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Fundamental form IV and curvature formulas of the hypersphere

Erhan Güler[1]*

Abstract
We study curvature formulas and the fourth fundamental form IV of hypersurfaces in the four dimensional Euclidean geometry $\mathbb{E}^4$. We calculate fourth fundamental form and curvatures for hypersurfaces, and also for hypersphere. Moreover, we give some relations of fundamentals forms, and curvatures of hypersphere.

Keywords
Euclidean spaces, four space, hypersurface, hypersphere, curvature, fourth fundamental form.

AMS Subject Classification
53B25, 53C40.

[1] *Bartın University, Faculty of Sciences, Department of Mathematics 74100 Bartın, Turkey.*
***Corresponding author**: [1] eguler@bartin.edu.tr; **ORCID**: https://orcid.org/0000-0003-3264-6239
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Contents

1. Introduction

Surfaces and hypersurfaces have been studied by mathematicians for years such as [1]-[37].

In this paper, we consider fourth fundamental form $IV = f_{ij}$, and i-th curvature formulas $\mathfrak{C}_i$ of hypersphere in the four dimensional Euclidean geometry $\mathbb{E}^4$. In Section 2, we give some basic notions of the four dimensional Euclidean geometry. Defining fourth fundamental form and i-th curvature for hypersurfaces, we calculate $\mathfrak{C}_i$ and fourth fundamental form of hypersphere in Section 3.

Let $\mathbb{E}^m$ denote the Euclidean m-space with the canonical Euclidean metric tensor given by $\widetilde{g} = \langle\ ,\ \rangle = \sum_{i=1}^{m} dx_i^2$, where $(x_1, x_2, \ldots, x_m)$ is a rectangular coordinate system in $\mathbb{E}^m$. Consider an m-dimensional Riemannian submanifold of the space $\mathbb{E}^m$. We denote the Levi-Civita connections of $\mathbb{E}^m$ and M by $\widetilde{\nabla}$ and ∇, respectively. We use letters X, Y, Z, W (resp., ξ, η) to denote vectors fields tangent (resp., normal) to M. The Gauss and Weingarten formulas are given, respectively, by

$$\widetilde{\nabla}_X Y = \nabla_X Y + h(X,Y), \tag{1.1}$$

$$\widetilde{\nabla}_X \xi = -A_\xi(X) + D_X \xi, \tag{1.2}$$

where h, D and A are the second fundamental form, the normal connection and the shape operator of M, respectively. For each $\xi \in T_p^\perp M$, the shape operator A_ξ is a symmetric endomorphism of the tangent space T_pM at $p \in M$. The shape operator and the second fundamental form are related by

$$\langle h(X,Y), \xi \rangle = \langle A_\xi X, Y \rangle.$$

The Gauss and Codazzi equations are given, respectively, by

$$\langle R(X,Y,)Z,W \rangle = \left\{ \begin{array}{c} \langle h(Y,Z), h(X,W) \rangle \\ -\langle h(X,Z), h(Y,W) \rangle \end{array} \right\}, \tag{1.3}$$

$$(\bar{\nabla}_X h)(Y,Z) = (\bar{\nabla}_Y h)(X,Z), \tag{1.4}$$

where R, R^D are the curvature tensors associated with connections ∇ and D, respectively, and $\bar{\nabla} h$ is defined by

$$(\bar{\nabla}_X h)(Y,Z) = D_X h(Y,Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z).$$

1.1 Hypersurfaces of Euclidean space

Now, let M be an oriented hypersurface in the Euclidean space $\mathbb{E}^{n+1}$, $\mathbf{S}$ its shape operator (i.e. Weingarten map) and x its position vector. We consider a local orthonormal frame field $\{e_1, e_2, \ldots, e_n\}$ of consisting of principal directions of M

corresponding from the principal curvature k_i for $i = 1, 2, \ldots n$. Let the dual basis of this frame field be $\{\theta_1, \theta_2, \ldots, \theta_n\}$. Then the first structural equation of Cartan is

$$d\theta_i = \sum_{i=1}^{n} \theta_j \wedge \omega_{ij}, \quad i, j = 1, 2, \ldots, n, \tag{1.5}$$

where ω_{ij} denotes the connection forms corresponding to the chosen frame field. We denote the Levi-Civita connection of M and $\mathbb{E}^{n+1}$ by ∇ and $\widetilde{\nabla}$, respectively. Then, from the Codazzi equation (1.3), we have

$$e_i(k_j) = \omega_{ij}(e_j)(k_i - k_j), \tag{1.6}$$
$$\omega_{ij}(e_l)(k_i - k_j) = \omega_{il}(e_j)(k_i - k_l) \tag{1.7}$$

for distinct $i, j, l = 1, 2, \ldots, n$.

