c\mathscr{F}}(G) = |\mathscr{F}|$.*

Proof. Suppose $\gamma_{c\mathscr{F}} < |\mathscr{F}|$. Let $S = \{u_1, u_2, \cdots, u_m\}$ be a $\gamma_{c\mathscr{F}}$ set. Then $m < |\mathscr{F}|$. Then at least one u_i is in more than one F in $\mathscr{F}$, a contradiction. Hence $\gamma_{c\mathscr{F}} \geq |\mathscr{F}|$.

On the other hand any subset S of $V(G)$ containing exactly one element from each $F \in \mathscr{F}$ forms a connected $\mathscr{F}$ dominating set. Hence the result. □

The following theorem gives a sufficient condition for a family of subsets of V to satisfy $\gamma_{c\mathscr{F}}(G) = \gamma(G)$.

Theorem 2.4. *Let G be a graph and a collection $\mathscr{F}$ of subsets of $V(G)$, which satisfies the condition $\bigcup_{F\in\mathscr{F}} F = V(G)$. If $\langle F \rangle$ is a connected and $|F| \geq n - \gamma(G) + 1$ for every $F \in \mathscr{F}$ then $\gamma_{c\mathscr{F}}(G) = \gamma(G)$.*

Proof. Let S be a γ-set of G. Given that $|F| \geq n - \gamma(G) + 1$ for every $F \in \mathscr{F}$. Therefore $|F^c| \leq \gamma(G) - 1$. Thus $F \cap S \neq \emptyset$, for every $F \in \mathscr{F}$. □

The converse of Theorem 2.4 need not be true. For example, consider the graph K_3 with vertex set $V(G) = \{v_1, v_2, v_3\}$ and $\mathscr{F} = \{\{v_1, v_2\}\{v_1, v_3\}\}$. Then $\gamma_{c\mathscr{F}}(G) = \gamma(G) = 1$, but $\mathscr{F}$ does not satisfy the hypothesis of Theorem 2.4.

Corollary 2.5. *Let G be a graph. If $|F| \geq n - \frac{n}{1+\Delta(G)} + 1$ for every F in a collection $\mathscr{F}$ of subsets of $V(G)$ then $\gamma_{c\mathscr{F}}(G) = \gamma(G)$.*

Theorem 2.6. *Let G be a graph and D be a minimal dominating set of G. Then there exists a collection $\mathscr{F}$ of subsets of V such that D is a minimal connected $\mathscr{F}$ dominating set.*

Proof. Let D be a minimal dominating set of G. Choose $\mathscr{F} = \{N[v]; v \in D\}$. □

Note that the minimal dominating set D with $\mathscr{F} = \{N[v]; v \in D\}$ need not be minimum connected $\mathscr{F}$ dominating set.

For example the set of all pendant vertices of the star graph $K_{1,n}$ $(n \geq 2)$ is a minimal dominating set but it is not a minimum connected $\mathscr{F}$ dominating set, where $\mathscr{F} = \{N[v] : v \in D\}$.

Theorem 2.7. *Let G be a graph and $\mathscr{F}$ be a collection of vertex disjoint subsets of $V(G)$. If each $F \in \mathscr{F}$ induces a star graph then $\gamma_{c\mathscr{F}}(G) = |F|$.*

Proof. If $\mathscr{F}$ satisfies the hypothesis of the theorem then by Remark 2, $\gamma_{c\mathscr{F}}(G) \geq |\mathscr{F}|$. On the otherhand the set of all central vertices of stars in $\mathscr{F}$ forms a connected $\mathscr{F}$ dominating set. □

Theorem 2.8. *Let G be a graph and $\mathscr{F}$ be a collection of subsets of $V(G)$. Then G has unique connected $\mathscr{F}$ dominating set if and only if $\{v\} \in \mathscr{F}$ for every $v \in V(G)$.*

Proof. Suppose that G has unique connected $\mathscr{F}$ dominating set. If possible, assume that there is a $v \in V(G)$ such that $\{v\} \notin \mathscr{F}$. Then v must not be an iolated vertex of G. Hence $V(G) \setminus \{v\}$ is a connected $\mathscr{F}$ dominating set, Also $D = V(G)$ itself forms a connected $\mathscr{F}$ dominating set, which is a contradiction. Converse is trivial. □

