



Spherical fuzzy graph

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Abstract

Classification of degrees and their properties are deliberated in the spherical fuzzy graph. Also the spherical fuzzy graph are interrogated using the order, size, completeness and their regularity.

Keywords

Effective degree, neighborhood degree, spherical complete fuzzy graph, regular spherical fuzzy graph.

AMS Subject Classification

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1. Introduction

In numerous reasonable circumstances such as operation management, networking and economical interpretation, the graph-theoretical portrayals of the data have been discovered to be more powerful and advantageous to manage the data embedded among various articles, characteristics, choices. The celebration of fuzzy graphs put forward by Kaufmann [2] grounded by the fuzzy relation [3]. The conceptual idea of fuzzy vertex and fuzzy edge progressed by Rosenfeld [4]. An intuitionistic fuzzy graph evolved from fuzzy graph, studied by Parvathi et. al., [5]. Pythagorean fuzzy graphs generalized out of intuitionistic fuzzy graph [6] and the spherical fuzzy graph acquainted by Akram et. al., [1]. In this paper, spherical fuzzy subgraph, complete spherical fuzzy graph, minimum and maximum degrees of spherical fuzzy graphs are demonstrated. The effective degree, neighborhood degree, closed neighborhood degree and their minimum and maximum neighborhood degrees are also defined in spherical fuzzy graph. The spherical regular fuzzy graph, the order and

their size of spherical fuzzy graph are elucidated with their properties.

2. Preliminaries

Definition 2.1. [1] Let \mathbb{V} be the non-empty set possess spherical fuzzy graph SFG is $\mathbb{G} = (P, Q)$, here P and Q are spherical fuzzy set and spherical fuzzy relation on \mathbb{V} in such

$$\begin{aligned}\alpha_Q(p, q) &\leq \min\{\alpha_P(p), \alpha_P(q)\}; \\ \gamma_Q(p, q) &\leq \min\{\gamma_P(p), \gamma_P(q)\}; \\ \beta_Q(p, q) &\leq \max\{\beta_P(p), \beta_P(q)\}.\end{aligned}$$

and $0 \leq \alpha_Q^2(p, q) + \gamma_Q^2(p, q) + \beta_Q^2(p, q) \leq 1 \forall p, q \in \mathbb{V}$. The spherical fuzzy vertex set P is of \mathbb{G} and the spherical edge set Q is of \mathbb{G} . The insubstantial edge is $Q(p, q) = 0 \forall (p, q) \in \mathbb{V} \times \mathbb{V} - \mathbb{E}$. The spherical fuzzy digraph has no symmetry relation in \mathbb{G} .

Example 2.2. [1] Let the vertex set $\mathbb{V} = \{p, q, r, s\}$ and the edge set $\mathbb{E} = \{pq, qr, rs, ps\}$ in $\mathbb{G}^* = (V, E)$. Take the spherical fuzzy set $P = (\alpha_P, \gamma_P, \beta_P)$ in \mathbb{V} and the spherical fuzzy edge set in $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ defined by

$$\begin{aligned}(\alpha_P(p), \gamma_P(p), \beta_P(p)) &= (0.6, 0.5, 0.3), \\ (\alpha_P(q), \gamma_P(q), \beta_P(q)) &= (0.7, 0.3, 0.6), \\ (\alpha_P(r), \gamma_P(r), \beta_P(r)) &= (0.3, 0.8, 0.4), \\ (\alpha_P(s), \gamma_P(s), \beta_P(s)) &= (0.6, 0.4, 0.5)\end{aligned}$$

and

$$\begin{aligned}(\alpha_Q(pq), \gamma_Q(pq), \beta_Q(pq)) &= (0.6, 0.3, 0.6), \\(\alpha_Q(qr), \gamma_Q(qr), \beta_Q(qr)) &= (0.2, 0.3, 0.6), \\(\alpha_Q(rs), \gamma_Q(rs), \beta_Q(rs)) &= (0.3, 0.4, 0.5), \\(\alpha_Q(ps), \gamma_Q(ps), \beta_Q(ps)) &= (0.5, 0.4, 0.5).\end{aligned}$$

Then, it is a spherical fuzzy graph.

