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Some results on E cordial labeling of hypercube related graphs

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Abstract

All the graphs considered in this article are finite, simple and undirected. In this paper we have proved that the hypercube graph, path union of the hypercube graphs, open star of hypercube graphs, one point union of path of hypercube graphs are E Cordial.

Keywords

E Cordial graphs, Hypercube graph, Path union of graphs, Open star of graphs, One point union of path graphs

AMS Subject Classification

05C78.

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Contents

1. Introduction

We begin with a finite, connected and undirected graph $G = (V(G), E(G))$ without loops and multiple edges. We denote the edge e with end vertices u and v by $e = uv$. For notation and theoretical terminology of any graph, we follow Balakrishnan and Ranganathan [1].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

In 1997 Cahit and Yilmaz [7] defined E-cordial labeling as a weaker version of edge- graceful labeling. They proved that K_n, C_n are E-cordial if and only if $n \not\equiv 2(\bmod 4), K_{m,n}$ admits E-cordial labeling if and only if $m+n \not\equiv 2(\bmod 4)$ and trees with n vertices are E Cordial. The brief summary of definitions and relevant results are given below.

Definition 1.1. *A mapping $f : V(G) \rightarrow \{0,1\}$ is called a binary vertex labelling of G. For an edge $e = uv$, the induced edge labelling is defined as $f^*(e = uv) = |f(u) - f(v)|$. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f. Let $e_f(0)$ and $e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*.*

A binary vertex labelling of graph G is called cordial labelling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ and a graph G is called a Cordial graph if it admits a Cordial labeling.

Definition 1.2. *Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. Let $f : E(G) \rightarrow \{0,1\}$ be defined on $V(G)$ as $f(v) = \sum\{f(uv); uv \in E(G)\}(\bmod 2)$. The function f is called E Cordial labeling of G if $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph is E Cordial if it admits E Cordial labeling.*

Definition 1.3. *Let G be a graph and $G_1, G_2, \ldots, G_n; n \geq 2$ be n copies of graph G. Then the graph obtained by adding an edge from G_i to $G_{i+1} (i = 1, 2, \ldots, n-1)$ is called path union of graph G.*

Definition 1.4. *A graph obtained by replacing each vertex of $K_{1,n}$ except the apex vertex by the graph $G_1, G_2, \ldots, G_n$ is known as an open star of graphs which is denoted by $S(G_1, G_2, \ldots, G_n)$ If we replace each vertex of $K_{1,n}$ except the apex vertex by a graph G. i.e. $G_1 = G_2 = \cdots = G_n$. Open star of graphs can be denoted by $S(n \cdot G)$.*

Definition 1.5. *A graph G is obtained by replacing each edge of $K_{1,t}$ by a path P_n of length n on $n+1$ vertices is called one point union for t copies of path P_n which is denoted by P_n^t.*

Definition 1.6. *A graph G is obtained by replacing each vertex of P_n^t except the central vertex by the graph $G_1, G_2, \ldots, G_{t_n}$ is known as one point union for path of graphs and is denoted by*

$$P_n^t\left(G_1, G_2, \ldots, G_{t_n}\right).$$

If we replace each vertex of P_n^t except the central vertex by graph H. i.e. $G_1 = G_2 = \cdots = G_n = H$, then such a one point union for path of graphs shall be denoted by $P_n^t(t_n \cdot H)$.

Definition 1.7. *Hypercube is an n-dimensional analogue of a square $(n=2)$ and a cube $(n=3)$ which is also known as an n-cube or n -dimensional cube which is denoted by Q_n. Four dimension cube Q_4 is known as a tesseract, five dimension cube Q_5 is known as a penteract, six dimension cube Q_6 is known as hexeract etc. Hypercubes are part of regular polytopes. Hypercube shapes represent compact, convex and closed figure.*

