Malaya Journal of Matematik, Vol. 8, No. 4, 1922-1925, 2020
https://doi.org/10.26637/MJM0804/0098

β-Baire space in fuzzy soft topological spaces

K. Kamala[1]* and V. Seenivasan [2]

Abstract
In this paper a property which can be used to measure and category named fuzzy soft β-Baire spaces in fuzzy soft Topological spaces is investigated. For this purpose fuzzy soft β-dense, nowhere fuzzy soft β-dense and fuzzy soft β-first category sets are defined and some results about these concepts are also obtained.

Keywords
Fuzzy soft β-dense set, fuzzy soft nowhere β-dense sets, fuzzy soft β-Baire space, fuzzy soft β- first category.

AMS Subject Classification
54A40,03E72.

[1,2] *Department of Mathematics, University College of Engineering Panruti, Panruti- 607106, Tamil Nadu, India.*
***Corresponding author**: [1] krsaut@gmail.com; [2]seenujsc@yahoo.co.in
Article History: Received **17** June **2020**; Accepted **27** September **2020**

Contents

1. Introduction

Zadeh[8] introduced the concepts of fuzzy sets. Soft sets theory was introduced by Molodtsov [4].The notion of fuzzy soft set is investigated and discussed[5]. Chang[2] introduced and developed the concepts of fuzzy topology. In recent years, B. Tanay and M. B. Kandemir [6] much attention has been used to generalize the basic notions of fuzzy topology in soft setting. Further, the concept of fuzzy soft β-open set is introduced by A. M. Abd El-latif[1]. The notion of baires space in fuzzy topology introduced and discussed G.Thangaraj etal [7] In this paper we define new concepts called fuzzy soft β-dense sets, fuzzy soft nowhere β-dense sets. Also we define of fuzzy soft β-first category sets and fuzzy soft β-Baire space. We obtain a characterization and investigated some properties of fuzzy soft β-Baire space.

2. Preliminaries

Throughout the present paper, X , Y, P and Q denote the fuzzy soft topological spaces. Let f_A be a fuzzy soft set of X. The closure (resp. the interior) of f_A is denoted by $(f_A)^-$(resp. $(f_A)^\circ$). A fuzzy soft setf_A is defined to be fuzzy soft β-open [1] if $f_A \sqsubseteq (((f_A)^-)^\circ)^-$, The complement of a fuzzy soft β -open(fsβOS) set is called fuzzy soft β-closed(fsβCS). The intersection of all fuzzy soft β-closed sets containing f_A is called the fuzzy soft β-closure of A and is denoted by $fs(f_A)^-_\beta$. The fuzzy soft β-interior of f_A is defined by the union of all fuzzy soft β -open sets contained in f_A and is denoted by fs$(f_A)^\circ_\beta$.

Definition 2.1. *[3] A fuzzy soft set f_A in a fsts (X,E,τ) is called fuzzy soft dense if there exists no fuzzy soft closed sets g_B in (X,E,τ) such that $F_A \sqsubseteq G_B \sqsubseteq E$.*

Definition 2.2. *[3] A fuzzy soft set f_A in a fsts (X,E,τ) is called fuzzy soft nowhere dense if there exists no nonempty fuzzy soft open sets g_B in (X,E,τ) such that $G_B \sqsubseteq fscl(F_A)$. That is, $fsint(fscl(F_A) = \phi$.*

Definition 2.3. *[3] Let f_A be a fuzzy soft set of X. f_A is defined to be of fuzzy soft first category if it can be represented as a countable union of fuzzy soft nowhere dense sets.i.e) $(\bigsqcup_{i=1}^{\infty} f_{A_i}) = f_A$, where f_{A_i}'s are fs nowhere dense set X. Otherwise fuzzy soft open f_A in X is said to be fuzzy soft second category. If f_A is fuzzy soft first category in X then ${f_A}^c$ is called fuzzy soft residual set in X.*

Definition 2.4. *[3] A fsts (X,E,τ) is called fuzzy soft Baire space if $fsint(\bigsqcup_{i=1}^{\infty} f_{A_i}) = \phi$ wheref_{A_i}'s are fs nowhere dense sets in (X,E,τ).*

3. Fuzzy soft β-dense and Fuzzy soft β-nowhere dense

In this section, we define fuzzy soft β -dense and fuzzy soft β-nowhere dense and we discuss with some properties.