Definition 2.3. [1] The spherical fuzzy graph $\mathbb{G} = (P, Q)$ is defined on $\mathbb{G}^* = (\mathbb{V}, \mathbb{E})$. The vertex degree of \mathbb{G} is designated by $d_{\mathbb{G}}(p) = (d_{\alpha}(p), d_{\gamma}(p), d_{\beta}(p))$ and the vertex degree of \mathbb{G} is elucidated as

$$d_{\mathbb{G}}(p) = (\sum_{p \neq q} \alpha_Q(p, q), \sum_{\alpha \neq \beta} \gamma_Q(p, q), \sum_{p \neq q} \beta_Q(p, q)),$$

here $\forall (p, q) \in \mathbb{E}$.

3. Classification of degrees in SFG

Definition 3.1. Consider the spherical fuzzy graph $\mathbb{G} = (P, Q)$. A spherical fuzzy graph $\mathbb{H} = (P', Q')$ is claimed to be spherical fuzzy subgraph of $\mathbb{G} = (P, Q)$ if $P' \subseteq P$ and $Q' \subseteq Q$. To be specifically, if $\alpha_{P'}(p) \leq \alpha_P(p)$; $\gamma_{P'}(p) \leq \gamma_P(p)$ and $\beta_{P'}(p) \geq \beta_P(p)$. $\alpha_{Q'}(p, q) \leq \alpha_Q(p, q)$; $\gamma_{Q'}(p, q) \leq \gamma_Q(p, q)$ and $\beta_{Q'}(p, q) \geq \beta_Q(p, q)$ for each $p, q \in P$.

Definition 3.2. A spherical fuzzy graph is complete if

$$\begin{aligned}\alpha_Q(p, q) &= \min\{\alpha_P(p), \alpha_P(q)\}, \\ \gamma_Q(p, q) &= \min\{\gamma_P(p), \gamma_P(q)\}, \\ \beta_Q(p, q) &= \max\{\beta_P(p), \beta_P(q)\}.\end{aligned}$$

Example 3.3. Let the vertex set $\mathbb{V} = \{p, q, r\}$ and the edge set $\mathbb{E} = \{pq, qr, pr\}$ in $\mathbb{G}^* = (\mathbb{V}, \mathbb{E})$. Take the spherical fuzzy set $P = (\alpha_P, \gamma_P, \beta_P)$ in \mathbb{V} and the spherical fuzzy edge set in $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ defined by

$$\begin{aligned}(\alpha_P(p), \gamma_P(p), \beta_P(p)) &= (0.8, 0.5, 0.6), \\(\alpha_P(q), \gamma_P(q), \beta_P(q)) &= (0.2, 0.3, 0.7), \\(\alpha_P(r), \gamma_P(r), \beta_P(r)) &= (0.5, 0.6, 0.8)\end{aligned}$$

and

$$\begin{aligned}(\alpha_Q(pq), \gamma_Q(pq), \beta_Q(pq)) &= (0.2, 0.3, 0.7), \\(\alpha_Q(qr), \gamma_Q(qr), \beta_Q(qr)) &= (0.2, 0.3, 0.8), \\(\alpha_Q(pr), \gamma_Q(pr), \beta_Q(pr)) &= (0.5, 0.5, 0.8).\end{aligned}$$

Then, it is a complete SFG.

Definition 3.4. The minimum degree of SFG, $\mathbb{G} = (P, Q)$ is designated $\delta(\mathbb{G}) = (\delta_{\alpha}(\mathbb{G}), \delta_{\gamma}(\mathbb{G}), \delta_{\beta}(\mathbb{G}))$ where,

$$\begin{aligned}\delta_{\alpha}(\mathbb{G}) &= \min\{d_{\alpha}(p) | p \in P\}; \\ \delta_{\gamma}(\mathbb{G}) &= \min\{d_{\gamma}(p) | p \in P\}; \\ \delta_{\beta}(\mathbb{G}) &= \min\{d_{\beta}(p) | p \in P\}.\end{aligned}$$

Definition 3.5. The maximum degree of SFG, $\mathbb{G} = (P, Q)$ is designated as $\Delta(\mathbb{G}) = (\Delta_{\alpha}(\mathbb{G}), \Delta_{\gamma}(\mathbb{G}), \Delta_{\beta}(\mathbb{G}))$ where,

$$\begin{aligned}\Delta_{\alpha}(\mathbb{G}) &= \max\{d_{\alpha}(p) | p \in P\}; \\ \Delta_{\gamma}(\mathbb{G}) &= \max\{d_{\gamma}(p) | p \in P\}; \\ \Delta_{\beta}(\mathbb{G}) &= \max\{d_{\beta}(p) | p \in P\}.\end{aligned}$$