2. Main Results

Theorem 2.1. *The hypercube graph Q_n is E Cordial.*

Proof. Let $G = Q_n$ be a graph with 2^n vertices and $n2^{n-1}$ edges. If we represent a graph $G = Q_n$ on a cartesian plane then we get a binary number as coordinates i.e. $u = (k, k, \ldots, k)$; $k = 0$ or 1. If we assign a number $n(u) = i + i + \cdots + i$ to a vertex of a graph G, then the two partite sets are $V = \left\{u_1, u_2, \ldots, u_{\frac{n}{2}}\right\}$ where $n(u_p) \equiv 0(\bmod 2); 1 \leq p \leq \frac{n}{2}$ and $V' = \left\{u'_1, u'_2, \ldots, u'_{\frac{n}{2}}\right\}$ where $n\left(u'_p\right) \equiv 1(\bmod 2); 1 \leq p \leq \frac{n}{2}$
Consider $f : E \to \{0,1\}$. Define a labeling as below.
Case-I: n is odd

$$e\left(u_i u'_j\right) = 1; 1 \leq i \leq \frac{n}{4}, 1 \leq j \leq \frac{n}{2}$$
$$e\left(u_i u'_j\right) = 0; \frac{n}{4} < i \leq \frac{n}{2}, 1 \leq j \leq \frac{n}{2}.$$

Case-II: n is even

$$e\left(u_i u'_j\right) = 1; i, j \equiv 1(\bmod 2); 0 < i = j \leq \frac{n}{2}$$
$$e\left(u_i u'_j\right) = 0; i, j \equiv 0(\bmod 2); 0 < i = j \leq \frac{n}{2}$$
$$e\left(u_i u'_j\right) = 1; 1 \leq i \leq \frac{n}{4}; 1 \leq j \leq \frac{n}{2}; i \neq j, \text{ if both are even}$$
$$e\left(u_i u'_j\right) = 0; \frac{n}{4} < i \leq \frac{n}{2}; 1 \leq j \leq \frac{n}{2}; i \neq j, \text{ if both are odd}$$

Above labeling pattern give rise E Cordial labeling to Hypercube graph Q_n. □

Example 2.2. *An E Cordial labeling of Tesseract (hypercube Q_4) is shown in following figure 1.*

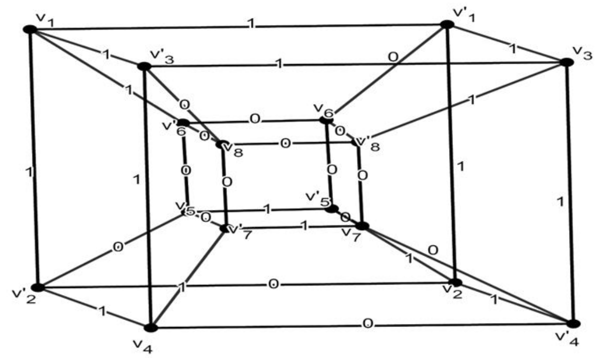

Figure 1. E-Cordial labeling of Tesseract Q_4

Theorem 2.3. *The path union of the hypercube graph Q_n is E cordial.*

Proof. Let G be a graph obtained by joining k copies of the hypercube Q_n by an edge. Let

$$V = \left\{u_{i,j,k}, u'_{i,j,k}; i, j \in [1,n], k \in \left[1, 2^{n-1}\right]\right\}$$

be the partite set of vertex set of the hypercube graph Q_n where i represents no of copy of hypercube and j represent the number of path of graph G.
To obtained a Path union of the hypercube graph Q_n, connect the vertices $u'_{i,j,2^{n-1}}$ and $u_{i+1,j,1}$ by an edge. Define the edge labeling of hypercubes and path union in graph G as below.
Edge labeling of path in G

$$e\left(u'_{i,j,2^{n-1}}, u_{i+1,j,1}\right) = \begin{cases} 1 & ; j \equiv 1(\bmod 2) \\ 0 & ; j \equiv 0(\bmod 2) \end{cases}$$

Edge labeling of hypercube in G
Let $l_1, l_2 \in k = \left\{1, 2, \ldots, 2^{n-1}\right\}$.
Case-I: For Q_n, let n be an odd natural number.
Subcase-I(a): $i \equiv 0(\bmod 2)$

$$e\left(u_{i,j,l_1}, u'_{i,j,l_2}\right) = \begin{cases} 1; 1 \leq l_1 \leq \frac{n}{4}, 1 \leq l_2 \leq \frac{n}{2} \\ 0; \frac{n}{4} < l_1 \leq \frac{n}{2}, 1 \leq l_2 \leq \frac{n}{2}. \end{cases}$$

Subcase-I (b): $i \equiv 1(\bmod 2)$

$$e\left(u_{i,j,l_1}, u'_{i,j,l_2}\right) = \begin{cases} 0; 1 \leq l_1 \leq \frac{n}{4}, 1 \leq l_2 \leq \frac{n}{2} \\ 1; \frac{n}{4} < l_1 \leq \frac{n}{2}, 1 \leq l_2 \leq \frac{n}{2}. \end{cases}$$