Definition 3.1. *A fuzzy soft set f_A in fuzzy soft topological space (X,E,τ) is called fuzzy soft β-dense(fsβ-dense) if there exists no fuzzy soft β-closed set f_B in (X,E,τ) such that $f_A \sqsubseteq f_B \sqsubseteq 1_E$. i.e)fs$(f_A)^-_\beta = 1_E$.*

Remark 3.2. *Every fuzzy soft dense is fsβ-dense set but the converse is not true in general.*

Example 3.3. *Let $X = \{a,b\}$, $E = \{e_1,e_2,e_3\}$ and $A = \{e_1,e_2\}$ $B = \{e_1,e_3\}$ and $C = \{e_2,e_3\}$ and let fuzzy soft sets $f_A = \{f(e_1) = \{a_{0.4},b_{0.8}\}, f(e_2) = \{a_{0.3},b_{0.5}\}, f(e_3) = \{a_0,b_0\}\}$ $f_B = \{f(e_1) = \{a_{0.4},b_{0.4}\}, f(e_2) = \{a_0,b_0\}, f(e_3) = \{a_{0.2},b_{0.6}\}\}$ Consider the fuzzy soft topology $\tau = \{0_E,1_E,f_A,f_B,f_A \sqcup f_B, f_A \sqcap f_B\}$ defined over (X,τ,E). Now let us consider $f_E = \{f(e_1) = \{a_{0.4},b_{0.8}\}, f(e_2) = \{a_{0.3},b_{0.5}\}, f(e_3) = \{a_{0.2},b_{0.6}\}\}$ be a fuzzy β soft open sets. Then $fs(f_E)^-_\beta = 1_E$. Therefore it is a fsβ dense set in (X,E,τ) but it is not a fuzzy soft dense set.*

Definition 3.4. *A fuzzy soft set f_A in fuzzy soft topological space (X,E,τ) is called fuzzy soft β-nowhere dense(fsβ-nowhere dense) if there exists no fuzzy soft β-open set f_B in (X,E,τ) such that $f_B \sqsubseteq$fs$(f_A)^-_\beta$. i.e)(fs(fs$(f_A)^-_\beta)^\circ_\beta) = 0_E$.*

Remark 3.5. *Every fs nowhere dense set is a fsβ-nowhere dense set. The converse is not true in general as shown in example.*

Example 3.6. *Let $X = \{a,b,c\}, E = \{e_1,e_2,e_3\}$ and $A = \{e_1,e_2\}$ $B = \{e_1,e_3\}$ and $C = \{e_2,e_3\}$ and let fuzzy soft sets $f_A = \{f(e_1) = \{a_{0.7},b_{0.8},c_{0.2}\}, f(e_2) = \{a_{0.6},b_{0.7}, c_{0.4}\}, f(e_3) = \{a_0,b_0,c_0\}\}$ $f_B = \{f(e_1) = \{a_{0.5},b_{0.4},c_{0.6}\}, f(e_2) = \{a_0,b_0,c_0\}, f(e_3) = \{a_{0.2},b_{0.3},c_{0.1}\}\}$, $f_C = \{f(e_1) = \{a_0,b_0,c_0\}, f(e_2) = \{a_{0.7},b_{0.8},c_{0.4}\}, f(e_3) = \{a_{0.9},b_{0.6}, c_{0.5}\}\}$*

Consider the fuzzy soft topology $\tau = \{0_E,1_E,f_A,f_B,f_A \sqcup f_B, f_A \sqcap f_B, f_A \sqcup f_C, f_B \sqcup f_C, f_A \sqcap f_C, f_B \sqcap f_C\}$ defined over (X,τ,E). Now let us consider $f_E = \{f(e_1) = \{a_{0.9},b_{0.6},c_{0.7}\}, f(e_2) = \{a_{0.7},b_{0.8},c_{0.4}\}, f(e_3) = \{a_{0.96},b_{0.7},c_{0.6}\}\}$ be a fuzzy β soft open sets. Then $fs(f_E^c)^-_\beta = (f_B \sqcup f_C)^c \Rightarrow fs((f_B \sqcup f_C)^c)^\circ = 0$. Therefore f_E^c is a fsβ nowhere dense set in (X,E,τ) but it is not a fuzzy soft nowhere dense set.

