Malaya Journal of Matematik, Vol. 8, No. 4, 2060-2062,
https://doi.org/10.26637/MJM0804/0124

A novel method of similarity measures of Trapezoidal intuitionistic fuzzy numbers

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Abstract
In this paper, we made an extensive analysis of the similarity relation between two trapezoidal intuitionistic fuzzy numbers. We proposed a two similarity measures for trapezoidal intuitionistic fuzzy number using the geometric distance and graded mean. Some numerical examples are listed for justification of the work.

Keywords
Trapezoidal Fuzzy Number, Intuitionistic Fuzzy Number, Geometric Distance, Graded mean, Similarity Measures.

AMS Subject Classification:
03E72, 03F55, 08A72.

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Article History: Received **17** September **2020**; Accepted **12** November **2020**

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1. Introduction

Many real - world applications make use of similarity measure to see how two objects are related together. The similarity measures have been introduced based on the concepts such graded mean integration[3]. A new method to measure the similarity between fuzzy numbers has been introduced by Chen and Chen [1]. Finding the degree of similarity between the patterns is very essential in order to reduce computation. Fuzzy similarity measures are predominantly employed to classify the resemblance between patterns. Chen and Chen [2] have proposed a similarity measure of generalized fuzzy number. Based on the concept of geometric distance the similarity measure has been proposed Liu [4]. The similarity measures between two TRIFNs using value and ambiguity has been proposed by Stephen Dinagar and Fany Helena [5]. The main objective of this paper is to brief out the conceptual theory behind then trapezoidal intuitionistic fuzzy numbers. In this paper we proposed two similarity measures of TRIFNs based on the geometric distance and graded mean integration. We adopt $\tilde{A}^i$ notion for intuitionistic fuzzy numbers. Some numerical examples are given for the proposed similarity measure.

The paper is organized as follows: Section 2 presents the basic definitions. In section 3 we proposed a similarity measures for trapezoidal intuitionistic fuzzy numbers based on geometric distance. In section 4, we proposed similarity measures using graded mean integration. Section 5 is followed by conclusion.

2. Preliminaries

Definition 2.1. *A fuzzy set is characterized by a membership function mapping the elements of a domain to the unit interval [0, 1]. A fuzzy set $\tilde{A}$ of x is defined by the following pair such as*

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x) / x \in X)\}$$

Definition 2.2. *A trapezoidal intuitionistic fuzzy number $\tilde{A}^i = [(a_1, a_2, a_3, a_4)(b_1, b_2, b_3, b_4)]$ is a intuitionistic fuzzy set on a* *set of real numbers R, with parameter $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ whose membership function and non- membership function are defined as follows*

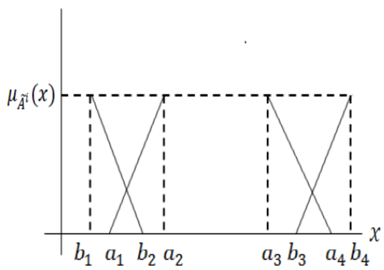

Figure 1. **Trapezoidal Intuitionistic Fuzzy Number.**

$$\mu_{\tilde{A}^i}(x) = \begin{cases} \left(\frac{x-a_1}{a_2-a_1}\right), & \text{if } a_1 \le x \le a_2 \\ 1, & \text{if } a_2 \le x \le a_3 \\ \left(\frac{a_4-x}{a_4-a_3}\right), & \text{if } a_3 \le x \le a_4 \\ 0, & \text{Otherwise.} \end{cases}$$

$$\nu_{\tilde{A}^i}(x) = \begin{cases} \left(\frac{b_2-x}{b_2-b_1}\right), & \text{if } b_1 \le x \le b_2 \\ 0, & \text{if } b_2 \le x \le b_3 \\ \left(\frac{x-b_3}{b_4-b_3}\right), & \text{if } b_3 \le x \le b_4 \\ 1, & \text{Otherwise.} \end{cases}$$