Case-II: For Q_n, let n be an even natural number

$$e\left(u_{ij,l_1}, u'_{i,j,l_2}\right) = \begin{cases} 1; l_1, l_2 \equiv 1(\bmod 2), 1 \leq l_1, l_2 \leq \frac{n}{2} \\ 0; l_1, l_2 \equiv 0(\bmod 2), 1 \leq l_1, l_2 \leq \frac{n}{2} \end{cases}$$

$$e\left(u_{ij,l_1}, u'_{i,j,l_2}\right) = \begin{cases} 1; 1 \leq l_1 \leq \frac{n}{4}; 1 \leq l_2 \leq \frac{n}{2}; l_1 \neq l_2 \text{ for both even} \\ 0; \frac{n}{4} < l_1 \leq \frac{n}{2}; 1 \leq l_2 \leq \frac{n}{2}; l_1 \neq l_2 \text{ for both odd} \end{cases}$$

Above labeling pattern give raise E Cordial labeling to the path union of hypercube graphs. □

Example 2.4. *Path union of three copies of hypercube Q_3 and its E Cordial labeling shown in the following figure 2.*

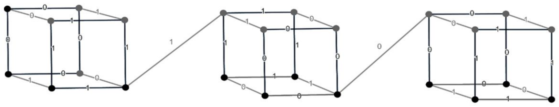

Figure 2. Path union of three copies of Q_3

Theorem 2.5. *Open star of graphs $S(t, Q_n)$ is E Cordial.*

Proof. Let G be a graph obtained by replacing each vertex of $K_{1,t}$ except the apex vertex by the graph Q_n. Let u_0 be the apex vertex of $K_{1,t}$ i.e. it is the central vertex of the graph G.

Let $V = \left\{u_{i,j}, u'_{i,j}; i \in [1,n], j \in \left[1, 2^{n-1}\right]\right\}$ be the partite set of vertex set of the hypercube graph Q_n where i represents the no of branch of $K_{1,t}$; and j represents the vertices of the graph G. Define the edge labeling of branch of $K_{1,n}$ in G as below.

$$e(u_0, u_{i,1}) = \begin{cases} 0; i \in \{4k/4k-3; k = 1,2,\ldots,t\} \\ 1; i \in \{4k-1/4k-2; n = 1,2,\ldots,t\}. \end{cases}$$

Define the Edge Labeling of the hypercube Q_n in G as below. Let $l_1, l_2 \in j = \left\{1, 2, \ldots, 2^{n-1}\right\}$.

Case-I: For Q_n, let n be an odd natural number.

Subcase-I(a): $i \equiv 1(\bmod 2)$ (i.e. labeling of Q_n which are connect with (consider the fragment) odd number of branch of $K_{1,n}$ in G.)

$$e\left(u_{i,l_1} u'_{i,l_2}\right) = \begin{cases} 0; 1 \leq l_1 \leq \frac{n}{4}, 1 \leq l_2 \leq \frac{n}{2} \\ 1; \frac{n}{4} < l_1 \leq \frac{n}{2}, 1 \leq l_2 \leq \frac{n}{2}. \end{cases}$$

Subcase-I(b): $i \equiv 0(\bmod 2)$ (i.e. labeling of Q_n which are connect with odd number of branch of $K_{1,n}$ in G.)

$$e\left(u_{i,l_1} u'_{i,l_2}\right) = \begin{cases} 1; 1 \leq l_1 \leq \frac{n}{4}, 1 \leq l_2 \leq \frac{n}{2} \\ 0; \frac{n}{4} < l_1 \leq \frac{n}{2}, 1 \leq l_2 \leq \frac{n}{2}. \end{cases}$$

Case-II: For Q_n, let n be an even natural number.

$$e\left(u_{i,l_1} u'_{i,l_2}\right) = \begin{cases} 1; l_1, l_2 \equiv 1(\bmod 2), 1 \leq l_1, l_2 \leq \frac{n}{2} \\ 0; l_1, l_2 \equiv 0(\bmod 2), 1 \leq l_1, l_2 \leq \frac{n}{2}. \end{cases}$$