Lemma 3.7. *A fuzzy soft set f_A in (X,E,τ) is fsβ-nowhere dense set if and only if fs$(f_A)^-_\beta$ has no interior points.*

Proof : Let f_A be a fsβ-nowhere dense set. Then there exists a fuzzy soft β-open set and fuzzy soft dense set f_B such that $f_B \sqsubseteq 1_E - f_A$. Since f_B is fsβ-open set fs$(f_B)^\circ_\beta = f_B \sqsubseteq 1 -$ fs$(f_A)^\circ_\beta$ $=1 -$ fs$(f_A)^-_\beta$. Then fs$(f_B)^- \sqsubseteq$ fs$(1_E -$ fs$(f_A)^-_\beta)^-$ $=1_E -$ fs(fs$(f_A)^-_\beta)^\circ$ Since f_B is a fuzzy soft dense set fs$(f_B)^-$ $=1$, hence fs(fs$\beta cl(f_A)^-_\beta)^\circ = 0_E$.

Proposition 3.8. *If f_A is fuzzy soft β-closed set in an fsts (X,E,τ) and fs$(f_A)^\circ_\beta = 0_E$, then f_A is fsβ-nowhere dense set in (X,E,τ).*

Proof : Let f_A be fsβCS in (X,E,τ). Then fs$(f_A)^-_\beta = f_A$. Now, fs(fs$(f_A)^-_\beta)^\circ_\beta =$ fs$(f_A)^\circ_\beta = 0_E$. Thus f_A is fsβ-nowhere dense set.

Proposition 3.9. *If fsβ-nowhere dense set f_A in (X,E,τ) is an fs closed set then f_A is fs nowhere dense set in (X,E,τ).*

Proof : Let f_A be an fsβ-nowhere dense set in (X,E,τ). Then fs(fs$(f_A)^-_\beta)^\circ_\beta = 0_E$ But fs$(f_A)^\circ \sqsubseteq$ fs$(f_A)^\circ_\beta \sqsubseteq$ fs$(f_A)^-_\beta = 0_E$. Thus fs$(f_A)^\circ_\beta = 0_E$. Since f_A is fuzzy soft closed set, fs$(f_A)^- = f_A$. So fs$(f_A)^\circ =$ fs(fs$(f_A)^-)^\circ \sqsubseteq 0_E$. This gives fs(fs$(f_A)^-)^\circ \sqsubseteq 0_E$. Hence f_A is fuzzy soft nowhere dense set.

Proposition 3.10. *If f_A is fs nowhere dense set in (X,E,τ), then fs$(f_A)^\circ_\beta = 0_E$.*

Proof : Letf_A be an fs nowhere dense set in (X,E,τ). Then fs(fs$(f_A)^-)^\circ = 0_E$. Also fs$(f_A)^\circ_\beta \sqsubseteq f_A$. Since fs$(f_A)^\circ_\beta$ is fsβOS, fs$(f_A)^\circ_\beta \sqsubseteq$ fs(fs(fs(fs$(f_A)^\circ_\beta)^-)^\circ)^- \sqsubseteq$ fs(fs(fs$(f_A)^-)^\circ)^-$. Thus, fs$(f_A)^\circ_\beta = f_A \sqcap$ fs$(f_A)^\circ_\beta \sqsubseteq f_A \sqcap$ fs(fs(fs$(f_A)^-)^\circ)^- = f_A \sqcap$ fs$(0_E)^- = 0_E$. Hence fs$(f_A)^\circ_\beta = 0_E$.

