Malaya Journal of Matematik, Vol. 8, No. 4, 1926-1929, 2020

https://doi.org/10.26637/MJM0804/0099

Fuzzy inventory model to maintain green environment by disposing wastage using incineration method

S. Jerline Zita[1]* and S. Rexlin Jeyakumari[2]

Abstract

This paper focuses on suggesting a mathematical model to minimize the total expenditure of the organization by avoiding the amount spent on storing the wastage. The unavoidable wastage during manufacturing process are disposed by incineration method which keeps the surrounding clean and green. It also provides more space for the storage of the inventory. Pentagonal fuzzy numbers are applied for the fuzzified approach of the problem. A special type of Graded Mean Integration Representation method is used for defuzzifying all the fuzzified quantities. The optimum order quantity is arrived by reducing the equation using differentiation. The results can be compared using numerical illustrations.

Keywords

Graded Mean Integration Representation Method, Pentagonal Fuzzy Number, Waste Disposal.

AMS Subject Classification

90B05.

[1,2] *Department of Mathematics, Holy Cross College (Autonomous), Tiruchirappalli-620002, Tamil Nadu, India.*

[1,2] *Affiliated to Bharathidasan University, Tiruchirappalli-620024, Tamil Nadu, India.*

***Corresponding author**: [1]jerlinezita.stephen@gmail.com.

Article History: Received **24** August **2020**; Accepted **19** October **2020**

Contents

1 Introduction 1926
2 Definitions and Methodologies 1927
3 Mathematical Model in Crisp Sense 1927
4 Mathematical Model in Fuzzy Sense 1928
5 Numerical Example 1928
References 1929

1. Introduction

Reading the customer's mindset, being updated with the current affairs and being social and environment concern are the key things for the successful run of any business by any organization. Nowadays, people have the awareness to avoid carbon consuming products and have the social responsibility to keep their surroundings green and clean. As the population increases, protecting the globe from all kind of pollutions and from global warming has become one of the basic needs for the survival of the human being. The industrial wastages during manufacturing process, the used materials and the defective products are to be disposed as it consumes the storage space of the inventory. When the industry has to do the waste disposal with environment concern, the method of incineration seems to be more economic.

The inventory model suggested by Chen et al [2] considers the waste disposal cost in the calculation of total cost. The recycling of wastages is taken into consideration in the mathematical model developed by Dobos & Richter's [1]. Further, it was developed by Jaber & Rosen [3] which included the entropy cost together with the repairing cost. The concept of remanufacturing of the defective items was introduced by A. Roy and K. Maity [3]. The waste disposal issue is approached in a different way by A.M.A. El Saandany and M.Y. Jaber [2] focused on renewable energy acquired from bio wastages. The benefits of incineration method were explained clearly by W. Ritha and Nivetha Martin [7]. The mathematical inventory model derived by W. Ritha and I. AntonitteVinolin [7] came up with another EOQ in which wastages are disposed with environment concern. R. K. Gupta et. al. [8] suggested an eco-friendly green inventory model under fuzzy logics. S. Rani et. al. [6] derived a mathematical model for deteriorating items in a green supply chain with carbon concerned demand.

The mathematical model suggested in this paper calculates the total cost together with the waste disposal cost. The minimization of the total expenditure can be done by reducing the waste disposal cost. This derivation ends up with a mathematical formulation which gives the economic order quantity. The same model is considered under fuzzy nature. For that, the inventory quantities are converted into fuzzy quantities using pentagonal fuzzy numbers. Again, a special type of graded mean integration representation method is applied for defuzzification. A Numerical example is given in order to compare the applicability of the model in both the senses.

2. Definitions and Methodologies

Pentagonal Fuzzy Number:
A pentagonal fuzzy number $\tilde{A} = (a,b,c,d,e)$ is represented with membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} L_1(x) = \frac{x-a}{b-a}, a \le x \le b \\ L_2(x) = \frac{x-b}{c-b}, b \le x \le c \\ R_1(x) = \frac{d-x}{d-c}, c \le x \le d \\ R_2(x) = \frac{e-x}{e-d}, d \le x \le e \\ 0, \text{ Otherwise} \end{cases}$$

Graded Mean Integration Representation Method:
The Graded mean integration representation method is a function that maps the set of all pentagonal numbers to the real line R. The real number that corresponds to the pentagonal fuzzy number using the graded mean representation method is given by

$$R(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + 2a_4 + a_5}{8}.$$

Arithmetic Operations under Function Principle:
Suppose $\tilde{A} = (a_1,a_2,a_3,a_4,a_5)$ and $\tilde{B} = (b_1,b_2,b_3,b_4,b_5)$ are two pentagonal fuzzy numbers, then the arithmetic operations are defined as follows.

