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# On decompositions of H̋-continuous functions in topological spaces

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**Abstract**

In this article, we introduce H̋-continuous and study their relations with various continuous functions and discuss some properties of H̋-continuous functions, H̋$_{\mathscr{A}}$-continuous functions , H̋-irresolute, strongly H̋-continuous, strongly H̋$_{\mathscr{A}}$-continuous discuss their basic properties.

**Keywords**

H̋-continuous functions, strongly H̋-continuous, H̋-irresolute and H̋$_{\mathscr{A}}$-continuous functions.

**AMS Subject Classification**

54C05, 54C08, 54C10.

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## Contents



## 1. Introduction

Maki [6], introduced $\Lambda$-set, Arenas introduced $\lambda$-closed and the introduced a notions of $\lambda$-continuous. In this article, we introduce H̋-continuous functions & study their relations among various continuous functions and discuss some properties of H̋-continuous functions, H̋$_{\mathscr{A}}$-continuous functions , H̋-irresolute, strongly H̋-continuous, strongly H̋$_{\mathscr{A}}$-continuous discuss their basic properties.

## 2. Preliminaries

Through out paper obtained in the Topological space $(X,\tau)$ (resp. $(X,\sigma)$ and $(X,\eta)$) is denoted by TS X (resp. TS Y and TS Z).

For a subset C of a TS X, int(C), cl(C) denoted the interior, closure of C respectively. And $\lambda$ symbol use this thesis $\mathscr{A}$. For so many author introduced sets and $\lambda$-closed [1], $\theta$-closed [8], $\lambda T_{\frac{1}{2}}$-space [3], H̋-set and H̋-closed [7], g-continuous [5], $\lambda$-continuous [1], faintly continuous [2], faintly $\lambda$-continuous [4], $\lambda$-irresolute [3]

**Definition 2.1.** *[7] Consent to S be a subset of a TS X $\Longrightarrow$ we define a*
$S\H{S} = \cap\{Q/Q \supset S, Q \in \mathscr{A}O(X,\tau)\}$.

**Lemma 2.2.** *[4] A TS X is locally indiscrete $\Longleftrightarrow$ each $\mathscr{A}$-open is open.*

**Theorem 2.3.** *[7] If $C_i$ is H̋-open for every $i \in I \Longrightarrow \bigcup_{i\in I} C_i$ is H̋-open.*

**Lemma 2.4.** *[7] In a TS X, $\mathscr{A}$-open set $\Longrightarrow$ H̋-open set.*

**Theorem 2.5.** *[7] For a TS X, the next conditions are equivalent.*

1. *X is a $\mathscr{A}T_{\frac{1}{2}}$-space.*
2. *each subset of X is H̋-closed.*

**Theorem 2.6.** *[7] In a TS X, every $\Lambda_r$-closed is H̋-closed.*

**Theorem 2.7.** *[7] For subsets C & of a TS X, the next properties hold:*

1. $C \subset \H{H}cl(C) \subset \mathscr{A}cl(C) \subset cl(C)$.
2. *If $C \subset B$ then $\H{H}cl(C) \subset \H{H}cl(M)$.*
3. *$\H{H}cl(C)$ is H̋-closed.*

## 3. On H̋-continuous functions

**Definition 3.1.** *A function $f : X \to Y$ is said to be a*

1. *H̋-continuous if $f^{-1}(L)$ is H̋-open in TS X, for any open set V in TS Y.*
2. *$\H̋_{\mathscr{A}}$-continuous if $f^{-1}(L)$ is H̋-open in TS X, for any $\mathscr{A}$-open set V in TS Y.*
3. *H̋-irresolute if $f^{-1}(L)$ is H̋-open in TS X, for any H̋-open set V in TS Y.*
4. *S̋-open if $f(L)$ is H̋-open in TS Y, for any H̋-open set V in TS X.*

**Theorem 3.2.** *A function f is H̋-continuous $\iff$ $f^{-1}(K)$ is H̋-closed in TS X whenever F is closed in TS Y.*

*Proof.* Let $F$ be any closed subset of Y. Then $Y - F$ is open in Y. Since $f$ is H̋-continuous, by Definition 3.1 $f^{-1}(Y - F)$ is H̋-open in X. But $f^{-1}(Y - F) = X - f^{-1}(F)$. So, $X - f^{-1}(F)$ is H̋-open in X. That is $f^{-1}(F)$ is H̋-closed in X.