Proposition 3.11. *If f_A is fsβ-dense and fsβOS in (X,E,τ) and if $f_B \sqsubseteq 1_E - f_A$ then f_B is fsβ-nowhere dense set f_A in (X,E,τ).*

Proof : Let f_A is fsβ-dense and fsβOS in (X,E,τ). Then fs$(f_A)^-_\beta = 1_E$ and fs$(f_A)^-_\beta = f_A$. Now $f_B \sqsubseteq 1_E - f_A arrowfs(f_B)^-_\beta \sqsubseteq$ fs$(1_E - f_A)^-_\beta$$= 1_E -$ fs$(f_A)^\circ_\beta$$= 1 - f_A$ which implies fs(fs$(f_B)^-_\beta)^\circ_\beta \sqsubseteq$ fs$(1_E - f_A)^\circ_\beta = 1_E -$ fs$(f_A)^-_\beta = 0_E$. Thus fs(fs$(f_B)^-_\beta)^\circ_\beta = 0_E$. Hence f_B is fsβ-nowhere dense set.

Proposition 3.12. *If fs nowhere dense set f_A in (X,E,τ) is an fuzzy soft βCS then f_A is fsβ-nowhere dense set in (X,E,τ).*

Proof : Let f_A be fuzzy soft nowhere dense set in (X,E,τ). Then fs(fs$(f_A)^-)^\circ = 0_E$. By Proposition3.11, fs$(f_A)^\circ_\beta = 0_E$. Since f_A is fuzzy soft βCSs, fs$(f_A)^-_\beta = f_A$. This implies fs(fs$(f_A)^-_\beta)^\circ_\beta =$ fs$(f_A)^\circ_\beta = 0_E$. Hence f_A is fsβ-nowhere dense set.

Proposition 3.13. *Let f_A, f_B be fuzzy soft set in (X,E,τ). The following statements hold:*
(i). If $f_A \sqsubseteq f_B$ and f_B is fsβ-nowhere dense then f_A is fsβ-nowhere dense.
(ii). If f_A is fsβ-nowhere dense then $f_A - f_B$ is fsβ-nowhere dense.
(iii). If f_A or f_B is fsβ-nowhere dense then $f_A \sqcap f_B$ is fsβ-nowhere dense.

Proof : (i). Now $f_A \sqsubseteq f_B$ which implies $fs(f_A)^-_\beta \sqsubseteq \text{fs}(f_B)^-_\beta$ a then $fs(fs(f_A)^-_\beta)^\circ_\beta \sqsubseteq \text{fs}(fs(f_B)^-_\beta)^\circ_\beta$. Since f_B is fs β-nowhere dense in (X,E,τ), then $fs(fs(f_A)^-_\beta)^\circ_\beta \sqsubseteq 0$. Hence f_A is fsβ-nowhere dense set in (X,E,τ). (ii) and (iii) proof is similar to that of (i)

Definition 3.14. *Let f_A be a fuzzy soft set of X. f_A is defined to be of fuzzy soft β-first category(fsβ-first category) if it can be represented as a countable union of fsβ-nowhere dense sets.i.e) $fs(\bigsqcup_{i=1}^{\infty} f_{A_i})^\circ_\beta = f_A$ where f_{A_i}'s are fsβ-nowhere dense set X. Otherwise fuzzy soft β-open f_A in X is said to be fuzzy soft β-second category. If f_A is fuzzy soft β-first category in X then ${f_A}^c$ is called fuzzy soft β-residual set in X.*

Remark 3.15. *Every fuzzy soft first category set is of fsβ-first category. But the converse is not true in general as seen in example*

Example 3.16. *Let $X = \{a,b,c\}$, $E = \{e_1,e_2,e_3\}$ and $A = \{e_1,e_2\}$ $B = \{e_1,e_3\}$, $C = \{e_2\}$ and $D = \{e_2,e_3\}$ and let fuzzy soft sets $f_A = \{f(e_1) = \{a_{0.8},b_{0.5},c_{0.1}\}, f(e_2) = \{a_{0.9},b_{0.6},c_{0.3}\}, f(e_3) = \{a_0,b_0,c_0\}\}$*
$f_B = \{f(e_1) = \{a_{0.9},b_{0.5},c_{0.6}\}, f(e_2) = \{a_0,b_0,c_0\}, f(e_3) = \{a_{0.7},b_{0.6},c_{0.8}\}\}$, $f_C = \{f(e_1) = \{a_0,b_0,c_0\}, f(e_2) = \{a_{0.7},b_{0.8},c_{0.6}\}, f(e_3) = \{a_0,b_0,c_0\}\}$.