Theorem 2.9. *Let G be the complete graph K_n. Then for any positive integer $k \leq n$, there exists a partition $\mathscr{F}$ of $V(G)$ such that $\gamma_{c\mathscr{F}}(G) = |\mathscr{F}| = k$.*

Proof. Let $\{v_1, v_2 \cdots, v_n\}$ be the vertex set of K_n. For $k = 1$, $\mathscr{F} = \{\{v_1, v_2, \cdots, v_n\}\}$ and for $k > 1$, $\mathscr{F} = \{\{v_1\}, \{v_2\}, \cdots, \{v_{k-1}\}, \{v_k, v_{k+1}, \cdots, v_n\}\}$ will serve the purpose. □

Theorem 2.10. *Let G be a graph of order n and $\gamma(G) = p$. Then for any integer q with $p \leq q \leq n$ there exist a collection $\mathscr{F}$ of subsets of V such that $\bigcup_{F\in\mathscr{F}} F = V(G)$ and $\gamma_{c\mathscr{F}} = q$.*

Proof. If $q = p$, choose $\mathscr{F} = \{N[v]; v \in V(G)\}$. If $q > p$, let $A = \{u_1, \cdots, u_p\}$ be a γ set of G and B be any set of $q - p$ elements $\{v_1, v_2, \cdots, v_{q-p}\}$ of A^c. In this case $\mathscr{F} = \{\{v\}; v \in A \cup B\} \cup \{N[u_i], u_i \in A\}$ forms a connected $\mathscr{F}$ dominating set with $\gamma_{c\mathscr{F}} = q$. □

Theorem 2.11. *Let G be a graph with n vertices and suppose that $\gamma(G) = p$. Then for every integer q such that $p \leq q \leq n$ there will be a partition $\mathscr{F}$ of subsets of V such that $\gamma_{c\mathscr{F}}(G) = q$.*

Proof. Let $D_1 = \{v_1, v_2, \cdots, v_p\}$ be a γ set of G.

For $1 \leq i \leq p$, let $E_i = (N(v_i) \cap D^c) \cup \{v_i\}$. Now let $F_1 = E_1$ and for $1 < i \leq p$, let $F_i = E_i \setminus (E_1 \cup \cdots \cup E_{i-1})$ and set $\mathscr{F} = \{F_1, F_2, \cdots F_p\}$. Then D forms a connected $\mathscr{F}$ dominating set. Hence the result is true for $p = q$.

Now let $q > p$, let D_2 be $q - p$ elements $\{v_{p+1}, \cdots, v_q\}$ of D_1^c. Now set $E_i' = (N(v_i) \cap (D_1 \cup D_2)^c) \cup \{v_i\}$ for $1 \leq i \leq p$ and set

$$F_i' = \begin{cases} E_1', & \text{for } i = 1 \\ E_i' \setminus (E_1' \cup \cdots \cup E_{i-1}'), & \text{for } 1 < i \leq p \\ \{v_i\}, & \text{for } p+1 \leq i \leq q. \end{cases}$$

If $\mathscr{F} = \{F_1', F_2', \cdots, F_q'\}$ then $\mathscr{F}$ satisfies the requirements of the theorem. □

Theorem 2.12. *Let $G = (V, E)$ be a graph and S be a connected $\mathscr{F}$ dominating set. Then S is a minimal connected $\mathscr{F}$ dominating set if and only if either S is a minimal dominating set or for every minimal dominating set $S' \subsetneq S$ and for every $u \in S \setminus S'$ there will be an $F \in \mathscr{F}$ such that $(S \setminus \{u\}) \cap F = \emptyset$.*

Proof. Suppose that S is minimal connected $\mathscr{F}$ dominating set. If S is a minimal dominating set there is nothing to prove. So suppose that S is not a minimal dominating set. If possible, let S' be a minimal dominating set, $S' \subsetneq S$ and a $u \in S \setminus S'$ such that for every $F \in \mathscr{F}$ $(S \setminus \{u\}) \cap F \neq \emptyset$. Then $S \setminus u$ is also a connected $\mathscr{F}$ dominating set, a contradiction to our assumption.

