Malaya Journal of Matematik, Vol. 8, No. 4, 2063-2070, 2020
https://doi.org/10.26637/MJM0804/0125

On pairwise fuzzy Baire dense sets

G. Thangaraj[1*] and E. Roseline Gladis[2]

Abstract
In this paper, the concept of pairwise fuzzy Baire dense sets in fuzzy bitopological spaces is introduced by means of pairwise fuzzy first category sets. Several characterizations of pairwise fuzzy Baire dense sets are established. The conditions for fuzzy bitopological spaces to become pairwise fuzzy Baire spaces, pairwise fuzzy second category spaces and pairwise fuzzy resolvable space, are obtained.

Keywords
Pair wise fuzzy dense set, pairwise fuzzy first category set, pairwise fuzzy G_δ-set, pairwise fuzzy F_σ-set, pairwise fuzzy P-space, pairwise fuzzy Baire space, pairwise fuzzy resolvable space.

AMS Subject Classification
54A40, 03E72.

[1] *Department of Mathematics, Thiruvalluvar University Vellore-632115, Tamil Nadu, India.*
[2] *Department of Mathematics, Voorhees College Vellore-632001, Tamil Nadu, India.*
***Corresponding author**: [1] g.thangaraj@rediffmail.com; [2]roselinegladis@gmail.com
Article History: Received **05** September **2020**; Accepted **25** November **2020**

Contents

1. Introduction

The idea of fuzzy sets , fuzzy set operations was introduced by L.A. Zadeh [15] in his classical paper in the year 1965. in order to deal with uncertainties. The fuzzy notion has successfully been applied in all branches of Mathematics. The concepts of fuzzy topology was defined by C. L. Chang [2] in the year 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In 1989, Kandil [3] introduced the concept of fuzzy bitopological spaces as a generalization of fuzzy topological spaces.
In this paper, the concept of pairwise fuzzy Baire dense sets in fuzzy bitopological spaces is introduced by means of pairwise fuzzy first category sets. Several characterizations of pairwise fuzzy Baire dense sets are established. The conditions under which fuzzy bitopological spaces become pairwise fuzzy Baire spaces, pairwise fuzzy second category spaces and pairwise fuzzy resolvable space, are obtained.

2. Preliminaries

In order to make the exposition self- contained, some basic notions and results used in the sequel are given. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to CHANG (1968). Let X be a non-empty set and I, the unit interval $[0,1]$. A fuzzy set λ in X is a function from X into I. The null set 0_X is the function from X into I which assumes only the value 0 and the whole fuzzy set 1_X is the function from X into I which takes 1 only. By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X,T_1,T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X.

Definition 2.1 ([15]). *Let λ and μ be fuzzy sets in X. Then, for all $x \in X$,*

(i) $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$.

(ii) $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$,

(iii) $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max\{\lambda(x), \mu(x)\}$

(iv) $\delta = \lambda \wedge \mu \Leftrightarrow \psi(x) = \min\{\lambda(x), \mu(x)\}$

(v) $\eta = \lambda^c \quad \Leftrightarrow \quad \eta(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X,T), the union $\psi = V_i(\lambda_i)$ and intersection $\delta = \Lambda_i(\lambda_i)$, are defined respectively as

(vi) $\psi(x) = \sup_i\{\lambda_i(x)/x \in X\}$

(vii) $\delta(x) = \inf_i\{\lambda_i(x)/x \in X\}$.

Definition 2.2 ([2]). *Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior and the closure of λ are defined respectively as follows:*

(i) $\mathrm{int}(\lambda) = \vee\{\mu/\mu \leq \lambda, \quad \mu \in T\}$ *and*

(ii) $\mathrm{cl}(\lambda) = \wedge\{\mu/\lambda \leq \mu, \quad 1-\mu \in T\}$.

Lemma 2.3. *For a fuzzy set λ of a fuzzy topological space X,*

(i) $1 - \mathrm{Int}(\lambda) = \mathrm{Cl}(1-\lambda)$

(ii) $1 - \mathrm{Cl}(\lambda) = \mathrm{Int}(1-\lambda)$.