We put $s_j = \sigma_j(k_1, k_2, \ldots, k_n)$, where σ_j is the j-th elementary symmetric function given by

$$\sigma_j(a_1, a_2, \ldots, a_n) = \sum_{1 \le i_1 < i_2 < \ldots, i_j \le n} a_{i_1} a_{i_2} \ldots a_{i_j}.$$

We use following notation

$$r_i^j = \sigma_j(k_1, k_2, \ldots, k_{i-1}, k_{i+1}, k_{i+2}, \ldots, k_n).$$

By the definition, we have $r_i^0 = 1$ and $s_{n+1} = s_{n+2} = \cdots = 0$. We call the function s_k as the k-th mean curvature of M. We would like to note that functions $H = \frac{1}{n} s_1$ and $K = s_n$ are called the mean curvature and Gauss-Kronecker curvature of M, respectively. In particular, M is said to be j-minimal if $s_j \equiv 0$ on M.

In $\mathbb{E}^{n+1}$, to find the i-th curvature formulas $\mathfrak{C}_i$ (Curvature formulas sometimes are shown as mean curvature H_i, or sometimes as Gaussian curvature K_i by different writers, such as [1] and [30]. We call it just i-th curvature $\mathfrak{C}_i$ in this paper.), where $i = 0, .., n$, firstly, we use the characteristic polynomial of **S:**

$$P_{\mathbf{S}}(\lambda) = 0 = \det(\mathbf{S} - \lambda I_n) = \sum_{k=0}^{n} (-1)^k s_k \lambda^{n-k}, \tag{1.8}$$

where $i = 0, .., n$, I_n denotes the identity matrix of order n. Then, we get curvature formulas $\binom{n}{i}\mathfrak{C}_i = s_i$,. That is, $\binom{n}{0}\mathfrak{C}_0 = s_0 = 1$ (by definition), $\binom{n}{1}\mathfrak{C}_1 = s_1, \ldots, \binom{n}{n}\mathfrak{C}_n = s_n = K$.

k-th fundamental form of M is defined by $I\left(\mathbf{S}^{k-1}(X), Y\right) = \left\langle \mathbf{S}^{k-1}(X), Y \right\rangle$. So, we get

$$\sum_{i=0}^{n} (-1)^i \binom{n}{i} \mathfrak{C}_i I\left(\mathbf{S}^{n-i}(X), Y\right) = 0. \tag{1.9}$$

1.2 Hypersphere

We will obtain a hypersphere in Euclidean 4-space. We would like to note that the definition of rotational hypersurfaces in Riemannian space forms were defined in [17]. A rotational hypersurface $M \subset \mathbb{E}^{n+1}$ generated by a curve $\mathscr{C}$ around an axis $\mathscr{C}$ that does not meet $\mathscr{C}$ is obtained by taking the orbit of $\mathscr{C}$ under those orthogonal transformations of $\mathbb{E}^{n+1}$ that leaves $\mathfrak{r}$ pointwise fixed (See [17, Remark 2.3]).

We shall identify a vector (a, b, c, d) with its transpose. Consider the case $n = 3$, and let $\mathscr{C}$ be the curve parametrized by

$$\gamma(w) = (r\cos w, 0, 0, r\sin w), \tag{1.10}$$

where $r \in \mathbb{R} - \{0\}$, $w \in [0, 2\pi]$. If $\mathfrak{r}$ is the x_4-axis, then an orthogonal transformations of $\mathbb{E}^{n+1}$ that leaves $\mathfrak{r}$ pointwise fixed has the form

$$\mathbf{O}(u, v) = \begin{pmatrix} \cos u \cos v & -\sin u & -\cos u \sin v & 0 \\ \sin u \cos v & \cos u & -\sin u \sin v & 0 \\ \sin v & 0 & \cos v & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