- $\tilde{A} \oplus \tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5)$
- $\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5)$
- $\tilde{A} \Theta \tilde{B} = (a_1-b_5, a_2-b_4, a_3-b_3, a_4-b_2, a_5-b_1)$
- $\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_5}, \frac{a_2}{b_4}, \frac{a_3}{b_3}, \frac{a_4}{b_2}, \frac{a_5}{b_1}\right)$
- $\alpha\tilde{A} = \begin{cases} \alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha a_5, \alpha \ge 0 \\ \alpha a_5, \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1, \alpha < 0 \end{cases}$

Notations:

A - Ordering cost

H - Holding cost

W - Waste disposing cost

S - Setup cost

D - Demand Rate

P - Production Rate

T_C - Total cost

m - Proportion of waste produced

Q - Economic Order Quantity

$\tilde{A}$ - Fuzzy Ordering cost

$\tilde{H}$ - Fuzzy Holding cost

$\tilde{W}$ - Fuzzy Waste disposing cost

$\tilde{S}$ - Fuzzy Setup cost

$\tilde{m}$ - Fuzzy Proportion of waste produced

$\tilde{Q}^*$ - Fuzzy economic order quantity

$\tilde{T}_C$ - Fuzzy total cost

Assumptions:

- The demand rate is a known constant.
- No shortage is allowed.
- No limit for time.

3. Mathematical Model in Crisp Sense

The total cost of this inventory model includes the waste disposal cost together with all the other inventory costs.

$$T_C = \frac{Q}{2}\left[HD + \left(1 - \frac{P}{D}\right)A\right] + \frac{1}{Q}[AD + S + Wm] \tag{3.1}$$

The above equation can be reduced by differentiating it with respect to Q and by equating it to zero.

$$\Rightarrow Q = \sqrt{\frac{2[AD+S+Wm]}{\left[H + \left(1-\frac{P}{D}\right)A\right]}}$$

So, the economic order quantity is given by

$$Q^* = \sqrt{\frac{2(AD+S+Wm)}{HD + \left(1-\frac{P}{D}\right)A}} \tag{3.2}$$

Hence, the total cost is given by equation (3.1) and the economic order quantity is given by equation (3.2).

4. Mathematical Model in Fuzzy Sense

To consider the same problem in fuzzy sense, all the inventory costs together with the waste disposal cost and the proportion of waste produced are taken as pentagonal fuzzy numbers. Let

$$\begin{aligned}\tilde{A} &= (a_1, a_2, a_3, a_4, a_5),\\ \tilde{S} &= (s_1, s_2, s_3, s_4, s_5),\\ \tilde{H} &= (h_1, h_2, h_3, h_4, h_5),\\ \tilde{W} &= (w_1, w_2, w_3, w_4, w_5),\\ \tilde{m} &= (m_1, m_2, m_3, m_4, m_5)\end{aligned}$$

be pentagonal fuzzy numbers. Now, the fuzzy total cost is given by

$$\tilde{T}_C = \frac{Q}{2} \otimes \left[\tilde{H}D \oplus \left(1\Theta\frac{P}{D}\right)\tilde{A}\right] \oplus \frac{1}{Q} \otimes [\tilde{A}D \oplus \tilde{S} \oplus \tilde{W}\tilde{m}]. \tag{4.1}$$

Substituting the pentagonal fuzzy numbers, we get

$$\Rightarrow \tilde{T}_C = \begin{bmatrix} \frac{Q}{2}\left[h_1 D + \left(1-\frac{P}{D}\right)a_1\right] + \frac{1}{Q}\left[a_1 D + s_1 + w_1 m_1\right], \\ \frac{Q}{2}\left[h_2 D + \left(1-\frac{P}{D}\right)a_2\right] + \frac{1}{Q}\left[a_2 D + s_2 + w_2 m_2\right], \\ \frac{Q}{2}\left[h_3 D + \left(1-\frac{P}{D}\right)a_3\right] + \frac{1}{Q}\left[a_3 D + s_3 + w_3 m_3\right], \\ \frac{Q}{2}\left[h_4 D + \left(1-\frac{P}{D}\right)a_4\right] + \frac{1}{Q}\left[a_4 D + s_4 + w_4 m_4\right], \\ \frac{Q}{2}\left[h_5 D + \left(1-\frac{P}{D}\right)a_5\right] + \frac{1}{Q}\left[a_5 D + s_5 + w_5 m_5\right] \end{bmatrix} \tag{4.2}$$

We use graded mean integration representation method for defuzzification. By defuzzifying the above equation using graded mean integration representation method, we get

$$\begin{aligned}\tilde{T}_C = &\left[\frac{Q}{16}\begin{bmatrix}(h_1+2h_2+2h_3+2h_4+h_5)D+\\ \left(1-\frac{P}{D}\right)(a_1+2a_2+2a_3+2a_4+a_5)\end{bmatrix}\right.\\ &\left.+\frac{1}{8Q}\begin{bmatrix}(a_1+2a_2+2a_3+2a_4+a_5)D+\\ (s_1+2s_2+2s_3+2s_4+s_5)+(w_1m_1+\\ 2w_2m_2+2w_3m_3+2w_4m_4+w_5m_5)\end{bmatrix}\right]\\ =&F(q) \text{ say}\end{aligned} \tag{4.3}$$