Subcase-II(a): $i \equiv 1(\bmod 2)$ (i.e. labeling of Q_n which are connect with odd number of branch of $K_{1,n}$ in G.)

$$e\left(u_{i,l_1} u'_{i,l_2}\right) = \begin{cases} 1; 1 \leq l_1 \leq \frac{n}{4}; 1 \leq l_2 \leq \frac{n}{2}; l_1 \neq l_2 \text{ for both even} \\ 0; \frac{n}{4} < l_1 \leq \frac{n}{2}; 1 \leq l_2 \leq \frac{n}{2}; l_1 \neq l_2 \text{ for both odd}. \end{cases}$$

Subcase-II(b): $i \equiv 0(\bmod 2)$ (i.e. labeling of Q_n which are connect with odd number of branch of $K_{1,n}$ in G)

$$e\left(u_{i,l_1} u'_{i,l_2}\right) = \begin{cases} 0; 1 \leq l_1 \leq \frac{n}{4}; 1 \leq l_2 \leq \frac{n}{2}; l_1 \neq l_2 \text{ for both even} \\ 1; \frac{n}{4} < l_1 \leq \frac{n}{2}; 1 \leq l_2 \leq n/2; l_1 \neq l_2 \text{ for both odd}. \end{cases}$$

Above labeling pattern give rise E Cordial labeling to Open star of hypercube graphs $S(t, Q_n)$. □

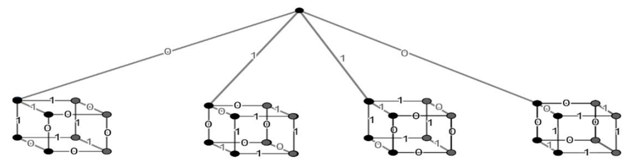

Figure 3. E Cordial labeling of Open star of graphs $S(4, Q_3)$

Example 2.6. *Open star of graphs $S(4, Q_3)$ and E Cordial labeling shown in the following figure 3.*

Theorem 2.7. *$P_n^t(t_n \cdot Q_n)$ is an E cordial graph.*

Proof. Let G be a graph obtained by replacing each vertex of P_n^t by the graph Q_n. Let the vertex set of graphs G be

$$V = \left\{u_0, u_{i,j,k,l}, u'_{i,j,k,l}\right\}$$

where u_0 is the apex vertex of graph G. Let $u_{i,j,k,l}$ and $u'_{i,j,k,l}$ be the partite sets of the hypercube graph Q_n where i represents the branch of the graph G, j represents the number of copy of path in path union, k represents number of copy of Q_n in path union and l is number of vertices in $Q_n, \forall i = 1,2,3,\ldots,t, \forall j = 1,2,3,\ldots,p, \forall k = 1,2,\ldots,q, 0 < l \leq 2^{n-1}, t,p,q \in N$. Here note that $k = j+1$.

To make a path union of Q_n, connect the vertices $u'_{i,j,k,2^{n-1}}$ and $u_{i,j,k+1,1}$ by an edge as mentioned below.

Case-I: If $i \in \{4n/4n-3; n = 1,2,\ldots,t\}$

$$e\left(u'_{i,j,k,2^{n-1}} u_{i,j,k+1,1}\right) = \begin{cases} 1; j \equiv 1(\bmod 2) \\ 0; j \equiv 0(\bmod 2). \end{cases}$$

Case-II: If $i \in \{4n-1/4n-2; n = 1,2,\ldots,t\}$

$$e\left(u'_{i,j,k,2^{n-1}} u_{i,j,k+1,1}\right) = \begin{cases} 0; j \equiv 1(\bmod 2) \\ 1; j \equiv 0(\bmod 2). \end{cases}$$

In order to create a one point union of path of graph Q_n, connect the apex vertex u_0 and $u_{i,j,1,1}$ as mentioned below

$$e\left(u_0 u_{i,j,1,1}\right) = \begin{cases} 1 \quad ; i \in \{4n-1/4n-2; n = 1,2,\ldots,t\} \\ 0 \quad ; i \in \{4n/4n-3; n = 1,2,\ldots,t\} \end{cases}$$

For the hypercube graph Q_n, of a path union of graph, define an edge labeling as below.

Case-I: For Q_n, let n be an odd natural number.