Example 3.6. Let the vertex set $\mathbb{V} = \{p, q, r, s\}$ and the edge set $\mathbb{E} = \{pq, qr, rs, ps\}$ in $\mathbb{G}^* = (\mathbb{V}, \mathbb{E})$. Take the spherical fuzzy set $P = (\alpha_P, \gamma_P, \beta_P)$ in \mathbb{V} and the spherical fuzzy edge set in $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ defined by

$$\begin{aligned}(\alpha_P(p), \gamma_P(p), \beta_P(p)) &= (0.7, 0.4, 0.6), \\(\alpha_P(q), \gamma_P(q), \beta_P(q)) &= (0.4, 0.3, 0.1), \\(\alpha_P(r), \gamma_P(r), \beta_P(r)) &= (0.6, 0.7, 0.1), \\(\alpha_P(s), \gamma_P(s), \beta_P(s)) &= (0.8, 0.9, 0.4).\end{aligned}$$

and

$$\begin{aligned}(\alpha_Q(pq), \gamma_Q(pq), \beta_Q(pq)) &= (0.3, 0.3, 0.5), \\(\alpha_Q(qr), \gamma_Q(qr), \beta_Q(qr)) &= (0.2, 0.3, 0.1), \\(\alpha_Q(rs), \gamma_Q(rs), \beta_Q(rs)) &= (0.5, 0.1, 0.2), \\(\alpha_Q(ps), \gamma_Q(ps), \beta_Q(ps)) &= (0.4, 0.3, 0.3).\end{aligned}$$

Then, it is a SFG.

The minimum degree of SFG, $\mathbb{G} = (P, Q)$ discovered by the Example (3.6) is $\delta(\mathbb{G}) = (1.3, 1.1, 0.9)$.

The maximum degree of SFG, $\mathbb{G} = (P, Q)$ discovered by the Example (3.6) is $\Delta(\mathbb{G}) = (0.8, 0.4, 0.7)$.

Proposition 3.7. The sum of the degree of membership value, abstinence value and the non-membership value of all vertices in SFG is equal to twice the sum of the membership value, abstinence value and the non-membership value of all edges in SFG respectively. To be specifically,

$$\begin{aligned}\Sigma d(p) &= (\Sigma d_{\alpha}(p), \Sigma d_{\gamma}(p), \Sigma d_{\beta}(p)) = \\ &[2 \sum_{p \neq q} \alpha_Q(p, q), 2 \sum_{p \neq q} \gamma_Q(p, q), 2 \sum_{p \neq q} \beta_Q(p, q)].\end{aligned}$$

Proposition 3.8. In SFG has p vertices and its maximum degree of any vertex is $p - 1$.

Proposition 3.9. A complete SFG must have at least one pair of vertices of \mathbb{G} whose α -degrees are identical and at least one pair of vertices of \mathbb{G} whose β and γ -degrees are identical.

Definition 3.10. The edge $\epsilon = (p, q)$ of $\mathbb{G} = (P, Q)$ be a SFG is called an effective edge of \mathbb{G} is defined as

$$\begin{aligned}\alpha_Q(p, q) &= \min\{\alpha_P(p), \alpha_P(q)\}; \\ \gamma_Q(p, q) &= \min\{\gamma_P(p), \gamma_P(q)\}; \\ \beta_Q(p, q) &= \max\{\beta_P(p), \beta_P(q)\}.\end{aligned}$$

Example 3.11. In Example (3.6), an edge cd is an effective edge of SFG.



Definition 3.12. The effective degree of a vertex p of SFFG , $\mathbb{G} = (P, Q)$ is elucidated by $d_{\mathcal{E}}(p) = (d_{\mathcal{E}\alpha}(p), d_{\mathcal{E}\gamma}(p), d_{\mathcal{E}\beta}(p))$ $\forall p \in \mathcal{E}$, here $d_{\mathcal{E}\alpha}(p)$ is the sum of the α -values of the effective edges of SFFG incident with p , $d_{\mathcal{E}\gamma}(p)$ is the sum of the γ -values of the effective edges of SFFG incident with p and $d_{\mathcal{E}\beta}(p)$ is the sum of the β -values of the effective edges of SFFG incident with p .