where $u, v \in \mathbb{R}$. Therefore, the parametrization of the hypersphere generated by a curve $\mathscr{C}$ around an axis $\mathfrak{r}$ is given by

$$\mathbf{x}(u, v, w) = \mathbf{O}(u, v)\gamma(w). \tag{1.11}$$

Let $\mathbf{x} = \mathbf{x}(u, v, w)$ be an isometric immersion from $M^3 \subset \mathbb{E}^3$ to $\mathbb{E}^4$. Triple vector product of $\overrightarrow{x} = (x_1, x_2, x_3, x_4)$, $\overrightarrow{y} = (y_1, y_2, y_3, y_4)$, $\overrightarrow{z} = (z_1, z_2, z_3, z_4)$ of $\mathbb{E}^4$ is given by

$$\overrightarrow{x} \times \overrightarrow{y} \times \overrightarrow{z} = \begin{pmatrix} e_1 & e_2 & e_3 & e_4 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{pmatrix}.$$

For a hypersurface $\mathbf{x}$ in $\mathbb{E}^4$, we get the fundamental form matrices

$$I = \begin{pmatrix} E & F & A \\ F & G & B \\ A & B & C \end{pmatrix}, \quad II = \begin{pmatrix} L & M & P \\ M & N & T \\ P & T & V \end{pmatrix},$$

$$III = \begin{pmatrix} X & Y & O \\ Y & Z & S \\ O & S & U \end{pmatrix}.$$

Then we have

$$\det I = (EG - F^2)C - EB^2 + 2FAB - GA^2,$$
$$\det II = \left(LN - M^2\right)V - LT^2 + 2MPT - NP^2,$$
$$\det III = \left(XZ - Y^2\right)U - ZO^2 + 2OSY - XS^2,$$

where $E = \langle \mathbf{x}_u, \mathbf{x}_u \rangle$, $F = \langle \mathbf{x}_u, \mathbf{x}_v \rangle$, $G = \langle \mathbf{x}_v, \mathbf{x}_v \rangle$, $A = \langle \mathbf{x}_u, \mathbf{x}_w \rangle$, $B = \langle \mathbf{x}_v, \mathbf{x}_w \rangle$, $C = \langle \mathbf{x}_w, \mathbf{x}_w \rangle$, $L = \langle \mathbf{x}_{uu}, \mathbf{G} \rangle$, $M = \langle \mathbf{x}_{uv}, \mathbf{G} \rangle$, $N = \langle \mathbf{x}_{vv}, \mathbf{G} \rangle$, $P = \langle \mathbf{x}_{uw}, \mathbf{G} \rangle$, $T = \langle \mathbf{x}_{vw}, \mathbf{G} \rangle$, $V = \langle \mathbf{x}_{ww}, \mathbf{G} \rangle$, $X = \langle \mathbf{G}_u, \mathbf{G}_u \rangle$, $Y = \langle \mathbf{G}_u, \mathbf{G}_v \rangle$, $Z = \langle \mathbf{G}_v, \mathbf{G}_v \rangle$, $O = \langle \mathbf{G}_u, \mathbf{G}_w \rangle$, $S = \langle \mathbf{G}_v, \mathbf{G}_w \rangle$, $U = \langle \mathbf{G}_w, \mathbf{G}_w \rangle$. Here,

$$\mathbf{G} = \frac{\mathbf{x}_u \times \mathbf{x}_v \times \mathbf{x}_w}{\|\mathbf{x}_u \times \mathbf{x}_v \times \mathbf{x}_w\|} \tag{1.12}$$

is unit normal (i.e. the Gauss map) of hypersurface $\mathbf{x}$. On the other side, $I^{-1} \cdot II$ gives shape operator matrix $\mathbf{S}$ of hypersurface $\mathbf{x}$ in 4-space. See [24–26] for details.

2. Curvatures and the Fourth Fundamental Form

Using characteristic polynomial $P_{\mathbf{S}}(\lambda)=a\lambda^3+b\lambda^2+c\lambda+d=0$, i.e.

$$P_{\mathbf{S}}(\lambda)=\det(\mathbf{S}-\lambda I_3)=0,$$

we obtain curvature formulas $\mathfrak{C}_0=1$ (by definition), $\binom{3}{1}\mathfrak{C}_1=\binom{3}{1}H=-\frac{b}{a}$, $\binom{3}{2}\mathfrak{C}_2=\frac{c}{a}$, $\binom{3}{3}\mathfrak{C}_3=K=-\frac{d}{a}$.

