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r_c-operator on topological spaces

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Abstract
In this paper a new operator called r_c-operator on topological spaces is introduced. Conditions for the operator to be an expansive,shrinking and invariant operator is determined. It is also shown that regular closed sets are fixed points of this operator.

Keywords
r_c-operator, Closure, Interior, regular closed sets.

AMS Subject Classification
54C10.

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Contents

1. Introduction

M.H. Stone introduced the concept of regular open set[4] in 1937. R.C.Jain[1] in 1980, worked on regularly open sets in Topology on his thesis. In this paper an attempt is done to find an operator for which complement of regular open set called regular closed set is a fixed point. In section 2 , preliminary ideas are given. In section 3, r_c-operator is defined. Section 4 , discusses about properties of r_c-operator and finds its fixed points.

2. Preliminary Ideas

Let (X, τ) be a topological space. (X, τ) is abbreviated as X . For a set A, $\overline{A}$ denotes the closure of A and A° denotes its interior.

2.1 Definition[4]

A subset A of X is

(i.) regular open, if $A = \overline{A}^\circ$.

(ii.) regular closed, if $A = \overline{A^\circ}$.

2.2 Properties of regular closed sets

(i.) Every regular closed set is closed.

(ii.) If A and B are regular closed sets, then $A \cup B$ is regular closed.

(iii.) If A and B are regular closed sets, then $A \cap B$ need not be regular closed.

3. r_c-operator

Definition 3.1. *Let (X, τ) be a topological space.The operator r_c defined on $P(X)$ by $r_c(A) = \overline{A^\circ}$ is known as r_c-operator.*

Example 3.2. *Let $X = \{a,b,c\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$.*
Then $r_c(\{a\}) = \{a,c\}$,$r_c(\{b\}) = \{b,c\}$,$r_c(\{a,b\}) = X$, $r_c(\{c\}) = \phi$
$r_c(\{b,c\}) = \{b,c\}$

Example 3.3. *Consider (R, τ), where R is the set of real numbers and τ is the usual topology. Then,*

1. *$r_c(\{(a,b)\}) = [a,b]$ for any open interval (a,b) in R.*
2. *$r_c(\{[a,b]\}) = [a,b]$ for any closed interval [a,b] in R.*
3. *$r_c(\{[a,b)\}) = [a,b] = r_c(\{(a,b]\})$ for any half open intervals in R*

4. Properties of r_c-operator

Theorem 4.1.

1. *For any subset A of X,* $A^\circ \subset r_c(A)$.
2. *If A is an open set, then r_c is an expansive operator. That is* $A \subset r_c(A)$ *for any open set A.*
3. *For any subset A of X,* $r_c(A) \subseteq \overline{A}$.
4. *If A is a closed set, then r_c is a shrinking operator. That is* $r_c(A) \subseteq A$, *for any closed set A.*
5. *The operator r_c is Idempotent. That is* $r_c(r_c(A)) = r_c(A)$

Proof. 1. $A^\circ \subset \overline{A^\circ}$ by definition of Closure of a set.

$\implies A^\circ \subset r_c(A)$

2. $A^\circ \subset r_c(A)$ .... (by (1)).

A open $\implies A^\circ = A$.

Hence, $A \subset r_c(A)$.

3. $A^\circ \subseteq A$, by definition of Interior .

$\implies \overline{A^\circ} \subseteq \overline{A}$.

$\implies r_c(A) \subseteq \overline{A}$

4. $r_c(A) \subseteq \overline{A}$ .... (by (3)).

A closed $\implies \overline{A} = A$

Hence, $r_c(A) \subseteq A$.

5. $r_c(r_c(A)) = \overline{\overline{A^\circ}^\circ}$

$\implies r_c(r_c(A)) = \overline{A^\circ}$

$\implies r_c(r_c(A)) = r_c(A)$

□

Theorem 4.2. 1. *For any subset A of X,* $r_c(A^\circ) = r_c(A)$.

2. *If* $A \subseteq B$ *, then* $r_c(A) \subseteq r_c(B)$ *, where* $A, B \subset X$.
3. $r_c(A \cup B) \supseteq r_c(A) \cup r_c(B)$,*where* $A, B \subset X$
4. $r_c(A \cap B) \subseteq r_c(A) \cap r_c(B)$,*where* $A, B \subset X$

Proof.

1. $r_c(A^\circ) = \overline{A^{\circ\circ}}$

$\implies r_c(A^\circ) = \overline{A^\circ}$

$\implies r_c(A^\circ) = r_c(A)$

2. $A \subseteq B \implies A^\circ \subseteq B^\circ$

$\implies \overline{A^\circ} \subseteq \overline{B^\circ}$

$\implies r_c(A) \subseteq r_c(B)$

3. $A \subseteq A \cup B \implies r_c(A) \subseteq r_c(A \cup B)$

$B \subseteq A \cup B \implies r_c(B) \subseteq r_c(A \cup B)$

$\implies r_c(A \cup B) \supseteq r_c(A) \cup r_c(B)$

4. $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

Then (2) $\implies r_c(A \cap B) \subseteq r_c(A)$, $r_c(A \cap B) \subseteq r_c(B)$

$\implies r_c(A \cap B) \subseteq r_c(A) \cap r_c(B)$

□

Theorem 4.3. 1. *Regular closed sets are fixed points of r_c-operator. That is,* $r_c(A) = A$.

2. *ϕ and X are fixed points r_c-operator. That is,* $r_c(\phi) = \phi$, $r_c(X) = X$.

Proof. 1. If A is a regular closed set $\overline{A^\circ} = A$.

$\implies r_c(A) = A$.

2. Trivial.

□

Theorem 4.4. 1. *If A and B are non empty regular closed sets, then* $r_c(A \cup B) = A \cup B$.

2. *If A and B are non empty regular closed sets, then* $r_c(A \cap B) \neq A \cap B$.

Proof. 1. Union of regular closed sets is regular closed. So $r_c(A \cup B) = A \cup B$.

2. Intersection of regular closed sets need not be regular closed.

Hence $r_c(A \cap B) \neq A \cap B$.

Example 4.5. *Let* $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$.

$A = \{b, c\}$ *and* $B = \{a, c\}$ *are regular closed sets.*

$A \cap B = \{c\}$

$r_c(A \cap B) = r_c(\{c\}) = \phi$

$r_c(A) = r_c(\{b, c\}) = \{b, c\}$ $r_c(B) = r_c(\{a, c\}) = \{a, c\}$

$r_c(A) \cap r_cB) = \{c\}$

Hence $r_c(A \cap B) \neq r_c(A) \cap r_c(B)$.

□

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