Conversely, let $Y - Q$ is closed in Y. By our assumption, $f^{-1}(Y - Q)$ is H̋-closed in X. But $f^{-1}(Y - Q) = X - f^{-1}(Q)$. So, $X - f^{-1}(Q)$ is H̋-closed in X. That is, $f^{-1}(Q)$ is H̋-open in X. Hence $f$ is H̋-continuous. □

**Theorem 3.3.** *A function f is H̋$_{\mathscr{A}}$-continuous $\iff$ $f^{-1}(K)$ is H̋-closed in TS X whenever K is $\mathscr{A}$-closed in TS Y.*

*Proof.* Follows by replace $\mathscr{A}$-closed set instead of closed set in Theorem 3.2. □

**Definition 3.4.** *Let a TS X, $x \in X$ and $\{x_e, e \in E\}$ be a net of X. We say that the net $\{x_e, e \in E\}$ H̋-converges to x if for each H̋-open set U containing x $\exists$ an element $e_0 \in E : e \geq e_0 \Longrightarrow x_e \in U$.*

**Theorem 3.5.** *Let a TS X and $C \subseteq X$. A point $x \in$ H̋$cl(C)$ $\iff$ if $\exists$ a net $\{x_e, e \in E\}$ of C which H̋-converges to x.*

*Proof.* The existence of such a net given every H̋-neighbourhood meets C and so $x \in$ H̋$cl(C)$. Suppose that $x \in$ H̋$cl(C)$ and let us denote by $\mathscr{U}$ the all H̋-open subsets $T$ of X such that $x \in T$ directed by the relation $\subseteq$ that is, let us define that $T_1 \leq T_2$ if $T_2 \subseteq T_1$. The net $\{x_T, T \in \mathscr{U}\}$, where $x_T$ is an arbitrary point of $C \cap T$, H̋-converges to $x$. □

**Corollary 3.6.** *A subset E of a TS X is H̋-closed $\iff$ limits of nets in E are in E.*

*Proof.* This follows from the Theorem 3.5 & the information that a set E is H̋-closed $\iff$ if $E =$ H̋$cl(E)$. □

**Corollary 3.7.** *A subset E of a TS X is H̋-open $\iff$ no net in the complement E can converge to a point in E.*

*Proof.* This follows by applying the Corollary 3.6 to complement of E. □

**Theorem 3.8.** *For a function $f : (X, \tau) \to (Y, \sigma)$ the next statements are equivalent:*

1. *f is H̋-continuous.*
2. *For every $x \in X$ and for every open L of Y containing $f(x)$ $\exists$ a H̋-open set T of X containing x and $f(T) \subseteq L$.*
3. *For every $x \in X$ and every net $\{x_e, e \in E\}$ in X which H̋-converges to x, the net $\{f(x_e), e \in E\}$ of Y converges to $f(x)$ in Y.*
4. *For any subset B of X, f(H̋$cl(B)) \subset cl(f(B))$ holds.*
5. *For every subset B of Y, H̋$cl(f^{-1}(B)) \subset f^{-1}(cl(B))$ holds.*

*Proof.* $(1) \Rightarrow (2)$ : Let $L \in \sigma$ and $f(x) \in L$ for each $x \in X$. As $f$ is H̋-continuous, $f^{-1}(L) \in$ H̋$O(X, \tau)$ & $x \in f^{-1}(L)$. Put $T = f^{-1}(L)$. At the time $x \in T$ and $f(T) \subset L$.

$(2) \Rightarrow (1)$ : Let L be an open set of Y; let $x$ be a point of $f^{-1}(L)$. At the time $f(x) \in L$, so that by hypothesis, $\exists$ a H̋-open set $T_x$ containing $x$ : $f(T_x) \subseteq L$.

$\therefore$, we've $x \in T_x \subset f^{-1}(L)$ and thus $f^{-1}(L) = \cup\{T_x / x \in f^{-1}(L)\}$. By Theorem 2.3, $f^{-1}(L)$ is H̋-open. Thus, $f$ is H̋-continuous.

$(2) \Rightarrow (3)$ : Let $x \in X$ and $\{x_e / e \in E\}$ be a net H̋ converging to $x$. For every open set of TS Y containing $f(x)$, by (2) there exist a H̋-open set $T$ of $X$ containing $x$ : $f(T) \subset L$. Since $\{x_e / e \in E\}$ converges to $x$, $\exists$ $e_0 \in E : e \geq e_0 \Longrightarrow x_e \in U$. $\therefore$ $f(x_e) \in L$ for any $e \geq e_0$ and the net $\{f(x_e) / e \in E\}$ converges to $f(x)$.