Consider the fuzzy soft topology $\tau = \{0_E, 1_E, f_A, f_B, f_A \sqcup f_B, f_A \sqcap f_B, f_A \sqcup f_C, f_B \sqcup f_C, f_A \sqcap f_C, f_B \sqcup (f_A \sqcap f_C), f_c \sqcup (f_A \sqcap f_B), f_A \sqcup f_B \sqcup f_C\}$ defined over (X,τ,E). Now let us consider $f_E = \{f(e_1) = \{a_{0.7},b_{0.5},c_{0.1}\}, f(e_2) = \{a_{0.9},b_{0.6},c_{0.2}\}, f(e_3) = \{a_{0.2},b_{0.5},c_{0.1}\}\}$ be a fuzzy β soft open sets. Then $fs(f_E^c) = (f_A \sqcup f_B)^c \sqcup (f_B \sqcup f_C)^c \sqcup f_E^c$. $(f_A \sqcup f_B)^c$, $(f_B \sqcup f_C)^c$, f_E^c are fsβ-nowhere dense sets. Therefore f_E^c is a fsβ-first category set in (X,E,τ) and F_E is fsβ-residual set in (X,E,τ) but it is not a fuzzy soft category set.

4. Fuzzy soft β-Baire space

In this section, we define fuzzy soft β-Baires space and we discuss with some characterization of this space.

Definition 4.1. *A fuzzy soft topological space (X,E,τ) is called fuzzy soft β-Baire space(fsβ-Baire space) if $fs(\bigsqcup_{i=1}^{\infty} f_{A_i})^\circ_\beta = 0_E$ where f_{A_i}'s are fsβ-nowhere dense sets in (X,E,τ).*

Example 4.2. *Let $X = \{a,b,c\}$, $E = \{e_1,e_2,e_3\}$ and $A = \{e_1,e_2\}$ $B = \{e_1,e_3\}$, $C = \{e_2\}$ and $D = \{e_2,e_3\}$ and let fuzzy soft sets $f_A = \{f(e_1) = \{a_{0.8},b_{0.5},c_{0.1}\}$, $f(e_2) = \{a_{0.9},b_{0.6}, c_{0.3}\}$, $f(e_3) = \{a_0,b_0,c_0\}\}$ $f_B = \{f(e_1) = \{a_{0.9},b_{0.5},c_{0.6}\}, f(e_2) = \{a_0,b_0,c_0\}, f(e_3) = \{a_{0.7},b_{0.6},c_{0.8}\}\}$, $f_C = \{f(e_1) = \{a_0,b_0,c_0\}, f(e_2) = \{a_{0.7},b_{0.8},c_{0.6}\}, f(e_3) = \{a_0,b_0,c_0\}\}$. Consider the fuzzy soft topology $\tau = \{0_E, 1_E, f_A,\ f_B, f_A \sqcup f_B, f_A \sqcap f_B, f_A \sqcap f_C, f_B \sqcup f_C, f_A \sqcup f_C, f_B \sqcap f_C\}$ defined over (X,τ,E). Now let us consider $f_E = \{f(e_1)\}, f(e_2), f(e_3)$, where $f(e_1) = \{a_{0.7},b_{0.5},c_{0.1}\}$, $f(e_2) = \{a_{0.9},b_{0.6},c_{0.2}\}$, $f(e_3) = \{a_{0.2},b_{0.5},c_{0.1}\}$ be a fs β-open sets. Then $f_E^c = (f_A \sqcup f_B)^c \sqcup (f_B \sqcup f_C)^c \sqcup f_E^c$, $(f_A \sqcup f_B)^c$, $(f_B \sqcup f_C)^c$, f_E^c are fsβ-nowhere dense sets Now fs$(f_E^c)^\circ_\beta$ is a fsβ-Baires space (X,E,τ)*

Proposition 4.3. *If fs$(\sqcup_{i=1}^{\infty} f_{A_i})^\circ_\beta = 0_E$ where fs$(f_{A_i})^\circ_\beta = 0_E$ and f_{A_i}'s are fsβ-closed sets in fsts (X,E,τ) then (X,E,τ) is fsβ-Baire space.*

Proof : Let f_{A_i}'s be fsβ-closed sets in (X,E,τ). Since fs$(f_{A_i})^\circ_\beta = 0_E$, by Proposition 3.8, f_{A_i}' s are fsβ-nowhere dense sets in X. Thus fs$(\sqcup_{i=1}^{\infty} f_{A_i})^\circ_\beta = 0_E$ where f_{A_i}'s are fsβ-nowhere dense sets in (X,E,τ). Hence (X,E,τ) is fsβ-Baire space.