Therefore, we get curvature folmulas depends on the coefficients of I and II fundamental forms in 4-space (It also can get depends on the coefficients of II and III, or III and IV):

Theorem 1. *Any hypersurface* $\mathbf{x}$ *in* $\mathbb{E}^4$ *has following curvature formulas,* $\mathfrak{C}_0=1$ *(by definition),*

$$\mathfrak{C}_1=\frac{\left\{\begin{array}{c}(EN+GL-2FM)C\\+(EG-F^2)V-LB^2-NA^2\\-2(APG-BPF-ATF\\+BTE-ABM)\end{array}\right\}}{3\left[(EG-F^2)C-EB^2+2FAB-GA^2\right]},\tag{2.1}$$

$$\mathfrak{C}_2=\frac{\left\{\begin{array}{c}(EN+GL-2FM)V\\+\left(LN-M^2\right)C-ET^2-GP^2\\-2(APN-BPM-ATM\\+BTL-PTF)\end{array}\right\}}{3\left[(EG-F^2)C-EB^2+2FAB-GA^2\right]},\tag{2.2}$$

$$\mathfrak{C}_3=\frac{\left(LN-M^2\right)V-LT^2+2MPT-NP^2}{(EG-F^2)C-EB^2+2FAB-GA^2}.\tag{2.3}$$

Proof. Solving $\det(\mathbf{S}-\lambda I_3)=0$ with some calculations, we get coefficients of polynomial $P_{\mathbf{S}}(\lambda)$.

Corollary 1. For *any hypersurface* $\mathbf{x}$ *in* $\mathbb{E}^4$*, the fourth fundamental form is related by*

$$\mathfrak{C}_0IV-3\mathfrak{C}_1III+3\mathfrak{C}_2II-\mathfrak{C}_3I=0.\tag{2.4}$$

Proof. Taking $n=3$ in (1.9), it is clear.

Definition 1. *With its shape operator* $\mathbf{S}$ *and the first fundamental form* $(g_{ij})=I$ *of any hypersurface* $\mathbf{x}$ *in 4-space, following relations holds:*

(a) *the second fundamental form* $(h_{ij})=II$ *is given by* $II=I{\cdot}\mathbf{S}$,

(b) *the third fundamental form* $(e_{ij})=III$ *is given by* $III=II{\cdot}\mathbf{S}$,

(c) *the fourth fundamental form* $(f_{ij})=IV$ *is given by* $IV=III{\cdot}\,\mathbf{S}$.

Corollary 2. *The fourth fundamental form of any hypersurface* $\mathbf{x}$ *in* $\mathbb{E}^4$ *is given by*

$$IV=III{\cdot}I^{-1}{\cdot}II.$$

Proof. From Definition 1**,** we see the result.

Corollary 3. *For any hypersurface* $\mathbf{x}$ *in* $\mathbb{E}^4$*, we have*

$$\det IV=\frac{\det II.\det III}{\det I}.$$

Proof. Computing the right side of $IV=III{\cdot}I^{-1}{\cdot}II$, it is seen, easily.

3. Fundamental Forms and Curvatures of Hypersphere

We consider hypersphere (1.11), that is

$$\mathbf{x}(u,v,w)=\left(\begin{array}{c}r\cos u\cos v\cos w\\r\sin u\cos v\cos w\\r\sin v\cos w\\r\sin w\end{array}\right),\tag{3.1}$$

where $r\in\mathbb{R}-\{0\}$ and $u,v\in\mathbb{R}$, $0\leq w\leq 2\pi$. Using the first differentials of hypersphere (3.1), we get the first quantities

$$I=\operatorname{diag}\left(r^2\cos^2v\cos^2w,r^2\cos^2w,r^2\right).\tag{3.2}$$

The Gauss map of the hypersphere is

$$\mathbf{G}=\left(\begin{array}{c}\cos u\cos v\cos w\\\sin u\cos v\cos w\\\sin v\cos w\\\sin w\end{array}\right).\tag{3.3}$$