$(3) \Rightarrow (2)$ : Consent to us suppose that $\exists$ point $x \in X$ and an open neighbourhood $L$ of $f(x)$ : for every H̋-open set $T$ of $X$ containing $x$ such that $f(T) \nsubseteq L$. Then for every H̋-open set $T$ of $X$ such that $x \in T$, we choose an element $x_T \in T$ such that $f(x_T) \notin L$. Consent to $\mathscr{T}$ be the set of all H̋-open set $T$ of $X$ containing $x$ and is directed by the relation $\subseteq$. That is, let us define that $T_1 \leq T_2$ if $T_2 \subseteq T_1$. Easily, the net $\{x_T, T \in \mathscr{T}\}$ : H̋-converges to $x$ but the net $\{f(x_T) : T \in \mathscr{T}\}$ does not converges to $f(x)$ which is a contradiction . Thus, $\exists$ a H̋-open set $T$ of $X$ : $x \in T$ and $f(T) \subseteq L$.

$(2) \Rightarrow (4)$ : Suppose (2) holds and consent $y \in$ f(H̋$cl(B))$ and consent to L be any open neighbourhood of $y$. Thus $\exists$ a point $x \in X$ & a H̋-open set T : $f(x) = y, x \in T, x \in$ H̋$cl(B)$ and $f(T) \subset L$. Since $x \in$ H̋$cl(B)$, $T \cap B \neq \phi$ holds and hence $f(B) \cap L \neq \phi$. $\therefore$ we've $y = f(x) \in cl(f(B))$.

$(4) \Rightarrow (2)$ : If (4) holds and consent $x \in X$ and let L be any open set containing $f(x)$. Let $B = f^{-1}(Y - L)$, then $x \notin B$. Since f(H̋$cl(B)) \subset cl(f(B)) \subset Y - L$, it is shown that H̋$cl(B) = B$. At the moment , since $x \notin$ H̋$cl(B)$, then there exists a H̋-open set T containing $x$ : $T \cap B = \phi$ and thus $f(T) \subset f(X - B) \subset L$.

$(4) \Rightarrow (5)$ : Suppose that (4) holds and consent to B be every subset of Y. Replacing B by $f^{-1}(C)$ we get from (4), f(H̋$cl(f^{-1}(C))) \subset cl(f(f^{-1}(C))) \subset cl(C)$. Thus H̋$cl(f^{-1}(C)) \subset f^{-1}(cl(C))$.

(5) ⇒ (4) Suppose that (5) holds, consent to $B = f(C)$ where C is a subset of X. After that $\H{H}cl(C) \subset \H{H}cl(f^{-1}(B)) \subset f^{-1}(cl(f(C)))$. Therefore f($\H{H}cl(C)) \subset cl(f(C))$. □

**Theorem 3.9.** *In support of a $f : (X,\tau) \to (Y,\sigma)$ the next statements are equivalent:*

1. *f is $\H{H}_{\mathscr{A}}$ continuous.*
2. *For any $x \in X$ and for any $\mathscr{A}$-open set L of Y containing $f(x)$ $\exists$ a $\H{H}$-open T of X containing x and $f(T) \subseteq L$.*
3. *For any subset B of X, f($\H{H}cl(B)) \subset \mathscr{A}cl(f(B))$ holds.*
4. *For any subset C of Y, $\H{H}cl(f^{-1}(C)) \subset f^{-1}(\mathscr{A}cl(C))$ holds.*

*Proof.* Follows by replacing $\mathscr{A}$-open sets in its place of open sets in Theorem 3.8. □

**Theorem 3.10.** *f is $\H{H}$-irresolute $\iff$ if $f^{-1}(F)$ is $\H{H}$-closed in TS X whenever F is $\H{H}$-closed in TS Y.*

*Proof.* Consent to K be any $\H{H}$-closed subset of Y. Thus $Y - K$ is $\H{H}$-open in Y. Since $f$ is $\H{H}$-irresolute, by Definition 3.1, $f^{-1}(Y-K)$ is $\H{H}$-open in X. But $f^{-1}(Y-K) = X - f^{-1}(K)$. So, $X - f^{-1}(K)$ is $\H{H}$-open. i.e., is $f^{-1}(K)$ is $\H{H}$-closed.