Definition 3.13. The minimum effective degree of $\mathbb{G} = (P, Q)$ in a SFFG is elucidated by $\delta_{\mathcal{E}}(\mathbb{G}) = (\delta_{\mathcal{E}\alpha}(\mathbb{G}), \delta_{\mathcal{E}\gamma}(\mathbb{G}), \delta_{\mathcal{E}\beta}(\mathbb{G}))$ where,

$$\begin{aligned} \delta_{\mathcal{E}\alpha}(\mathbb{G}) &= \min\{d_{\mathcal{E}\alpha}(p) | p \in P\}; \\ \delta_{\mathcal{E}\gamma}(\mathbb{G}) &= \min\{d_{\mathcal{E}\gamma}(p) | p \in P\}; \\ \delta_{\mathcal{E}\beta}(\mathbb{G}) &= \min\{d_{\mathcal{E}\beta}(p) | p \in P\}. \end{aligned}$$

Definition 3.14. The maximum effective degree of $\mathbb{G} = (P, Q)$ in a SFFG is elucidated by $\Delta_{\mathcal{E}}(\mathbb{G}) = (\Delta_{\mathcal{E}\alpha}(\mathbb{G}), \Delta_{\mathcal{E}\gamma}(\mathbb{G}), \Delta_{\mathcal{E}\beta}(\mathbb{G}))$ where,

$$\begin{aligned} \Delta_{\mathcal{E}\alpha}(\mathbb{G}) &= \max\{d_{\mathcal{E}\alpha}(p) | p \in P\}; \\ \Delta_{\mathcal{E}\gamma}(\mathbb{G}) &= \max\{d_{\mathcal{E}\gamma}(p) | p \in P\}; \\ \Delta_{\mathcal{E}\beta}(\mathbb{G}) &= \max\{d_{\mathcal{E}\beta}(p) | p \in P\}. \end{aligned}$$

Example 3.15. In Example (3.6),

$$\begin{aligned} d_{\mathcal{E}}(p) &= (0, 0, 0) \\ d_{\mathcal{E}}(q) &= (0, 0, 0) \\ d_{\mathcal{E}}(r) &= (0.6, 0.7, 0.4) \\ d_{\mathcal{E}}(s) &= (0.6, 0.7, 0.4) \\ \delta_{\mathcal{E}}(\mathbb{G}) &= (0, 0, 0) \\ \Delta_{\mathcal{E}}(\mathbb{G}) &= (0.6, 0.7, 0.4) \end{aligned}$$

Definition 3.16. The neighborhood of any vertex p in $\mathbb{G} = (P, Q)$ of a SFFG is elucidated as $N(p) = (N_{\alpha}(p), N_{\gamma}(p), N_{\beta}(p))$ where,

$$\begin{aligned} N_{\alpha}(p) &= \{q \in P : \alpha_Q(p, q) = \alpha_P(p) \wedge \alpha_P(q)\}; \\ N_{\gamma}(p) &= \{q \in P : \gamma_Q(p, q) = \gamma_P(p) \wedge \gamma_P(q)\}; \\ N_{\beta}(p) &= \{q \in P : \beta_Q(p, q) = \beta_P(p) \wedge \beta_P(q)\} \end{aligned}$$

and $N[p] = N(p) \cup p$ is called the closed neighbourhood of p .

Definition 3.17. The neighborhood degree of a vertex in $\mathbb{G} = (P, Q)$ of a SFFG is elucidated as

$$d_{\mathcal{N}}(p) = (d_{\mathcal{N}\alpha}(p), d_{\mathcal{N}\gamma}(p), d_{\mathcal{N}\beta}(p))$$

where,

$$\begin{aligned} d_{\mathcal{N}\alpha}(p) &= \sum_{p \in \mathcal{N}(p)} \alpha_P(p); \\ d_{\mathcal{N}\gamma}(p) &= \sum_{p \in \mathcal{N}(p)} \gamma_P(p); \\ d_{\mathcal{N}\beta}(p) &= \sum_{p \in \mathcal{N}(p)} \beta_P(p). \end{aligned}$$

Definition 3.18. The minimum neighborhood degree of $\mathbb{G} = (P, Q)$ in a SFFG is elucidated as

$$\delta_{\mathcal{N}}(\mathbb{G}) = (\delta_{\mathcal{N}\alpha}(\mathbb{G}), \delta_{\mathcal{N}\gamma}(\mathbb{G}), \delta_{\mathcal{N}\beta}(\mathbb{G}))$$

where,

$$\begin{aligned} \delta_{\mathcal{N}\alpha}(\mathbb{G}) &= \wedge\{d_{\mathcal{N}\alpha}(p) | p \in P\}, \\ \delta_{\mathcal{N}\gamma}(\mathbb{G}) &= \wedge\{d_{\mathcal{N}\gamma}(p) | p \in P\}, \\ \delta_{\mathcal{N}\beta}(\mathbb{G}) &= \wedge\{d_{\mathcal{N}\beta}(p) | p \in P\}. \end{aligned}$$