Conversely assume that S is either minimal dominating or for every minimal dominating set $S' \subsetneq S$ and for each $u \in S \setminus S'$ there exists an $F \in \mathscr{F}$ such that $(S \setminus \{u\}) \cap F = \emptyset$. If possible assume that S is not minimal connected $\mathscr{F}$ dominating set then, there exists a $v \in S$ such that $S \setminus \{v\}$ is a connected $\mathscr{F}$ dominating set. If S is minimal dominating, $S \setminus \{v\}$ is not connected $\mathscr{F}$ dominating set. Therefore S cannot be a minimal dominating set. Hence there exists $S' \subsetneq S$ and S' is minimal dominating. If $v \notin S'$ then there exists an $F \in \mathscr{F}$ such that $(S \setminus \{v\}) \cap F = \emptyset$. Thus $S \setminus \{v\}$ is not a connected $\mathscr{F}$ dominating set, a contradiction. If $v \in S'$, since $S \setminus \{v\}$ is connected $\mathscr{F}$ dominating set there exists a minimal dominating set S'' which is contained in $S \setminus \{v\}$. Then $v \in S \setminus S''$ so $(S \setminus \{v\}) \cap F = \emptyset$ for some $F \in \mathscr{F}$, a contradiction. Hence the result. □

Proposition 2.13. *For a collection $\mathscr{F}$ of subsets of $V(G)$ of the graph G with n vertices $\gamma_{c\mathscr{F}}$ is equal to 1 if and only if $K_{1,n}$ is a subgraph of G such that its central vertex belongs to every $F \in \mathscr{F}$.*

3. Bounds of Connected $\mathscr{F}$ domination number

Theorem 3.1. *Let G be a graph and $\mathscr{F}$ be a collection of subsets of $V(G)$. Then $\gamma_{c\mathscr{F}}(G) \leq \sum_{F \in \mathscr{F}} \gamma(F)$, where $\gamma(F)$ is the domination number of the connected subgraph induced by F.*

Proof. As the union of all dominating sets of each $F \in \mathscr{F}$ is a dominating set of G, we have the result. □

Theorem 3.2. *Let G be a graph, $S \subseteq V(G)$ satisfying the condition $\bigcup_{v \in S} N[v] = V(G)$ and $\mathscr{F} = \{N[v]; v \in S\}$. Then* $\gamma_{c\mathscr{F}}(G) \leq |S|$.

Theorem 3.3. *If all elements of a collection $\mathscr{F}$ of subsets of $V(G)$ of a graph G have a vertex in common then $\gamma_{c\mathscr{F}}(G) \leq$* $\gamma(G) + 1$.

Theorem 3.4. *For any graph G on n vertices and for any family $\mathscr{F}$ of subsets of V, $\lceil \frac{n}{1+\Delta(G)} \rceil \leq \gamma_{c\mathscr{F}}(G) \leq n$.*

Proof. Since $\lceil \frac{n}{1+\Delta(G)} \rceil \leq \gamma(G)$ we get $\lceil \frac{n}{1+\Delta(G)} \rceil \leq \gamma_{c\mathscr{F}}(G)$. Hence the result. □

Theorem 3.5. *Let G be any graph of order n and let v be a vertex with maximum degree such that $F \nsubseteq N(v)$ for every $F \in \mathscr{F}$. Then $\lceil \frac{n}{1+\Delta(G)} \rceil \leq \gamma_{c\mathscr{F}}(G) \leq n - \Delta(G)$.*

Proof. Let $D = V(G) - N(v)$ then D is a connected $\mathscr{F}$ dominating set. □

4. Homomorphism and connected $\mathscr{F}$ domination

Definition 4.1 ([2]). *Let G and G' be any two graphs, a homomorphism of G to G' is a function $\phi : V(G) \to V(G')$ such that $\phi(x)\phi(y) \in E(G')$ when ever $xy \in E(G)$. If $\phi : V(G) \to V(G')$ is onto and u', v' are adjacent in G' then there are two adjacent vertices u and v in G such that $\phi(u) = u'$ and $\phi(v) = v'$, we say that ϕ is a homomorphism of G onto G'.*

Remark 4.2. *Let G and G' be two simple graphs. Then any graph homomorphism ϕ from G to G' maps ends of edges of G to distinct vertices of G'.*