Theorem 4.4. *Let (X,E,τ) be fsts. Then the following properties are equivalent:*
(i) (X,E,τ) is a fsβ-Baire space.
(ii) fs$(f_A)^\circ_\beta = 0_E$ for every fsβ-first category set in (X,E,τ)
(iii) fs$(f_A)^-_\beta = 1_E$ for every fsβ-residual in (X,E,τ).

Proof : $(i) \Rightarrow (ii)$: Let (X,E,τ) be a fsβ-Baire space and let f_A, fsβ-first category set in (X,E,τ). Then f_A=fs$(\sqcup_{i=1}^{\infty} f_{A_i})$, where f_{A_i}'s are fsβ-nowhere dense set(X,E,τ). Then fs$(\sqcup_{i=1}^{\infty} f_{A_i})^\circ_\beta = 0_E$,since (X,E,τ) is a fsβ-Baire space. Therefore fs$(f_A)^\circ_\beta = 0_E$.
$(ii) \Rightarrow (iii)$: Let f_B be an fsβ-residual in (X,E,τ). Then $(f_B)^c$ is an fsβ-first category set in (X,E,τ). By hypothesis, fs$({f_B}^c)^\circ_\beta = 0_E$ which implies that $(\text{fs}(f_B)^-_\beta)^c = 0_E$. Hence fs$(f_B)^-_\beta = 1_E$.
$(iii) \Rightarrow (i)$: Let f_{A_i} be an fsβ-first category set in (X,E,τ). f_A=fs$(\sqcup_{i=1}^{\infty} f_{A_i})$ where f_{A_i}'s are fsβ-nowhere dense sets in (X,E,τ). Since f_A is an fsβ-first category set in X, ${f_A}^c$ is an fsβ-residual in X. By hypothesis, fs$({f_A}^c)^-_\beta = 1_E$. Then $(\text{fs}(f_A)^\circ_\beta)^c = 1_E$ which implies fs$(f_A)^\circ_\beta = 0_E$. Thus fs$(\sqcup_{i=1}^{\infty} f_{A_i})^\circ_\beta = 0_E$. Hence (X,E,τ) is an fsβ-Baire space.

Definition 4.5. *Let f_A be a fuzzy soft set of (X,E,τ). f_A is defined to be of fuzzy soft β-first category space if fs$(\bigsqcup_{i=1}^{\infty} f_{A_i})^\circ_\beta = 1_E$ where f_{A_i}'s are fsβ-nowhere dense set X. Otherwise fuzzy soft β-open set f_A in X is said to be fuzzy soft β-second category space.*

Remark 4.6. *The fsβ-first category space does not necessarily follow from the fsβ-Baire space which illustrates in the following example*

Example 4.7. *From example 3.16, the fsβ-nowhere dense sets $(f_A \sqcup f_C)^c \sqcup {f_D}^c \sqcup f_B^c = 1_E$. Now fs$(1_E)^\circ_\beta = 1_E$, which is a fs$\beta$-first category space but it is not fsβ-Baire space. Now the fsβ-nowhere dense sets $(f_B \sqcup (f_A \sqcap f_C))^c = (f_B \sqcup f_C)^c \sqcup (f_A \sqcup (f_B \sqcup f_C))^c \sqcup (f_B \sqcup (f_A \sqcap f_C))^c$ Now fs$((f_B \sqcup (f_A \sqcap f_C))^c)^\circ_\beta = 0_E$ which is a fs β-Baire space but it is not fsβ-first category space.*

Remark 4.8. *Every fsβ-Baire space is a fsβ-second category space but the converse not necessarily true.From example 3.16, the fsβ-nowhere dense sets $fs((f_A \sqcup f_B \sqcup f_C)^c \sqcup f_E{}^c \sqcup f_D^c)^\circ_\beta \neq 1_E$ which is a fsβ-second category space but it is not fsβ-Baire space.*