Definition 2.4 ([5]). *A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy open set if $\lambda \in T_i (i=1,2)$. The complement of pairwise fuzzy open set in (X,T_1,T_2) is called a pairwise fuzzy closed set.*

Definition 2.5 ([4]). *A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy dense set if $cl_{T_1}cl_{T_2}(\lambda) = cl_{T_2}cl_{T_1}(\lambda) = 1$ in (X,T_1,T_2).*

Definition 2.6 ([8]). *A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy nowhere dense set if int $_{T_1}cl_{T_2}(\lambda) = \mathrm{int}_{T_2}\, cl_{T_1}(\lambda) = 0$ in (X,T_1,T_2).*

Definition 2.7 ([8]). *Let (X,T_1,T_2) be a fuzzy bitopological space. A fuzzy set λ in (X,T_1,T_2) is called a pairwise fuzzy first category set if $\lambda = V_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are pairwise fuzzy nowhere dense sets in (X,T_1,T_2). Any other fuzzy set in (X,T_1,T_2) is said to be a pairwise fuzzy second category set in (X,T_1,T_2).*

Definition 2.8 ([8]). *If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X,T_1,T_2), then the fuzzy set $(1-\lambda)$ is called a pairwise fuzzy residual set in (X,T_1,T_2).*

Definition 2.9 ([5]). *A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy G_δ-set if*

$$\lambda = \Lambda_{i=1}^{\infty}(\lambda_i),$$

where $(\lambda_i)'s$ are pairwise fuzzy open sets in (X,T_1,T_2).

Definition 2.10 ([5]). *A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy F_σ-set if*

$$\lambda = V_{i=1}^{\infty}(\lambda_i),$$

where $(\lambda_i)'s$ are pairwise fuzzy closed sets in (X,T_1,T_2).

Definition 2.11 ([7]). *A fuzzy set λ defined on X in a fuzzy bitopological space (X,T_1,T_2) is called an pairwise fuzzy σ-nowhere dense set if λ is an pairwise fuzzy F_σ-set in (X,T_1,T_2) such that int_{T_1} int $T_2(\lambda) = \mathrm{int}_{T_2}$ int $_{T_1}(\lambda) = 0$.*

Definition 2.12 ([12]). *A fuzzy bitopological space (X,T_1,T_2) is called an pairwise fuzzy globally disconnected space if each pairwise fuzzy semi-open set is an pairwise fuzzy open set in (X,T_1,T_2).*

Definition 2.13 ([4]). *A fuzzy bitopological space (X,T_1,T_2) is called an pairwise fuzzy resolvable space if there exists an pairwise fuzzy dense set λ in (X,T_1,T_2) such that $1-\lambda$ is also an pairwise fuzzy dense set in (X,T_1,T_2). Otherwise (X,T_1,T_2) is called an pairwise fuzzy irresolvable space.*

Definition 2.14 ([6]). *The fuzzy bitopological space (X,T_1,T_2) is said to be an pairwise fuzzy almost resolvable space, if $V_{k=1}^{\infty}(\lambda_k) = 1_X$ where the fuzzy sets $(\lambda_k)'s$ are such that int $_{T_i}(\lambda_k) = 0$, $i = 1,2$, in (X,T_1,T_2). The fuzzy bitopological space (X,T_1,T_2) which is not an pairwise fuzzy almost resolvable space, is said to be an pairwise fuzzy almost irresolvable space.*

Definition 2.15 ([11]). *A fuzzy bitopological space (X,T_1,T_2) is called an pairwise fuzzy extraresolvable space if λ_m and $\lambda_n(\ m \neq n)$, are pairwise fuzzy dense sets in (X,T_1,T_2), then $\lambda_m \wedge \lambda_n$ is an pairwise fuzzy nowhere dense set in (X,T_1,T_2).*

Definition 2.16 ([5]). *A fuzzy bitopological space (X,T_1,T_2) is said to be an pairwise fuzzy weakly Volterra space if*

$$\Lambda_{k=1}^{N}(\lambda_k) \neq 0,$$

where $(\lambda_k)'s$ are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X,T_1,T_2).

Definition 2.17 ([8]). *A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy Baire space if*

$$\mathrm{int}_{T_i} V_{k=1}^{\infty}(\lambda_k) = 0,\ (i=1,2),$$

where $(\lambda_k)'s$ are pairwise fuzzy nowhere dense sets in (X,T_1,T_2).

Definition 2.18 ([7]). *A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy-σ Baire space if*

$$\mathrm{int}_{T_i} V_{k=1}^{\infty}(\lambda_k) = 0,\ (i=1,2),$$

where $(\lambda_k)'s$ are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2).