Using the second differentials and $\mathbf{G}$ of hypersphere (3.1), we have the second quantities

$$II=\operatorname{diag}\left(-r\cos^2v\cos^2w,-r\cos^2w,-r\right).\tag{3.4}$$

With the first differentials of (3.3), we find the third fundamental form matrix

$$III=\operatorname{diag}\left(\cos^2v\cos^2w,\cos^2w,1\right).\tag{3.5}$$

We calculate $I^{-1}.II$, then obtain shape operator matrix

$$\mathbf{S}=-\frac{1}{r}I_3,\tag{3.6}$$

where $I_3=$diag$(1,1,1)$. Therefore, we have following theorem.

Theorem 2. *Hypersphere* (3.1) *has following curvatures*

$$\mathfrak{C}_1=-\frac{1}{r},\ \mathfrak{C}_2=\frac{1}{r^2},\ \mathfrak{C}_3=-\frac{1}{r^3}.$$

Proof. Using (2.1),(2.2),(2.3), (3.2), (3.4), (3.5) of (3.1), we have curvatures.

Next, we see some results of the fourth fundamental form of (3.1).

Corollary 4. *The fourth fundamental form matrix* (f_{ij}) *of hypersphere* (3.1) *is as follows*

$$IV=\operatorname{diag}\left(-\frac{1}{r}\cos^2v\cos^2w,-\frac{1}{r}\cos^2w,-\frac{1}{r}\right).\tag{3.7}$$

Proof. Using Corollary 2 with hypersphere (3.1), we find the fourth fundamental form matrix.

Corollary 5. *Hypersphere* (3.1) *has following relations*

$$I=r^2III,\ \ II=-rIII,\ IV=-\frac{1}{r}III.$$

References

[1] Alias, L.J., Gürbüz, N.: An extension of Takashi theorem for the linearized operators of the highest order mean curvatures, Geom. Dedicata 121, 113–127 (2006).

[2] Arslan, K., Bayram, B.K., Bulca, B., Kim, Y.H., Murathan, C., Öztürk, G.: Vranceanu surface in $\mathbb{E}^4$ with pointwise 1-type Gauss map. Indian J. Pure Appl. Math. 42(1), 41–51 (2011).

[3] Arslan, K., Milousheva, V.: Meridian surfaces of elliptic or hyperbolic type with pointwise 1-type Gauss map in Minkowski 4-space. Taiwanese J. Math. 20(2) 311–332 (2016).

[4] Arvanitoyeorgos, A., Kaimakamis, G., Magid, M.: Lorentz hypersurfaces in $\mathbb{E}^4_1$ satisfying $\Delta H = \alpha H$. Illinois J. Math. 53(2), 581–590 (2009).

[5] Barros, M., Chen, B.Y.: Stationary 2-type surfaces in a hypersphere. J. Math. Soc. Japan 39(4), 627–648 (1987).

[6] Barros, M., Garay, O.J.: 2-type surfaces in S^3. Geom. Dedicata 24(3), 329–336 (1987).

[7] Bektaş, B.; Canfes, E.Ö; Dursun, U.: Classification of surfaces in a pseudo-sphere with 2-type pseudo-spherical Gauss map. Math. Nachr. 290(16), 2512–2523 (2017).

[8] Chen, B.Y.: On submanifolds of finite type. Soochow J. Math. 9, 65–81 (1983).

[9] Chen, B.Y.: Total mean curvature and submanifolds of finite type. World Scientific, Singapore (1984).

[10] Chen, B.Y.: Finite type submanifolds and generalizations. University of Rome, 1985.

[11] Chen, B.Y.: Finite type submanifolds in pseudo-Euclidean spaces and applications. Kodai Math. J. 8(3), 358–374 (1985).

[12] Chen, B.Y., Piccinni, P.: Submanifolds with finite type Gauss map. Bull. Austral. Math. Soc. 35, 161–186 (1987).

[13] Cheng, Q.M., Wan, Q.R.: Complete hypersurfaces of $\mathbb{R}^4$ with constant mean curvature. Monatsh. Math. 118, 171–204 (1994).

[14] Cheng, S.Y., Yau, S.T.: Hypersurfaces with constant scalar curvature. Math. Ann. 225, 195–204 (1977).