Proposition 4.9. *If $fs(\overset{n}{\underset{i=1}{\sqcap}} f_{A_i})^-_\beta = 1_E$ where f_{A_i}'s are fsβ-dense and fsβ-open sets in fsts (X,E,τ) iff (X,E,τ) is fsβ-Baire space.*

Proof : Let f_{A_i}'s are fsβ-dense sets in (X,E,τ). Then $\text{fs}(\overset{n}{\underset{i=1}{\sqcap}} f_{A_i})^-_\beta = 1_E$ Which implies $(\text{fs}(\overset{n}{\underset{i=1}{\sqcap}} f_{A_i})^-_\beta)^c = 0_E$. That is $\text{fs}((\overset{n}{\underset{i=1}{\sqcap}} f_{A_i})^c)^\circ_\beta = 0_E \Rightarrow \text{fs}(\overset{\infty}{\underset{i=1}{\sqcup}} f_{A_i}{}^c)^\circ_\beta = 0_E$ since f_{A_i}'s are fsβ-dense, $\text{fs}(f_{A_i})^-_\beta = 1_E$. Hence $\text{fs}(f_{A_i}{}^c)^\circ_\beta = (\text{fs}(f_{A_i})^-_\beta)^c = 0_E$ Consequently $\text{fs}(\overset{\infty}{\underset{i=1}{\sqcup}} f_{A_i}{}^c)^\circ_\beta = 0_E$, where $\text{fs}(f_{A_i}{}^c)^\circ_\beta = 0_E$ and f_{A_i}'s are fsβ-closed sets in (X,E,τ). By Proposition 4.3, (X,E,τ) is fsβ-Baire space.
Conversely, Let f_{A_i}'s are fsβ-dense sets and fsβ-open sets in (X,E,τ). By proportion 3.11, $f_{A_i}{}^c$'s are fsβ-nowhere dense sets in X. Then $f_A = \overset{\infty}{\underset{i=1}{\sqcup}} f_{A_i}{}^c$ is a fsβ-first category set in (X,E,τ). Now $\text{fs}(f_A)^\circ_\beta$=$\text{fs}(\overset{\infty}{\underset{i=1}{\sqcup}} f_{A_i}{}^c)^\circ_\beta = \text{fs}((\overset{\infty}{\underset{i=1}{\sqcap}} f_{A_i})^c)^\circ_\beta = (\text{fs}(\overset{\infty}{\underset{i=1}{\sqcup}} f_{A_i})^-_\beta)^c$. Since (X,E,τ) is fsβ-Baire space, by Theorem 4.4, we get $\text{fs}(f_A)^\circ_\beta = 0_E$. Then $(\text{fs}(\overset{\infty}{\underset{i=1}{\sqcap}} f_{A_i})^-_\beta)^c = 0_E$. This implies that $(\text{fs}(\overset{\infty}{\underset{i=1}{\sqcap}} f_{A_i})^-_\beta) = 1_E$.

Definition 4.10. *A surjective function $\varphi : (X,E,\tau) \to (X,E,\sigma)$ is defined to be*
(i). fsβ-slightly continuous if $fs(\varphi^{-1}(f_A))^\circ_\beta \neq 0_E$ whenever $fs(f_A)^\circ_\beta \neq 0_E$ for a fs set f_A of σ.
(ii). fsβ-slightly open if $fs(\varphi(f_B))^\circ_\beta \neq 0_E$ whenever $fs(f_B)^\circ_\beta \neq 0_E$ for a fs set f_B of τ.

Theorem 4.11. *Let $\varphi : (P,E_1,\tau) \to (Q,E_2,\sigma)$ be a surjective function. The following statements hold:*
(i). If φ is fsβ-slightly continuous and f_A is fsβ-dense in P, then $\varphi(f_A)$fsβ-dense in Q .
(ii)2. If φ is fsβ-slightly open and f_B is fsβ-dense in Q , then $\varphi^{-1}(f_B)$ is fsβ-dense in P.