Definition 3.19. The maximum neighborhood degree of $\mathbb{G} = (P, Q)$ in a SFFG is elucidated as

$$\Delta_{\mathcal{N}}(\mathbb{G}) = (\Delta_{\mathcal{N}\alpha}(\mathbb{G}), \Delta_{\mathcal{N}\gamma}(\mathbb{G}), \Delta_{\mathcal{N}\beta}(\mathbb{G}))$$

where,

$$\begin{aligned} \Delta_{\mathcal{N}\alpha}(\mathbb{G}) &= \vee\{d_{\mathcal{N}\alpha}(p) | p \in P\}, \\ \Delta_{\mathcal{N}\gamma}(\mathbb{G}) &= \vee\{d_{\mathcal{N}\gamma}(p) | p \in P\}, \\ \Delta_{\mathcal{N}\beta}(\mathbb{G}) &= \vee\{d_{\mathcal{N}\beta}(p) | p \in P\}. \end{aligned}$$

Example 3.20. In Example (3.3),

$$\begin{aligned} d_{\mathcal{N}}(p) &= (0.7, 0.9, 1.5) \\ d_{\mathcal{N}}(q) &= (1.3, 1.1, 1.4) \\ d_{\mathcal{N}}(r) &= (1.0, 0.8, 1.3) \\ \delta_{\mathcal{N}}(\mathbb{G}) &= (0.7, 0.8, 1.3) \\ \Delta_{\mathcal{N}}(\mathbb{G}) &= (1.3, 1.1, 1.5) \end{aligned}$$

Definition 3.21. The closed neighborhood degree of a vertex p of $\mathbb{G} = (P, Q)$ in a SFFG is elucidated as

$$d_{\mathcal{N}}[p] = (d_{\mathcal{N}\alpha}[p], d_{\mathcal{N}\gamma}[p], d_{\mathcal{N}\beta}[p])$$

where,

$$\begin{aligned} d_{\mathcal{N}\alpha}[a] &= \sum_{q \in \mathcal{N}(p)} \alpha_P(q) + \alpha_P(p), \\ d_{\mathcal{N}\gamma}[a] &= \sum_{q \in \mathcal{N}(p)} \gamma_P(q) + \gamma_P(p), \\ d_{\mathcal{N}\beta}[a] &= \sum_{q \in \mathcal{N}(p)} \beta_P(q) + \beta_P(p). \end{aligned}$$

Definition 3.22. The minimum closed neighborhood degree of $\mathbb{G} = (P, Q)$ in a SFFG is elucidated as

$$\delta_{\mathcal{N}}[\mathbb{G}] = (\delta_{\mathcal{N}\alpha}[\mathbb{G}], \delta_{\mathcal{N}\gamma}[\mathbb{G}], \delta_{\mathcal{N}\beta}[\mathbb{G}])$$

where,

$$\begin{aligned} \delta_{\mathcal{N}\alpha}[\mathbb{G}] &= \wedge\{d_{\mathcal{N}\alpha}(p) | p \in P\} \\ \delta_{\mathcal{N}\gamma}[\mathbb{G}] &= \wedge\{d_{\mathcal{N}\gamma}(p) | p \in P\} \\ \delta_{\mathcal{N}\beta}[\mathbb{G}] &= \wedge\{d_{\mathcal{N}\beta}(p) | p \in P\}. \end{aligned}$$

Definition 3.23. The maximum neighborhood degree of $\mathbb{G} = (P, Q)$ in a SFFG is elucidated as

$$\Delta_{\mathcal{N}}(\mathbb{G}) = (\Delta_{\mathcal{N}\alpha}(\mathbb{G}), \Delta_{\mathcal{N}\gamma}(\mathbb{G}), \Delta_{\mathcal{N}\beta}(\mathbb{G}))$$

where,

$$\begin{aligned} \Delta_{\mathcal{N}\alpha}(\mathbb{G}) &= \vee\{d_{\mathcal{N}\alpha}(p) | p \in P\}, \\ \Delta_{\mathcal{N}\gamma}(\mathbb{G}) &= \vee\{d_{\mathcal{N}\gamma}(p) | p \in P\}, \\ \Delta_{\mathcal{N}\beta}(\mathbb{G}) &= \vee\{d_{\mathcal{N}\beta}(p) | p \in P\}. \end{aligned}$$



4. Regular Spherical Fuzzy Graph

Definition 4.1. Regular of a SFG, $\mathbb{G} = (P, Q)$ is defined by all the vertices have the same closed neighborhood degree. To be specifically, $\delta_{\mathcal{N}}(\alpha)[\mathbb{G}] = \Delta_{\mathcal{N}}(\alpha)[\mathbb{G}]$; $\delta_{\mathcal{N}}(\gamma)[\mathbb{G}] = \Delta_{\mathcal{N}}(\gamma)[\mathbb{G}]$; $\delta_{\mathcal{N}}(\beta)[\mathbb{G}] = \Delta_{\mathcal{N}}(\beta)[\mathbb{G}]$.