Theorem 4.3. *Let G and G' be two graphs and $\phi : V(G) \to V(G')$ be a graph homomorphism from G onto G'. If D is a connected $\mathscr{F}$ dominating set of G then $\phi(D)$ is a connected $\phi(\mathscr{F})$ dominating set of G', where $\phi(\mathscr{F}) = \{\phi(F) : F \in \mathscr{F}\}$.*

Proof. Let $v \in (\phi(D))^c$. Since ϕ is surjective there exists a $u \in D^c$ such that $\phi(u) = v$. Since D is a dominating set there will be a $w \in D$ such that w is adjacent to u. Therefore $\phi(w) \in \phi(D)$ and $\phi(u)$ are adjacent. Hence $\phi(D)$ is a dominating set of G'.

By the definition of homomorphism, we get $\phi(F)$ induces a connected subgraph of G', for each $F \in \mathscr{F}$. As every $F \in \mathscr{F}$ intersects D, every $\phi(F)$ in $\phi(\mathscr{F})$ intersects $\phi(D)$. Hence the theorem. □

Remark 4.4. *A graph homomorphism ϕ from a graph G onto a graph H may not map a minimal connected $\mathscr{F}$ dominating set in G onto a minimal connected $\phi(\mathscr{F})$ dominating set in H.*

For example, let G be the cycle $C_7 = (v_1, v_2, \cdots, v_7, v_1)$ and H be the cycle $C_3 = (u_1, u_2, u_3)$. Let $\mathscr{F} = \{\{v_1, v_2, v_7\}, \{v_2, v_3, v_4\}, \{v_4, v_5, v_6\}\}$ be a family of subsets of $V(C_7)$. Now $D = \{v_1, v_3, v_5\}$ is a minimal connected $\mathscr{F}$ dominating set in G. Let $\phi : V(G) \to V(H)$ be defined by

$$\phi(v_i) = \begin{cases} u_1, & \text{for i=1,3,6} \\ u_2, & \text{for i=2,4,7} \\ u_3, & \text{for i=5} \end{cases}$$

Then ϕ is a graph homomorphism from C_7 onto C_3. We get $\phi(\mathscr{F}) = \{\{u_1, u_2\}, \{u_2, u_3, u_1\}\}$ and $\phi(D) = \{u_1, u_3\}$. Here $\phi(D)$ is a connected $\phi(\mathscr{F})$ dominating set. But $\phi(D) = \{u_1, u_3\}$ is not a minimal $\phi(\mathscr{F})$ dominating set of H.

5. Connected $\mathscr{F}$ domination in Graph Operations

Definition 5.1. *[2] Let G and G' be any two graphs. Their join $G + G'$ consists of $G \cup G'$ and all edges joining $V(G)$ and $V(G')$.*

Theorem 5.2. *Let G and G' be any two graphs. If $\mathscr{F}_1$ is a collection of subsets of $V(G)$ and $\mathscr{F}_2$ is a that of $V(G')$, then* $\gamma_{c(\mathscr{F}_1 \cup \mathscr{F}_2)}(G + G') \leq \gamma_{c\mathscr{F}_1}(G) + \gamma_{c\mathscr{F}_2}(G')$

Proof. Let D_1 be a $\gamma_{c\mathscr{F}_1}$ set of G and D_2 be a $\gamma_{c\mathscr{F}_2}$ set of G'. Clearly $D_1 \cup D_2$ dominates $G+G'$, Also $(D_1 \cup D_2) \cap F \neq \emptyset$ for every $F \in \mathscr{F}_1 \cup \mathscr{F}_2$. Therefore $\gamma_{c(\mathscr{F}_1 \cup \mathscr{F}_2)}(G+G') \leq \gamma_{c\mathscr{F}_1}(G_1) + \gamma_{c\mathscr{F}_2}(G_2)$. □

Theorem 5.3. *Let G_1 and G_2 be any two connected graphs. Let $\mathscr{F}_1$ be a collection of subsets of $V(G_1)$ and $\mathscr{F}_2$ be that of $V(G_2)$. If there exists a u in $V(G_1)$ and a $v \in V(G_2)$ such that $u \in F_1$ for every $F_1 \in \mathscr{F}_1$ and $v \in F_2$ for every $F_2 \in \mathscr{F}_2$. Then $\gamma_{c(\mathscr{F}_1 \cup \mathscr{F}_2)}(G_1+G_2) \leq 2$.*