Conversely, consent to $Y - D$ is $\H{H}$-closed. By our assumption, $f^{-1}(Y-D)$ is $\H{H}$-closed. But $f^{-1}(Y-D) = X - f^{-1}(D)$. So, $X - f^{-1}(D)$ is $\H{H}$-closed. That is, $f^{-1}(D)$ is $\H{H}$-open. Hence $f$ is $\H{H}$-irresolute. □

**Theorem 3.11.** *For a $f : (X,\tau) \to (Y,\sigma)$ the next statements are equivalent:*

1. *f is $\H{H}$-irresolute;*
2. *For any $x \in X$ and for each $\H{H}$-open set L of Y containing $f(x)$ $\exists$ a $\H{H}$-open set T of X containing x and $f(T) \subseteq L$;*
3. *f($\H{H}cl(B)) \subset \H{H}cl(f(B))$ for any subset B of X;*
4. *$\H{H}cl(f^{-1}(C)) \subset f^{-1}(\H{H}cl(C))$ for any subset C of Y.*

*Proof.* (1) ⇒ (2) : Let V be $\H{H}$-open in TS Y & $f(x) \in L$ for any $x \in X$. Since $f$ is $\H{H}$-irresolute, $f^{-1}(L) \in \H{H}O(X,\tau)$ & $x \in f^{-1}(L)$. Put $U = f^{-1}(L)$. Thus $x \in T$ & $f(T) \subset L$.

(2) ⇒ (1) : Let L be $\H{H}$-open set of Y; let $x$ be a point of $f^{-1}(L)$. After that $f(x) \in L$, so that by hypothesis, $\exists$ a $\H{H}$-open set $T_x$ containing $x : f(T_x) \subseteq L$. Therefore, we've $x \in T_x \subset f^{-1}(L)$ and thus $f^{-1}(L) = \cup\{T_x / x \in f^{-1}(L)\}$. By Theorem 2.3, $f^{-1}(L)$ is $\H{H}$-open in TS X. Thus, $f$ is $\H{H}$-continuous.

(1) ⇒ (3) : Consent to B be a subset of X. Since $f(B)$ in Y, by Theorem 2.7, $\H{H}cl(f(B))$ is $\H{H}$-closed in Y. By our assumption and by Theorem 3.10, $f^{-1}(\H{H}cl(f(B)))$ is $\H{H}$-closed in TS X. Since $B \subset f^{-1}(\H{H}cl(f(B)))$, by Theorem 2.7, we have, $\H{H}cl(B) \subset \H{H}cl(f(B)) = f^{-1}(\H{H}cl(f(B)))$ & hence f($\H{H}cl(B)) \subset \H{H}cl(f(B))$.

(3) ⇒ (4) : Consent to C be any subset of Y. By (4), we've f($\H{H}cl(f^1(C))) \subset \H{H}cl(f(f^{-1}(C))) = \H{H}cl(C)$ & hence $\H{H}cl(f^{-1}(C)) \subset f^{-1}(\H{H}cl(C))$.

(4) ⇒ (1) : Consent to Q be every $\H{H}$-closed set in TS Y. By Theorem 2.7(3), $Q = \H{H}cl(Q)$. Then $\H{H}cl(f^{-1}(Q)) \subset f^{-1}(\H{H}cl(Q)) = f^{-1}(Q)$ & $\H{H}cl(f^{-1}(Q)) \subset f^{-1}(Q)$. Therefore we taken $\H{H}cl(f^{-1}(Q)) = f^{-1}(Q)$. This shows that $f^{-1}(Q)$ is $\H{H}$-closed in TS X. □

**Proposition 3.12.** *If the function $f : (X,\tau) \to (Y,\sigma)$ is $\mathscr{A}$-continuous, then the function f is $\H{H}$-continuous.*

*Proof.* Consent to L be an open set in TS Y. Since $f$ is $\mathscr{A}$-continuous, $f^{-1}(L)$ is $\mathscr{A}$-open in TS X. By Lemma 2.4, $f^{-1}(L)$ is $\H{H}$-open in TS X. Thus $f$ is $\H{H}$-continuous. □

**Example 3.13.** *Let $X = \{1,2,3,4\} = Y$, $\tau$ =$\{\phi$, {1, 2}, {1, 2, 3, X}} and $\sigma$ =$\{\phi$, {4}, {1, 2}, {1, 2, 4, Y}}.*

*Define a $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = d$ and $f(d) = c$. Thus f is $\H{H}$-continuous but f is not $\mathscr{A}$-continuous because of $f^{-1}(\{d\}) = \{c\}$ is not $\mathscr{A}$-open.*

