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An approach for multi criteria decision making with intuitionistic fuzzy set by integrating H-max distance measure, CODAS method and PROMETHEE techniques

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Abstract

The goal of this paper work is to Combine Preference ranking organization method for enrichment evaluation (PROMETHEE), Combinative Distance Based Assessment (CODAS) and H-max distance measure for Multi-criteria Decision Making under Intuitionistic fuzzy set. The role of H-max distance measure is to derive the weight vectors of criteria and CODAS method is to choose the alternatives in the PROMETHEE technique. Finally, Multi-criteria Problem is adopted to state the reliability of the generated combined algorithm.

Keywords

Intuitionistic fuzzy set, PROMETHEE, CODAS, H-max distance measure.

AMS Subject Classification

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Contents

1. Introduction

In 1986, Atanassov introduced the Intuitionistic fuzzy set as an generalization of fuzzy set . Zadeh introduced the fuzzy set which has the membership degree only. But in the intuitionistic fuzzy set frame, it has membership , non-membership and hesitation degree with the condition , that the addition of membership degree and non-membership degree does not over the range 0 to 1. Intuitionistic fuzzy set applied in numerous fields including artificial intelligence, Particularly in Multi-criteria decision making problems [9].

In day to day life, several decision making problems are faced by the single person and Institution or company. Likewise engineering or business management whatever the problem, it may be effortless like choosing the right mobile phone or tricky like choosing the suitable site for construction. According to the problem, the procedure of decision are taken into account to select the finest alternative and gathered the criteria for analyzing the alternative. In that case, Multi-criteria decision making gives appropriate techniques to build right decision. The main intention in the decision making problem is to identify the finest alternative among the other alternative depend on the knowledge of the subject experts [8].

PROMETHEE concept was developed by brans in 1982. Organizing the alternative is the significance role of PROMETHEE technique by partially or completely depend on the outranking flows. The concern outranking flows are positive outranking flow, negative outranking flow and finally net outranking flow. PROMETHEE is one of the outranking method in Multicriteria decision making. In recent years, the extension of PROMETHEE technique is Proposed and utilized in various situation [7].

In this study, the PROMETHEE technique is processed based on the combination of CODAS method and H-max distance measure. As an attempt H-max distance measure is utilized to finding the importance of the criteria and CODAS method is applied in the PROMETHEE technique to prioritize the alternative. According to the value $\rho(\phi^{+}(\alpha))$ and $\rho(\phi^{-}(\alpha))$ from the PROMETHEE technique is used for determine the relative assessment matrix in CODAS method and depend on the least value of the RAM, the alternatives are arranged. To check the validity of the integrated algorithm, a MCDM problem the choice regarding the classification of bricks is taken.

2. Preliminaries

Definition 2.1 (Intuitionistic fuzzy set [3,4])**.** *Intuitionistic fuzzy set is defined as*

$$A = \{< x, \mu_A(x), \gamma_A(x) >\} : x \in E\},$$

where E is universal set. The functions $\mu_A : E \to [0,1]$ *and* $\gamma_A : E \to [0,1]$ *express the value of membership degree and non-membership degree of x which belong to the universal set E with the following conditions:* $0 \leq \mu_A + \gamma_A \leq 1$.

Definition 2.2 (H-max distance measure [1])**.** *Consider the two intuitionistic fuzzy set A and B on X (universal set). Then the H-max distance measure is defined as* $d_{HM}(A,B)$

$$d_{HM}(A,B) = \frac{1}{3m}\sum_{i=1}^{m}\left(|\mu_1(x_i) - \mu_2(x_i)| + |v_1(x_i) - v_2(x_i)| + |\max\{\mu_1(x_i), v_2(x_i)\} - \max\{\mu_2(x_i), v_1(x_i)\}|\right)$$

where μ_1, μ_2 *are membership functions and* v_1, v_2 *are non-membership functions. Then the*

$$\max\{\mu_1(x_i), v_2(x_i)\} - \max\{\mu_2(x_i), v_1(x_i)\}$$

is called the cross evaluation between A and B.

3. H-max distance measure and CODAS based PROMETHEE approach for IFMCDM

This part formulate the Integrating Procedure of H-max distance measure [1] and CODAS [4] method in PROMETHEE [6,7] approach for IFMCDM.

Step 1: Obtain the linguistic variables for the alternatives regarding the criteria and convert the given linguistic variable into the form of Intuitionistic fuzzy value.