Proof. From the given conditions, every member of $\mathscr{F}_1 \cup \mathscr{F}_2$ intersects $D = \{u, v\}$. By the definition of join of two graphs $D = \{u, v\}$ forms a dominating set of $G_1 + G_2$. □

Definition 5.4 ([4]). *Lexicographic product $G \circ G'$ of two graphs G and G' has vertex set $V(G) \times V(G')$ and two vertices (x_1, y_1) and (x_2, y_2) are adjacent if and only if either $x_1x_2 \in E(G)$ or $x_1 = x_2$ and $y_1y_2 \in E(G')$.*

Note that if G_1 and G_2 are any two connected graphs then $G_1 \circ G_2$ is also a connected graph. Also if F_1 induces a connected subgraph of G_1 and F_2 induces a connected subgraph of G_2 then $F_1 \times F_2$ induces a connected subgraph of $G_1 \circ G_2$.

Theorem 5.5. *Let G_1 and G_2 be any two subgraphs and $\mathscr{F}_1$ be a collection of subsets of $V(G_1)$ and $\mathscr{F}_2$ be that of $V(G_2)$ then $\gamma_{c(\mathscr{F}_1 \times \mathscr{F}_2)}(G_1 \circ G_2) \leq \gamma_{c\mathscr{F}_1}(G_1)\gamma_{c\mathscr{F}_2}(G_2)$.*

Proof. Let D_1 be a $\gamma_{\mathscr{F}_1}$ set of G_1 and D_2 be a $\gamma_{\mathscr{F}_2}$ set of G_2. Then we will show that $D_1 \times D_2$ form a connected $\mathscr{F}_1 \times \mathscr{F}_2$ domination set of $G_1 \circ G_2$. Let $(u_i, v_j) \in (D_1 \times D_2)^c$. Then either $u_i \notin D_1$ or $v_j \notin D_2$. If $u_i \notin D_1$ and $v_j \in D_2$ then there exists a $u_k \in D_1$ which is adjacent to u_i. Therefore (u_k, v_j) is adjacent to (u_i, v_j). If $u_i \in D_1$ and $v_j \notin D_2$ there exists a v_k in D_2 such that v_k is adjacent to v_j. Thus $(u_i, v_k) \in D_1 \times D_2$ and (u_i, v_k) is adjacent to (u_i, v_j). If $u_i \notin D_1$ and $v_j \notin D_2$ then there exists a $u_k \in D_1$ which is adjacent to u_i and there exists a $v_l \in D_2$ such that v_l is adjacent to v_j. Hence (u_i, v_j) is adjacent to $(u_k, v_l) \in D_1 \times D_2$. Also it can be easily seen that $(D_1 \times D_2) \cap (F_i \times F_j) \neq \emptyset$ for every $F_i \in \mathscr{F}_1$ and $F_j \in \mathscr{F}_2$. □

Theorem 5.6. *Let G and H be any two simple graphs and let D be a $\gamma_{c\mathscr{F}}$ set of $G \circ H$ for $\mathscr{F} = \{F_1, F_2, \ldots, F_n\}$. Then $D_1 = \{u \in V(G) : (u, v) \in D \quad \text{for some} \quad v \in V(H)\}$ is a connected $\mathscr{F}_1$ dominating set in G, where $\mathscr{F}_1 = \{F_1', \ldots, F_n'\}$ and $F_i' = \{u \in V(G); (u, v) \in F_i, F_i \in \mathscr{F}\}$, for $i =, 2, \ldots, n$.*

Proof. Let $u' \in D_1^c$ and $v \in V$ then $(u', v) \in D^c$ for every $v \in V(H)$. Consider a fixed $v \in V(H)$ then there will be a $(u, v') \in D$ such that (u, v) dominates (u', v). Thus u and u' are adjacent and $F_i' \cap D_1 \neq \emptyset$. Hence the result. □

Acknowledgment

The first author gratefully acknowledge the funding agency, Council of Scientific and Industrial Research(CSIR) of Government of India, for providing financial support to carry out this research work.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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