$F(q)$ is minimum when $\frac{\partial F(q)}{\partial Q} = 0$ and when $\frac{\partial^2 F(q)}{\partial Q^2} > 0$. Differentiating the above equation w.r.t Q, we get

$$\frac{\partial \tilde{T}_C}{\partial Q} = \begin{bmatrix} \frac{1}{16}\left[(h_1+2h_2+2h_3+2h_4+h_5)D \right. \\ \left. + \left(1-\frac{P}{D}\right)(a_1+2a_2+2a_3+2a_4+a_5)\right] \\ \frac{1}{8Q^2}\left[(a_1+2a_2+2a_3+2a_4+a_5)D \right. \\ \left. +(s_1+2s_2+2s_3+2s_4+s_5)+ \right. \\ \left. (w_1m_1+2w_2m_2+2w_3m_3+2w_4m_4+w_5m_5)\right] \end{bmatrix}$$

$$\frac{\partial \tilde{T}_C}{\partial Q} = 0$$

$$Q = \sqrt{\frac{2\begin{bmatrix}(a_1+2a_2+2a_3+2a_4+a_5)D+\\ (s_1+2s_2+2s_3+2s_4+s_5)+(w_1m_1+\\ 2w_2m_2+2w_3m_3+2w_4m_4+w_5m_5)\end{bmatrix}}{\begin{bmatrix}(h_1+2h_2+2h_3+2h_4+h_5)D+\\ \left(1-\frac{P}{D}\right)(a_1+2a_2+2a_3+2a_4+a_5)\end{bmatrix}}} \tag{4.4}$$

Hence, equation (4.4) gives the fuzzy optimal order quantity and equation (4.3) gives the fuzzy total cost.

5. Numerical Example

Crisp Sense

Let

$$\begin{aligned}D &= 1500 \text{ per year}\\ P &= 1000 \text{ per year}\\ A &= Rs.500 \text{ per unit per year}\\ H &= Rs.2 \text{ per unit}\\ S &= Rs.50000\\ W &= Rs.50 \text{ per unit}\\ m &= 500 \text{ units per year}\end{aligned}$$

Then, the optimum order quantity is given by

$$Q^* = 22.83.$$

And the total cost is given by

$$T_C = 72445.60.$$

Fuzzy Sense

Let

$$\begin{aligned}D &= 1500 \text{ per year}\\ P &= 1000 \text{ per year}\\ \tilde{A} &= (300, 400, 500, 600, 700)\\ \tilde{S} &= (30000, 40000, 50000, 60000, 70000)\\ \tilde{H} &= (1.0, 1.5, 2, 2.5, 3)\\ \tilde{W} &= (30, 40, 50, 60, 70)\\ \tilde{m} &= (300, 400, 500, 600, 700)\end{aligned}$$

Then, the fuzzy economic order quantity is given by

$$\Rightarrow \tilde{Q}^* = 22.85.$$

And, the fuzzy total cost is given by

$$\Rightarrow \tilde{T}_C^* = 69235.81/-$$

This paper suggests a mathematical inventory model that has taken the waste disposal cost into consideration for the calculation of total cost. It comes up with a conclusion that the method of incineration is the best economic method to dispose the wastages without spoiling the green atmosphere. The model is discussed in both crisp and fuzzy senses. For fuzzy sense of discussion, all the inventory costs including

the waste disposal cost are taken as pentagonal fuzzy numbers.The graded mean integration representation method for pentagonal numbers is used for defuzzification. The derived model can be compared and verified in both the senses using numerical examples. This paper can be developed further by future researchers by considering different issues in the inventory management.

References

[1] I. Dobos and K. Richter, A production/recycling model with quality consideration, *International Journal of Production Economics*, 104(2)(2006), 571–579.

[2] A. M. El Saadany and M. Y. Jaber, A production/re manufacture model with returns' subassemblies managed differently, *International Journal of Production Economics,* 133(1)(2011), 119–126.

[3] M. Y. Jaber and A. M. El Saadany, The production, remanufacture and waste disposal model with lost sales, *International Journal of Production Economics*, 120(1)(2009), 115–124.

[4] I. Konstantaras, K. Skouri, M. Y. Jaber, Lot sizing for a recoverable product with inspection and sorting, *Computers & Industrial Engineering*, 58(3)(2010), 452–462.

[5] Y. P. Li and G. H. Huang, An inexact two-stage mixed integer linear programming method for solid waste management in the City of Regina, *Journal of Environmental Management,* 81(3)(2006), 188–209.

[6] Rani, Smita, Ali, Rashid, Agarwal, Anchal, Fuzzy inventory model for deteriorating items in a green supply chain with carbon concerned demand, *Op. Search*, 56. (2019). 10.1007/s12597-019-00361-8.

[7] W. Ritha, I. Antonitte Vinoline, Environmentally responsible repair and waste Disposal Inventory Models, *Aryabhatta Journal of Mathematics and Informatics,* 6(1)(2014), 1–8.

[8] S. Saxena, R. K. Gupta, V. Singh, P. Singh and N. K. Mishra, Environmental Sustainability with eco-friendly green inventory model under Fuzzy logics considering carbon emission, *JETIR*, 5(11)(2018), 1–12.

★★★★★★★★★
ISSN(P):2319 – 3786
Malaya Journal of Matematik
ISSN(O):2321 – 5666
★★★★★★★★★