**Theorem 3.14.** *If the function $f : (X,\tau) \to (Y,\sigma)$ is $\mathscr{A}$-irresolute $\Longrightarrow$ the function f is $\H{H}_{\mathscr{A}}$-continuous.*

*Proof.* Consent to L be a $\mathscr{A}$-open set in TS Y. Because $f$ is $\mathscr{A}$-irresolute, $f^{-1}(L)$ is $\mathscr{A}$-open in TS X. By Lemma 2.4, $f^{-1}(L)$ is $\H{H}$-open in TS X. Thus $f$ is $\H{H}_{\mathscr{A}}$-continuous. □

**Example 3.15.** *Consent to $X = \{1,2,3,4\} = Y$, $\tau$ =$\{\phi$, {1}, {1, 3}, {1, 3, 4}, X} and $\sigma$ = $\{\phi$, {4}, {3, 4}, {1, 3, 4}, Y}.*

*Define a $f : (X,\tau) \to (Y,\sigma)$ by $f(1) = 4$, $f(2) = 1$, $f(3) = 2$ and $f(4) = 3$. Thus f is $\H{H}_{\mathscr{A}}$-continuous but not $\mathscr{A}$-irresolute, since $f^{-1}(\{b\}) = \{c\}$ is not $\mathscr{A}$-open.*

**Proposition 3.16.** *If $f : (X,\tau) \to (Y,\sigma)$ is $\Lambda_r$-continuous $\Longrightarrow$ f is $\H{H}$-continuous.*

*Proof.* Consent to L be an open set in TS Y. Since $f$ is $\Lambda_r$-continuous, $f^{-1}(L)$ is $\Lambda_r$-open in TS X. By Proposition 2.6, $f^{-1}(L)$ is $\H{H}$-open in TS X. Hence $f$ is $\H{H}$-continuous. □

**Remark 3.17.** *The reverse of above Proposition 3.16 is need not be a true.*

**Example 3.18.** *Let $X = \{1,2,3\} = Y$, $\tau$ = $\{\phi$, {1}, {2}, {1, 2}, X} and $\sigma$ =$\{\phi$, Y, {1}}.*

*Define $f : (X,\tau) \to (Y,\sigma)$ by f(1)=2, f(2)=3 and f(3) = 1. Here f is $\mathscr{A}$-continuous and hence $\H{H}$-continuous but not $\Lambda_r$-continuous, since of $f^{-1}(\{1\}) = \{3\}$ is not $\Lambda_r$-open in X.*

| | | |
|---|---|---|
| **$\Lambda_r$-continuous** | ⟵ | **continuous** |
| ↓ | | ↓ |
| **$\H{H}$-continuous** | ⟵ | **$\mathscr{A}$-continuous** |

**Theorem 3.19.** *If a $f : (X,\tau) \to (Y,\sigma)$ is faintly $\mathscr{A}$-continuous $\Longrightarrow$ $f : (X,\tau\H{S}) \to (Y,\sigma)$ is faintly continuous.*

*Proof.* Consent to L be $\theta$-open in TS Y. Since $f$ is faintly $\mathscr{A}$-continuous, $f^{-1}(L)$ is $\mathscr{A}$-open in TS X. Because every $\mathscr{A}$-open is S̋-set, $f^{-1}(L)$ is open in $(X, \tau\text{S̋})$. Hence $f$ is faintly continuous. □

**Theorem 3.20.** *If $f : (X,\tau) \to (Y,\sigma)$ is faintly $\mathscr{A}$-continuous and if $g : (Y,\sigma) \to (Z,\eta)$ is quasi-$\theta$-continuous then $g \circ f : (X,\tau) \to (Z,\eta)$ is faintly $\mathscr{A}$-continuous.*

*Proof.* Consent to T be $\theta$-open in TS Z. Because $g$ is quasi-$\theta$-continuous, $g^{-1}(T)$ is $\theta$-open in TS Y. Since $f$ is faintly $\mathscr{A}$-continuous, $f^{-1}(g^{-1}(T))$ is $\mathscr{A}$-open in TS X. Thus $(g \circ f)^{-1}(T)$ is $\mathscr{A}$-open in TS X, for each $\theta$-open set T in TS Z. Hence $g \circ f : (X,\tau) \to (Z,\eta)$ is faintly $\mathscr{A}$-continuous. □