Step 2: Determine the weight vector from the Definition 2.2.

Step 3: Convert the Intuitionistic Fuzzy Decision Making into Fuzzy Decision matrix using

$$\mu_A(x) = \mu_A(x) + \frac{\pi_A(x)}{2}$$

That is,

$$\mu_A(x) = \frac{\mu_A(x) + (1 - v_A(x))}{2}.$$

Step 4: Determine $D_k(\alpha_i, \alpha_j)$ for the alternatives over different criteria.

$$D_k(\alpha_i, \alpha_j) = \beta_k(\alpha_i) - (\alpha_j).$$

Evaluate the preference of each alternative that is the alternative α_i against the alternative $\propto_j$ regarding the criteria by utilizing V-shape indifference criterion.

$$p(d) = \begin{cases} 0 & ; \quad d \leq q \\ \frac{d-q}{p-q} & ; \quad q < d \leq p \\ 1 & ; \quad d > p \end{cases}$$

In that, the parameters P represent the strict preference threshold and q represent the indifference threshold. In this paper, the indifference threshold is consider as Zero.

Step 5: Calculate the Intuitionistic fuzzy preference relation $I_R^{(k)} = \left(r_{ij}^{(k)}\right)_{pxp}$ over the criteria

$$\beta_k(k = 1, 2, \ldots . q).$$

Step 6: Calculate the collective intuitionistic fuzzy preference relation $I_R^{(k)} = \left(r_{ij}^{(k)}\right)_{pxp}$, that is

$$r_{ij} = r(\alpha_i, \alpha_j) = (\mu_{ij}, v_{ij}) = \left(\sum_{k=1}^{q} w_k \mu_{ij}^{(k)}, \Sigma_{k=1}^{q} w_k v_{ij}^{(k)}\right)$$

Step 7: Obtain the Intuitionistic fuzzy positive outranking flow $\phi^{+}(\alpha_i)$ and the intuitionistic fuzzy negative outranking flow $\phi^{-}(\alpha_i)$ for each alternative (α_i) by using the equation

$$\phi^{+}(\alpha_i) = \frac{1}{p-1}\sum_{\delta \in U}\Pi(\alpha_i, \delta)$$

and

$$\phi^{-}(\alpha_i) = \frac{1}{p-1}\sum_{\delta \in U}\Pi(\delta, \alpha_i)$$

Step 8 : Calculate the value of $\rho(\varphi^{+}(\alpha))$ and $\rho(\varphi^{-}(\alpha))$ from the following equation:

$$\rho(\varphi(\alpha)) = 0.5\left(1 + \Pi_\phi(\alpha)\left(1 - \mu_\phi(\alpha)\right)\right).$$

Step 9 : Identify the relative assessment matrix , $RA = [P_{ik}]_{nxn}$

$$P_{ik} = \left(\rho\left(\varphi^{+}(\alpha)\right)_i - \rho\left(\varphi^{+}(\alpha)\right)_k\right) + t\left(\rho\left(\varphi^{+}(\alpha)\right)_i - \rho\left(\varphi^{+}(\alpha)\right)_k\right) \times \left(\rho\left(\varphi^{-}(\alpha)\right)_i - \rho\left(\varphi^{-}(\alpha)\right)_k\right)$$

where $K = \{1, 2, \ldots, n\}$ and t is a threshold function

$$t(x) = \begin{cases} 1 \text{ if } |x| \geq \theta \\ 0 \text{ if } |x| < \theta \end{cases}$$

Step 10: Evaluate the assessment score $AS_i = \sum_{k=1}^{n} P_{ik}$ for each alternative.

Step 11: Depend on the value of AS_i, the alternatives are arranged from the least value to the highest one. The least value is consider as the best alternative of the problem

4. Application of the Integrated Procedure and Discussion

Consider if there is a person who is willing to construct his/her own house. So that the person come to know the classification of bricks and want to know the required one. For that, the classification of bricks are taken as alternatives which are Burnt Clay Bricks, sand lime Bricks,Fire Bricks, Concrete Bricks and Fly Ash Bricks and the criteria are Strength and Density, Compressive strength, Water Absorption and Soluble Salts. The subject expert gives the information in the form of linguistic variable due to the lack of knowledge in IFS which is latterly changed into the Intuitionistic fuzzy value. The