[15] Choi, M., Kim, Y.H.: Characterization of the helicoid as ruled surfaces with pointwise 1-type Gauss map. Bull. Korean Math. Soc. 38 , 753–761 (2001).

[16] Dillen, F., Pas, J., Verstraelen, L.: On surfaces of finite type in Euclidean 3-space. Kodai Math. J. 13, 10–21 (1990).

[17] Do Carmo, M., Dajczer, M.: Rotation Hypersurfaces in Spaces of Constant Curvature. Trans. Amer. Math. Soc. 277, 685–709 (1983).

[18] Dursun, U.: Hypersurfaces with pointwise 1-type Gauss map. Taiwanese J. Math. 11(5), 1407–1416 (2007).

[19] Dursun, U., Turgay, N.C.: Space-like surfaces in Minkowski space $\mathbb{E}^4_1$ with pointwise 1-type Gauss map. Ukrainian Math. J. 71(1), 64–80 (2019).

[20] Ferrandez, A., Garay, O.J., Lucas, P.: On a certain class of conformally at Euclidean hypersurfaces. In Global Analysis and Global Differential Geometry; Springer: Berlin, Germany 48–54 (1990).

[21] Ganchev, G., Milousheva, V.: General rotational surfaces in the 4-dimensional Minkowski space. Turkish J. Math. 38, 883–895 (2014).

[22] Garay, O.J.: On a certain class of finite type surfaces of revolution. Kodai Math. J. 11, 25–31 (1988).

[23] Garay, O.: An extension of Takahashi's theorem. Geom. Dedicata 34, 105–112 (1990).

[24] Güler, E., Hacısalihoğlu, H.H., Kim, Y.H.: The Gauss map and the third Laplace-Beltrami operator of the rotational hypersurface in 4-space. Symmetry 10(9), 1–12 (2018).

[25] Güler, E., Magid, M., Yaylı, Y.: Laplace-Beltrami operator of a helicoidal hypersurface in four-space. J. Geom. Symm. Phys. 41, 77–95 (2016).

[26] Güler, E., Turgay, N.C.: Cheng-Yau operator and Gauss map of rotational hypersurfaces in 4-space. Mediterr. J. Math. 16(3), 1–16 (2019).

[27] Hasanis, Th., Vlachos, Th.: Hypersurfaces in $\mathbb{E}^4$ with harmonic mean curvature vector field. Math. Nachr. 172, 145–169 (1995).

[28] Kim, D.S., Kim, J.R., Kim, Y.H.: Cheng-Yau operator and Gauss map of surfaces of revolution. Bull. Malays. Math. Sci. Soc. 39(4), 1319–1327 (2016).

[29] Kim, Y.H., Turgay, N.C.: Surfaces in $\mathbb{E}^4$ with L_1-pointwise 1-type Gauss map. Bull. Korean Math. Soc. 50(3), 935–949 (2013).

[30] Kühnel, W.: Differential geometry. Curves-surfaces-manifolds. Third ed. Translated from the 2013 German ed. AMS, Providence, RI, 2015.

[31] Levi-Civita, T.: Famiglie di superficie isoparametriche nellordinario spacio euclideo. Rend. Acad. Lincei 26, 355–362 (1937).

[32] Moore, C.: Surfaces of rotation in a space of four dimensions. Ann. Math. 21, 81–93 (1919).

[33] Moore, C.: Rotation surfaces of constant curvature in space of four dimensions. Bull. Amer. Math. Soc. 26, 454–460 (1920).

[34] Senoussi, B., Bekkar, M.: Helicoidal surfaces with $\Delta^J r = Ar$ in 3-dimensional Euclidean space. Stud. Univ. Babeş-Bolyai Math. 60(3), 437–448 (2015).

[35] Stamatakis, S., Zoubi, H.: Surfaces of revolution satisfying $\Delta^{III} x = Ax$. J. Geom. Graph. 14(2), 181–186 (2010).

[36] Takahashi, T.: Minimal immersions of Riemannian manifolds. J. Math. Soc. Japan 18, 380–385 (1966).

[37] Turgay, N.C.: Some classifications of Lorentzian surfaces with finite type Gauss map in the Minkowski 4-space. J. Aust. Math. Soc. 99(3), 415–427 (2015).

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