**Theorem 3.21.** *If $f : (X,\tau) \to (Y,\sigma)$ is faintly continuous and $g : (Y,\sigma) \to (Z,\eta)$ is quasi-$\theta$-continuous then $g \circ f : (X,\tau) \to (Z,\eta)$ is faintly continuous.*

*Proof.* Consent to T be $\theta$-open in TS Z. Since $g$ is quasi-$\theta$-continuous, $g^{-1}(T)$ is $\theta$-open in TS Y. Becausee $f$ is faintly continuous, $f^{-1}(g^{-1}(T))$ is open in TS X. Thus $(g \circ f)^{-1}(T)$ is open, for every $\theta$-open set T in TS Z. Hence $g \circ f : (X,\tau) \to (Z,\eta)$is faintly continuous. □

**Theorem 3.22.** *If $f : (X,\tau) \to (Y,\sigma)$ is a faintly $\mathscr{A}$-continuous function & X is locally indiscrete $\Longrightarrow$ f is a faintly continuous.*

*Proof.* Consent to $f$ be faintly $\mathscr{A}$-continuous. Let L be $\theta$-open in Y. Because $f$ is faintly $\mathscr{A}$-continuous, $f^{-1}(L)$ is $\mathscr{A}$-open. By Lemma 2.2, $f^{-1}(L)$ is open in X. Hence $f$ is faintly continuous. □

**Theorem 3.23.** *If $f : (X,\tau) \to (Y,\sigma)$ is any function & TS X is a $\mathscr{A}T_{\frac{1}{2}}$-space $\Longrightarrow$ f is H̋-continuous.*

*Proof.* Consent to L be each closed set in TS Y. By Lemma 2.5, each subset of X in H̋-closed. Hence $f^{-1}(L)$ should be H̋-closed, for any closed set in TS Y. □

**Theorem 3.24.** *Let a TS X, Y and Z. Then the composition $g \circ f : (X,\tau) \to (Z,\eta)$ is H̋-continuous, where $g : (Y,\sigma) \to (Z,\eta)$ is a contra $\mathscr{A}$-continuous function and $f : (X,\tau) \to (Y,\sigma)$ is H̋-irresolute.*

*Proof.* Consent to L be closed in TS Z. Because $g$ is contra $\mathscr{A}$-continuous, $g^{-1}(L)$ is $\mathscr{A}$-open in TS Y and hence$g^{-1}(L)$ is H̋-closed in TS Y. Since $f$ is H̋-irresolute, $f^{-1}(g^{-1}(L))$ is H̋-closed in TS X. That is $(g \circ f)^{-1}(L)$ is H̋-closed in TS X. Thus $g \circ f$ is H̋-continuous. □

**Theorem 3.25.** *If $f : (X,\tau) \to (Y,\sigma)$ is a surjective S̋-open function & $g : (Y,\sigma) \to (Z,\eta)$ : $g \circ f : (X,\tau) \to (Z,\eta)$ is H̋-continuous $\Longrightarrow$ g is H̋-continuous.*

*Proof.* Consent to T be every open set in TS Z. Since $g \circ f : (X,\tau) \to (Z,\eta)$ is H̋-continuous, $(g \circ f)^{-1}(T) = f^{-1}(g^{-1}(T))$ is H̋-open in TS X. Since $f : (X,\tau) \to (Y,\sigma)$ is a surjective S̋-open function, $f(f^{-1}(g^{-1}(T)))$ is H̋-open in TS Y and hence $g^{-1}(T)$ is H̋-open in TS Y. Thus $g$ is H̋-continuous. □

**Theorem 3.26.** *Consent to $f : (X,\tau) \to (Y,\sigma)$ be H̋$_{\mathscr{A}}$-continuous & $g : (Y,\sigma) \to (Z,\eta)$ is $\mathscr{A}$-irresolute $\Longrightarrow$ $g \circ f : (X,\tau) \to (Z,\eta)$ is H̋$_{\mathscr{A}}$-continuous.*

*Proof.* Consent to L be $\mathscr{A}$-open in TS Z. Since $g$ is $\mathscr{A}$-irresolute, $g^{-1}(L)$ is $\mathscr{A}$-open in TS Y. Because $f$ is H̋$_{\mathscr{A}}$-continuous, $(f^{-1} \circ g^{-1})(L) = f^{-1}(g^{-1}(L))$ is H̋-open in TS X. Thus $g \circ f$ is H̋$_{\mathscr{A}}$-continuous.

□

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