Table 1. Linguistic variable for Decision Matrix

Linguistic Variable	IFV
Very Good	¡0.8,0.15¿
Good	¡0.65,0.2¿
Acceptable	¡0.40,0.35¿
Poor	¡0.15,0.4¿
Very poor	¡0.05,0.5¿

weight vectors of the criteria's are

$$\omega_1 = 0.18 \omega_2 = 0.31, \omega_3 = 0.33 \text{ and } \omega_4 = 0.17.$$

According to the step 4, the Fuzzy decision matrix is obtained as

$$\begin{bmatrix} 0.725 & 0.275 & 0.825 & 0.725 & 0.725 \\ 0.825 & 0.375 & 0.825 & 0.375 & 0.375 \\ 0.725 & 0.375 & 0.825 & 0.375 & 0.375 \\ 0.725 & 0.725 & 0.825 & 0.375 & 0.725 \end{bmatrix}_{5\times 5}$$

Obtained preference matrix of each alternative with respect to thecriterion using V-shaped indifference criterion

$$C^1 = \begin{bmatrix} - & 1 & 0 & 0 & 0 \\ 0 & - & 0 & 0 & 0 \\ 0.6 & 0.6 & - & 0.6 & 0.6 \\ 0 & 1 & 0 & - & 0 \\ 0 & 1 & 0.6 & 0 & - \end{bmatrix}$$

$$C^2 = \begin{bmatrix} - & 1 & 0 & 1 & 1 \\ 0 & - & 0 & 0 & 0 \\ 0 & 1 & - & 1 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & - \end{bmatrix}$$

$$C^3 = \begin{bmatrix} - & 1 & 0 & 1 & 1 \\ 0 & - & 0 & 0 & 0 \\ 0.3 & 1 & - & 1 & 1 \\ 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & - \end{bmatrix}$$

$$C^4 = \begin{bmatrix} - & 0 & 0 & 1 & 0 \\ 0 & - & 0 & 1 & 0 \\ 0.6 & 0.6 & - & 1 & 0.6 \\ 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 & - \end{bmatrix}$$

By applying step 5, the Intuitionistic fuzzy preference relation $I_R^{(k)} = \left(r_{ij}^{(k)}\right)_{pxp}$ over the criteria are:

$$I_R^{(1)} = \begin{bmatrix} - & (1,0) & (0,0.6) & (0,0) & (0,0) \\ (0,1) & - & (0,1) & (0,1) & (0,1) \\ (0.6,0) & (1,0) & - & (1,0) & (1,0.6) \\ (0,0) & (1,0) & (0,1) & - & (0,0) \\ (0,0) & (1,0) & (0.6,1) & (0,0) & - \end{bmatrix}$$

$$I_R^{(2)} = \begin{bmatrix} - & (1,0) & (0,0) & (1,1) & (1,1) \\ (0,1) & - & (0,1) & (0,0) & (0,0) \\ (0,0) & (1,0) & - & (1,0) & (1,0) \\ (1,1) & (0,0) & (0,1) & - & (0,0) \\ (1,1) & (0,0) & (0,1) & (0,0) & - \end{bmatrix}$$

$$I_R^{(3)} = \begin{bmatrix} - & (1,0) & (0,0.3) & (1,0) & (1,0) \\ (0,1) & - & (0,1) & (0,0) & (0,0) \\ (0.3,0) & (1,0) & - & (1,0) & (1,0) \\ (0,1) & (0,0) & (0,1) & - & (0,0) \\ (0,01) & (00) & (0,1) & (0,0) & - \end{bmatrix}$$

$$I_R^{(4)} = \begin{bmatrix} - & (0,0) & (0,0.6) & (1,0) & (0,0) \\ (0,0) & - & (0,0.6) & (1,0) & (0,0) \\ (0.6,0) & (0.6,0) & - & (1,0) & (0.6,0) \\ (0,1) & (0,1) & (0,1) & - & (0,0) \\ (0,0) & (0,0) & (0,0.6) & (0,0) & - \end{bmatrix}$$

By applying step 6, Collected intuitionistic fuzzy preference relation $I_R^{(k)} = \left(r_{ij}^{(k)}\right)_{pxp}$.