3. Proposed Similarity Measure between two Trapezoidal Intuitionistic Fuzzy Number based on Geometric Distance

Let $\tilde{A}_P^i = [(a_1,a_2,a_3,a_4)(b_1,b_2,b_3,b_4)]$ and $\tilde{B}_P^i = [(c_1,c_2,c_3,c_4)(d_1,d_2,d_3,d_4)]$ be two trapezoidal intuitionistic fuzzy numbers with a parameter $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ and $d_1 \le c_1 \le d_2 \le c_2 \le c_3 \le d_3 \le c_4 \le d_4$. Using the similarity measure of trapezoidal fuzzy number we proposed for trapezoidal intuitionistic fuzzy number (i.e) the similarity measure $S_1\left(\tilde{A}_P^i, \tilde{B}_P^i\right)$ between two TRIFNs based on geometric distance is given by the formulae

$$S_1\left(\tilde{A}_P^i, \tilde{B}_P^i\right) = 1 - \frac{\left| \frac{\sum_{i=1}^4 \left[\mu_{\tilde{A}_P^i}(x) + \nu_{\tilde{A}_P^i}(x)\right]}{2} - \frac{\sum_{i=1}^4 \left[\mu_{\tilde{B}_P^i}(x) + \nu_{\tilde{B}_P^i}(x)\right]}{2} \right|}{4} \tag{3.1}$$

Where $\mu_{\tilde{A}_P^i}(x) = a_1 + a_2 + a_3 + a_4; \nu_{\tilde{A}_P^i}(x) = b_1 + b_2 + b_3 + b_4$
$\mu_{\tilde{B}_P^i}(x) = c_1 + c_2 + c_3 + c_4; \nu_{\tilde{B}_P^i}(x) = d_1 + d_2 + d_3 + d_4$

3.1 Numerical Examples

Example 3.1. *Let $\tilde{A}_P^i = [(0.5, 0.6, 0.65, 0.7)(0.35, 0.55, 0.7, 0.8)]$ and $\tilde{B}_P^i = [(0.45, 0.75, 0.8, 0.85)(0.4, 0.7, 0.85, 0.9)]$ be two trapezoidal intuitionistic fuzzy numbers. Find the similarity measure based on geometric mean.*

$$S_1\left(\tilde{A}_P^i, \tilde{B}_P^i\right) = 1 - \frac{\left| \frac{\sum_{i=1}^4 \left[\mu_{\tilde{A}_P^i}(x) + \nu_{\tilde{A}_P^i}(x)\right]}{2} - \frac{\sum_{i=1}^4 \left[\mu_{\tilde{B}_P^i}(x) + \nu_{\tilde{B}_P^i}(x)\right]}{2} \right|}{4}$$
$$= 0.89375$$

Example 3.2. *Let $\tilde{A}_P^i = [(-0.7, -0.5, -0.4, -0.3)(-0.8, -0.53, -0.3, -0.2)]$ and $\tilde{B}_P^i = [(-0.57, -0.43, -0.38, -0.35)(-0.62, -0.54, -0.3, -0.2)]$ be two trapezoidal intuitionistic fuzzy numbers. Find the similarity measure based on geometric mean.*

$$S_1\left(\tilde{A}_P^i, \tilde{B}_P^i\right) = 1 - \frac{\left| \frac{\sum_{i=1}^4 \left[\mu_{\tilde{A}_P^i}(x) + \nu_{\tilde{A}_P^i}(x)\right]}{2} - \frac{\sum_{i=1}^4 \left[\mu_{\tilde{B}_P^i}(x) + \nu_{\tilde{B}_P^i}(x)\right]}{2} \right|}{4}$$
$$= 0.9575$$

4. New Approach

Let $\mu_{\tilde{A}_P^i}(\alpha)$ and $\nu_{\tilde{A}_P^i}(\alpha)$ are the alpha- cuts set of a trapezoidal intuitionistic fuzzy number $\tilde{A}^i = [(a_1,a_2,a_3,a_4)(b_1,b_2,b_3,b_4)]$ then the graded mean integration representation for the membership function $\mu_\alpha(x)$ and non-membership function $\nu_\alpha(x)$ of the TRIFN $\tilde{A}^i$ are defined as

$$\mu_{\tilde{A}^i}(\alpha) = \int_0^1 \{[a_1 + \alpha(a_2 - a_1)] + [a_4 - \alpha(a_4 - a_3)]\} f(\alpha)\, d\alpha$$
$$\nu_{\tilde{A}^i}(\beta) = \int_0^1 \{[b_2 - \alpha(b_2 - b_1)] + [b_3 + \alpha(b_4 - b_3)]\} g(\beta)\, d\beta$$