Example 4.2. In Example (3.3),

$$d_{\mathcal{N}}[p] = (1.5, 1.4, 2.1),$$

$$d_{\mathcal{N}}[q] = (1.5, 1.4, 2.1),$$

$$d_{\mathcal{N}}[r] = (1.5, 1.4, 2.1)$$

which implies

$$\delta_{\mathcal{N}}[\mathbb{G}] = (1.5, 1.4, 2.1),$$

$$\Delta_{\mathcal{N}}[\mathbb{G}] = (1.5, 1.4, 2.1).$$

Proposition 4.3. Every spherical complete fuzzy graph in \mathbb{G} is spherical regular fuzzy graph.

Proof. Let SFG, $\mathbb{G} = (P, Q)$ has completeness implies closed neighborhood degree are elucidated in Definition (3.2) and in Definition (3.21) respectively. This leads to every vertex of a minimum and maximum closed neighborhood degree are equal in \mathbb{G} . To be specifically, $\delta_{\mathcal{N}}(\alpha)[\mathbb{G}] = \Delta_{\mathcal{N}}(\alpha)[\mathbb{G}]$; $\delta_{\mathcal{N}}(\gamma)[\mathbb{G}] = \Delta_{\mathcal{N}}(\gamma)[\mathbb{G}]$; $\delta_{\mathcal{N}}(\beta)[\mathbb{G}] = \Delta_{\mathcal{N}}(\beta)[\mathbb{G}]$. This tends to \mathbb{G} is spherical regular fuzzy graph. Hence the proof. \square

5. Elucidation of Order and Size in SFG

Definition 5.1. The order of SFG, \mathbb{G} is elucidated by $\Theta(\mathbb{G}) = (\Theta_{\alpha}(\mathbb{G}), \Theta_{\gamma}(\mathbb{G}), \Theta_{\beta}(\mathbb{G}))$ where

$$\Theta_{\alpha}(\mathbb{G}) = \sum_{p \in P} \alpha_p(p),$$

$$\Theta_{\gamma}(\mathbb{G}) = \sum_{p \in P} \gamma_p(p),$$

$$\Theta_{\beta}(\mathbb{G}) = \sum_{p \in P} \beta_p(p).$$

Definition 5.2. The size of SFG, \mathbb{G} is elucidated by $\Omega(\mathbb{G}) = (\Omega_{\alpha}(\mathbb{G}), \Omega_{\gamma}(\mathbb{G}), \Omega_{\beta}(\mathbb{G}))$ where

$$\Omega_{\alpha}(\mathbb{G}) = \sum_{p \neq q} \alpha_Q(p, q),$$

$$\Omega_{\gamma}(\mathbb{G}) = \sum_{p \neq q} \gamma_Q(p, q),$$

$$\Omega_{\beta}(\mathbb{G}) = \sum_{p \neq q} \beta_Q(p, q).$$

Example 5.3. In Example (3.3), $\Theta(\mathbb{G}) = (1.5, 1.4, 2.1)$; $\Omega(\mathbb{G}) = (0.9, 1.4, 2.3)$.

Proposition 5.4. Every vertex of a closed neighborhood degree of a complete SFG has same order in \mathbb{G} . To be specifically, $\Theta_{\alpha}(\mathbb{G}) = (d_{N\alpha}[p] : p \in P)$; $\Theta_{\gamma}(\mathbb{G}) = (d_{N\gamma}[p] : p \in P)$; $\Theta_{\beta}(\mathbb{G}) = (d_{N\beta}[p] : p \in P)$.

6. Conclusion

In the present study, grasped some contemporary concepts of spherical fuzzy graphs which is the addendum of intuitionistic, Pythagorean and picture fuzzy graphs. Considerably more work should be possible to explore the structure of spherical fuzzy graphs. It is much convenient in the application field such as pattern recognition, decision making and network analysis.

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