Proof :(i). Let φ be a fsβ-fslightly continuous function and f_A be a fsβ-dense set in P. Suppose that $\varphi(f_A)$ is not fsβ-dense. Then $1_{E_1} \neq fs(\varphi(f_A))^-_\beta$ and $0_{E_2} \neq 1_{E_2} - fs(\varphi(f_A))^-_\beta$. Let $f_c = 1_{E_2} - fs(\varphi(f_A))^-_\beta$. Then f_c is a nonzero fsβ-open set. Since f is fsβ-slightly continuous $\text{fs}(f^{-1}(f_c))^\circ_\beta \neq 0_{E_2}$. Also $\text{fs}(\varphi^{-1}(f_c))^\circ_\beta \sqcap f_A \sqsubseteq \varphi^{-1}(f_c) \sqcap \varphi^{-1}(\varphi(f_A))$= $\varphi^{-1}(f_c \sqcap \varphi(f_A)) \sqsubseteq \varphi^{-1}(f_C \sqcap \text{fs}(\varphi(f_A))^-_\beta) = 0_{E_2}$. This is a contradiction sincef_A is fsβ-dense. Hence $\varphi(f_A)$ is fsβ-dense.
(ii). Let f be fsβ-slightly open and $f_B$be fsβ-dense in Q. Suppose that $\varphi^{-1}(f_B)$ is not fsβ-dense in P. Then there exists a nonempty fsβ-open set f_D of P such that $f_D \sqcap \varphi^{-1}(f_B) = 0_{E_1}$. Since φ is fsβ-slightly open $\text{fs}(\varphi(f_D))^\circ_\beta \neq 0_{E_2}$. Moreover, we have $\text{fs}(\varphi(f_D))^\circ_\beta \sqcap f_B \sqsubseteq \varphi(f_D) \sqcap f_B = 0_{E_2}$. This is a contradiction since f_B is fsβ-dense. Hence $\varphi^{-1}(f_B)$ is fsβ-dense.

Theorem 4.12. *Let $\varphi : P \to Q$ be a fsβ-slightly continuous and fsβ-slightly open surjection. If P is a fsβ-Baire space , then Q is a fsβ-Baire space.*

Proof : Let P be a fsβ-Baire space and $f_{B_i} \sqsubseteq Q$ be a fsβ-dense set for each $i \in I$, where I is the set of natural numbers. Since φ is fsβ-slightly open $\varphi^{-1}(f_{B_i})$ is fsβ-dense in P. Since P is a fs β-Baire space, $\underset{i\in I}{\sqcap} \varphi^{-1}(f_{B_i})$ is fsβ-dense in P. By theorem 4.11, φ is fsβ-slightly continuity, $\varphi(\underset{i\in I}{\sqcap} \varphi^{-1}(f_{B_i})) = \underset{i\in I}{\sqcap} f_{B_i}$ is fsβ-dense in Q . This shows that Q is a fsβ-Baire space.

5. Conclusion

Thus in this paper the concepts of fuzzy soft β -dense and fuzzy soft β-nowhere dense were introduced. Also the concepts of fuzzy soft β-Baire space were being introduced and discussed. Some characterizations of these spaces and some basic interesting properties of such fuzzy baires space were obtained

References

[1] A. M. Abd El-latif, Fuzzy soft separation axioms based on fuzzy β-open soft sets, *Annals of Fuzzy Mathematics and Informatics,* 11(2016), 223-239.

[2] C. L. Chang, Fuzzy topological spaces,*J. Math. Anal. Appl.,* 24(1968), 182-190.

[3] S. M. Khalil , M. Ulrazaq, S A.Ghani, and A. F. Al-Musawi, Algebra and Baire in Fuzzy Soft Setting *Advances in Fuzzy Systems,* 2018, 5731682, 1-10,

[4] D. Molodtsov, Soft set theory-first results,*Comput. Math. Appl.,* 37(1999),19-31.

[5] P. K. Maji, A. R. Roy, R. Biswas, Fuzzy soft sets, *J. Fuzzy Math.,* 9(2001), 589-602.

[6] B. Tanay and M. B. Kandemir, Topological structure of fuzzy soft sets, *Computers and Mathematics with Applications,* 61(2011), 2952–2957.

[7] G. Thangaraj and S.Anjalmose, On fuzzy Baires spaces, *Journal of Fuzzy Mathematics,* 21(2013), 667–676.

[8] L.A. Zadeh, Fuzzy sets, *Information and Control,* 8(1965), 338–353.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