Respectively where we shall choose $f(\alpha) = \alpha$ for both the $\mu_{\tilde{A}^i}(\alpha)$ and $\nu_{\tilde{A}^i}(\alpha)$.
The graded mean integration representation for the membership and non-membership functions can be calculated as follows.

$$\mu_{\tilde{A}^i}(\alpha) = \int_0^1 \{[a_1 + \alpha(a_2 - a_1)] + [a_4 - \alpha(a_4 - a_3)]\}\alpha d\alpha$$
$$= \frac{a_1 + 2a_2 + 2a_3 + a_4}{6} \tag{4.1}$$

$$\nu_{\tilde{A}^i}(\beta) = \int_0^1 \{[b_2 - \alpha(b_2 - b_1)] + [b_3 + \alpha(b_4 - b_3)]\}(1-\beta)\, d\beta$$
$$= \frac{b_1 + 2b_2 + 2b_3 + b_4}{6} \tag{4.2}$$

4.1 Proposed Similarity Measure for Trapezoidal Intuitionistic Fuzzy Number using Graded Mean Integration Representation

Let $\tilde{A}_P^i = [(a_1,a_2,a_3,a_4)(b_1,b_2,b_3,b_4)]$ be the TRIFN, the graded mean integration representation is defined as

$$P\left(\widetilde{A}^i\right) = \left[\frac{\frac{a_1+2a_2+2a_3+a_4}{6} + \frac{b_1+2b_2+2b_3+b_4}{6}}{2}\right]$$

Then the similarity measure $S_2\left(\tilde{A}_P^i, \tilde{B}_P^i\right)$ between two trapezoidal intuitionistic fuzzy number $\tilde{A}_P^i = [(a_1,a_2,a_3,a_4)(b_1,b_2,

$b_3, b_4)]$ and $\tilde{B}^i_P = [(c_1, c_2, c_3, c_4)(d_1, d_2, d_3, d_4)]$ based on graded mean integration is given by the formulae.

$$S_2\left(\tilde{A}^i_P, \tilde{B}^i_P\right) = \frac{1}{1 + d\left(\tilde{A}^i, \tilde{B}^i\right)} \tag{4.3}$$

Where, $d\left(\tilde{A}^i, \tilde{B}^i\right) = |P\left(\tilde{A}^i_P\right) - P\left(\tilde{B}^i_P\right)|$ with

$$P(\tilde{A}^i_P) = \left[\frac{\frac{a_1+2a_2+2a_3+a_4}{6} + \frac{b_1+2b_2+2b_3+b_4}{6}}{2}\right]$$

and

$$P(\tilde{B}^i_P) = \left[\frac{\frac{C_1+2C_2+2C_3+C_4}{6} + \frac{d_1+2d_2+2d_3+d_4}{6}}{2}\right]$$

4.2 Numerical Examples

Example 4.1. *The two TRIFNs are* $\tilde{A}^i_P = [(0.3, 0.6, 0.75, 0.85)(0.25, 0.5, 0.8, 0.9)]$ *and* $\tilde{B}^i_P = [(0.4, 0.5, 0.7, 0.8)(0.3, 0.45, 0.8, 0.9)]$ *then the similarity measure based on graded mean representation is*

$$\begin{aligned} S_2\left(\tilde{A}^i_P, \tilde{B}^i_P\right) &= \frac{1}{1 + d\left(\tilde{A}^i, \tilde{B}^i\right)} \\ &= 0.9756 \end{aligned}$$

Example 4.2. *The two TRIFNs are* $\tilde{A}^i_P = [(-0.7, -0.5, -0.4, -0.3)(-0.8, -0.53, -0.3, -0.2)]$ *and* $\tilde{B}^i_P = [(-0.57, -0.43, -0.38, -0.35)(-0.62, -0.54, -0.3, -0.2)]$ *then the similarity measure based on graded mean representation is*

$$\begin{aligned} S_2\left(\tilde{A}^i_P, \tilde{B}^i_P\right) &= \frac{1}{1 + d\left(\tilde{A}^i, \tilde{B}^i\right)} \\ &= 0.9662 \end{aligned}$$

5. Conclusion

We have discussed about the similarity measures between two trapezoidal fuzzy numbers based on geometric distance and graded mean integration representations. Also we extend this similarity measure to trapezoidal intuitionistic fuzzy number and we have provided the numerical illustration for the similarity measure.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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