Subcase-I(a): If $k \equiv 1(\bmod 2)$ and $l_1, l_2 \in l = \left\{1, 2, \ldots, 2^{n-1}\right\}$

$$e\left(u_{i,j,k,l_1} u'_{i,j,k,l_2}\right) = \begin{cases} 0 \quad ; 1 \leq l_1 \leq \frac{n}{4}; 1 \leq l_2 \leq \frac{n}{2} \\ 1 \quad ; \frac{n}{4} < l_1 \leq \frac{n}{2}; 1 \leq l_2 \leq n/2 \end{cases}$$

Subcase-I(b): If $k \equiv 0(\bmod 2)$ and $l_1, l_2 \in l = \left\{1, 2, \ldots, 2^{n-1}\right\}$

$$e\left(u_{i,j,k,l_1} u'_{i,j,k,l_2}\right) = \begin{cases} 1 \quad ; 1 \leq l_1 \leq \frac{n}{4}; 1 \leq l_2 \leq \frac{n}{2} \\ 0 \quad ; \frac{n}{4} < l_1 \leq \frac{n}{2}; 1 \leq l_2 \leq n/2 \end{cases}$$

Case-II: For Q_n, let n be an even natural number

$$e\left(u_{i,j,k,l_1} u'_{i,j,k,l_2}\right) = \begin{cases} 1; l_1, l_2 \equiv 1(\bmod 2), 1 \leq l_1, l_2 \leq \frac{n}{2} \\ 0; l_1, l_2 \equiv 0(\bmod 2), 1 \leq l_1, l_2 \leq \frac{n}{2}. \end{cases}$$

Subcase-II(a): If $k \equiv 1(\bmod 2)$ and $l_1, l_2 \in l = \{1, 2, \ldots, 2^{n-1}\}$

$$e\left(u_{i,j,k,l_1} u'_{i,j,k,l_2}\right) = \begin{cases} 1; 1 \le l_1 \le \frac{n}{4}; 1 \le l_2 \le \frac{n}{2}; l_1 \ne l_2 \text{ for both even} \\ 0; \frac{n}{4} < l_1 \le \frac{n}{2}; 1 \le l_2 \le \frac{n}{2}; l_1 \ne l_2 \text{ for both odd} \end{cases}$$

Subcase-II(b): If $k \equiv 0(\bmod 2)$ and $l_1, l_2 \in l = \{1, 2, \ldots, 2^{n-1}\}$

$$e\left(u_{i,j,k,l_1} u'_{i,j,k,l_2}\right) = \begin{cases} 0; 1 \le l_1 \le \frac{n}{4}; 1 \le l_2 \le \frac{n}{2}; l_1 \ne l_2 \text{ for both even} \\ 1; \frac{n}{4} < l_1 \le \frac{n}{2}; 1 \le l_2 \le n/2; l_1 \ne l_2 \text{ for both odd} \end{cases}$$

Above labeling pattern give rise E Cordial labeling to One point union for path of graphs $P_n^t(t_n, Q_n)$. □

Example 2.8. *An E Cordial labeling of $P_3^3(3_3, Q_3)$ is shown in following figure 4.*

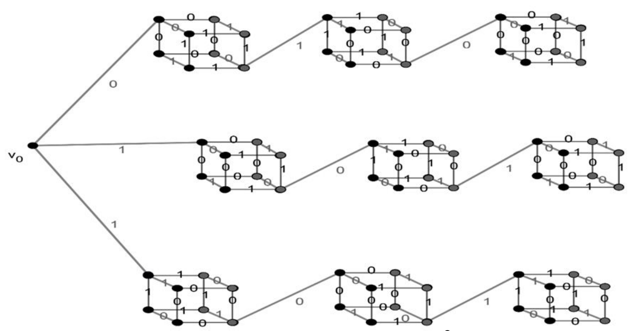

Figure 4. E Cordial labeling of $P_3^3(3_3, Q_3)$

3. Conclusion

In this paper we discussed E Cordial labeling of n-dimensional Cube (hypercube) and its related graph. We also proved Path union of the hypercube graphs, Open star of the hypercube graphs and One point for path of graphs are E Cordial. Labeling pattern is shown in illustration. Labeling on hypercube is very useful in coding decoding theory, so using these technique we can generate bigger encrypted data with easiest way. Labeling on hypercube is emerging concept and will be useful in so many areas because it includes n dimension.

4. Further Scope of Research

The study of different labeling technique on similar graph of families including graceful labeling, harmonious labeling, magic labeling and mean labeling are open area of research.

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