$I_R =$

$$\begin{bmatrix} - & (0.82,0) & (0,0.31) & (0.81,0.31) & (0.64,0.31) \\ (0,0.82) & - & (0,0.92) & (0.17,0.18) & (0,0.18) \\ (0.31,0) & (0.92,0) & - & (0.99,0) & (0.92,0.11) \\ (0.31,0.81) & (0.18,0.17) & (0,0.99) & - & (0,0) \\ (0.31,0.64) & (0.18,0) & (0.11,0.92) & (0,0) & - \end{bmatrix}$$

By applying step 7, Intuitionistic fuzzy positive outranking flow $\phi^+(\alpha_i)$'s and Intuitionistic fuzzy negative outranking flow $\phi^-(\alpha_i)$'s are obtained as follows:

$$\phi^+(\alpha_1) = (0.56,0.23), \phi^+(\alpha_2) = (0.04,0.52),$$
$$\phi^+(\alpha_3) = (0.78,0.02), \phi^+(\alpha_4) = (0.12,0.49),$$
$$\phi^+(\alpha_5) = (0.15,0.39) \phi^-(\alpha_1) = (0.23,0.56),$$
$$\phi^-(\alpha_2) = (0.52,0.04), \phi^-(\alpha_3) = (0.02,0.78)$$
$$\phi^-(\alpha_4) = (0.49,0.12) \phi^-(\alpha_5) = (0.39,0.15)$$

By applying step 8, Calculated value of $\rho(\varphi^+(\alpha))$ and $\rho(\varphi^-(\alpha))$ is as follow:

$$\rho\left(\phi^+(\alpha_1)\right) = 0.26, \rho\left(\phi^+(\alpha_2)\right) = 0.69,$$
$$\rho\left(\phi^+(\alpha_3)\right) = 0.13, \rho\left(\phi^+(\alpha_4)\right) = 0.61$$
$$\rho\left(\phi^+(\alpha_5)\right) = 0.62, \rho\left(\phi^-(\alpha_1)\right) = 0.46,$$
$$\rho\left(\phi^-(\alpha_2)\right) = 0.34, \rho\left(\phi^-(\alpha_3)\right) = 0.58,$$
$$\rho\left(\phi^-(\alpha_4)\right) = 0.35, \rho\left(\phi^-(\alpha_5)\right) = 0.44$$

The alternatives (Bricks) are arranged according to the non-increasing order as follows: Fire Bricks $<$ Burnt Clay

Table 2. Converted Intuitionistic Fuzzy Decision matrix from the linguistic variable

	Burnt Clay Bricks	sand lime Bricks	Fire Bricks	Concrete Bricks	Fly Ash Bricks
Strength and Density	$< 0.65, 0.2 >$	$< 0.05, 0.5 >$	$< 0.8, 0.15 >$	$< 0.65, 0.2 >$	$< 0.65, 0.2 >$
Compressive strength	$< 0.8, 0.15 >$	$< 0.15, 0.4 >$	$< 0.8, 0.15 >$	$< 0.15, 0.4 >$	$< 0.15, 0.4 >$
Water Absorption	$< 0.65, 0.2 >$	$< 0.15, 0.4 >$	$< 0.8, 0.15 >$	$< 0.15, 0.4 >$	$< 0.15, 0.4 >$
Soluble Salts	$< 0.65, 0.2 >$	$< 0.65, 0.2 >$	$< 0.8, 0.15 >$	$< 0.15, 0.4 >$	$< 0.65, 0.2 >$

Table 3. Relative Assessment Matrix

	Burnt BricksClay	sand lime Bricks	Fire Bricks	Concrete Bricks	Fly Ash Bricks	ASi
Burnt Clay Bricks	-	-0.31	0.01	-0.24	-0.34	-0.88
sand lime Bricks	0.31	-	0.32	007	-0.03	0.67
Fire Bricks	-0.01	-0.32	-	0.25	-0.35	-0.93
Concrete Bricks	0.24	-0.07	0.25	-	-0.01	0.41
Fly Ash Bricks	0.34	0.03	0.35	0.01	-	0.73

Bricks $<$ Concrete Bricks $<$ Sand lime Bricks $<$ Fly Ash Bricks.

5. Conclusion

Multicriteria decision making has several techniques. In that, PROMETHEE is one of the finest techniques which plays a significance role in outranking methods. In this paper, PROMETHEE technique is worked based on CODAS method and H-max distance measure. CODAS method is used to arranged the order of the alternative and the part of H-max distance measure is to identify the weight vectors of each criterion in PROMETHEE technique. The choice of BRICKS for constructing home is taken as a Multicriteria decision making problem . So the Proposed integrated algorithm is applied in that MCDM problem which gives the finest result that is fire brick which is good to construct the home other than the four bricks.

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