Theorem 2.19 ([8]). *Let (X,T_1,T_2) be an fuzzy bitopological space. Then the following are equivalent :*

1. *(X,T_1,T_2) is an pairwise fuzzy Baire space.*
2. *$int_{T_i}(\lambda) = 0$, $(i=1,2)$, for every pairwise fuzzy first category set λ in (X,T_1,T_2).*
3. *$cl_{T_i}(\mu) = 0$, $(i=1,2)$, for every pairwise fuzzy residual set μ in (X,T_1,T_2).*

Theorem 2.20 ([7]). *If λ is an pairwise fuzzy dense and pairwise fuzzy G_δ-set in an fuzzy bitopological space (X,T_1,T_2), then $1-\lambda$ is an pairwise first category set in (X,T_1,T_2).*

Theorem 2.21 ([9]). *If the fuzzy bitopological space (X,T_1,T_2). is an pairwise fuzzy Baire space, then no non-zero pairwise fuzzy open set is an pairwise fuzzy first category set in (X,T_1,T_2).*

Theorem 2.22 ([8]). *If the fuzzy bitopological space (X,T_1,T_2) is an pairwise fuzzy Baire space, then (X,T_1,T_2) is an pairwise fuzzy second category space.*

Theorem 2.23 ([9]). *If $\mu < \lambda$, where λ is an pairwise fuzzy first category set in the fuzzy bitopological space (X,T_1,T_2) and μ is an fuzzy set defined on X, then μ is an pairwise fuzzy first category set in (X,T_1,T_2).*

Theorem 2.24 ([13]). *If λ is an pairwise fuzzy residual set in the pairwise fuzzy submaximal space (X,T_1,T_2), then λ is an pairwise fuzzy G_δ-set in (X,T_1,T_2).*

Theorem 2.25 ([13]). *If λ is an pairwise fuzzy residual set in a pairwise fuzzy submaximal and pairwise fuzzy P-space (X,T_1,T_2), then λ is a pairwise fuzzy open set in (X,T_1,T_2).*

Theorem 2.26 ([13]). *If λ is an pairwise fuzzy residual set in the pairwise fuzzy globally disconnected space (X,T_1,T_2), then λ is an pairwise fuzzy G_δ-set in (X,T_1,T_2).*

Theorem 2.27 ([14]). *If λ is an pairwise fuzzy residual set in the pairwise fuzzy globally disconnected and pairwise fuzzy ∂ -space (X,T_1,T_2), then λ is an pairwise fuzzy simply open set in (X,T_1,T_2).*

Theorem 2.28 ([9]). *If the fuzzy bitopological space (X,T_1,T_2) is an pairwise fuzzy Baire space, then (X,T_1,T_2) is an pairwise fuzzy almost irresolvable space.*

Theorem 2.29 ([10]). *If the fuzzy bitopological space (X,T_1,T_2) is an pairwise fuzzy resolvable space, then (X,T_1,T_2) is an pairwise fuzzy almost resolvable space.*

Theorem 2.30 ([11]). *If $\mu \leq \lambda$, where λ is an pairwise fuzzy residual set in the pairwise fuzzy extraresolvable space (X,T_1,T_2), then $\operatorname{int}_{T_1}\operatorname{int}_{T_2}(\mu) = 0$ and $\operatorname{int}_{T_2}\operatorname{int}_{T_1}(\mu) = 0$ in (X,T_1,T_2).*

Theorem 2.31 ([11]). *If the fuzzy bitopological space (X,T_1,T_2) is an pairwise fuzzy Baire space, then (X,T_1,T_2) is not an pairwise fuzzy extra resolvable space.*

Theorem 2.32 ([6]). *If each pairwise fuzzy first category set is an pairwise fuzzy closed set in a pairwise fuzzy second category space (X,T_1,T_2) then (X,T_1,T_2) is an pairwise fuzzy weakly Volterra space.*

Theorem 2.33 ([9]). *If the fuzzy bitopological space (X,T_1,T_2) is an pairwise fuzzy Baire space, then (X,T_1,T_2) is an pairwise fuzzy almost irresolvable space.*

Theorem 2.34 ([6]). *If the fuzzy bitopological space (X,T_1,T_2) is an pairwise fuzzy weakly Volterra space, then (X,T_1,T_2) is not an pairwise fuzzy σ-Baire space.*

3. Pairwise Fuzzy Baire Dense Sets

Definition 3.1. *Let (X,T_1,T_2) be an fuzzy bitopological space A fuzzy (X,T_1,T_2) if there exists no pairwise fuzzy first category set μ in (X,T_1,T_2) such that $\lambda < \mu < 1$.*

Example 3.2. *Let $X = \{a,b,c\}$. Consider the fuzzy sets λ,μ,γ and δ, are defined on X as follows:*

$\lambda : X \to [0,1]$ *is defined as* $\lambda(a) = 1;\ \lambda(b) = 0.2;\ \lambda(c) = 0.7$
$\mu : X \to [0,1]$ *is defined as* $\mu(a) = 0.3;\ \mu(b) = 1;\ \mu(c) = 0.2$
$\gamma : X \to [0,1]$ *is defined as* $\gamma(a) = 0.7;\ \gamma(b) = 0.4;\ \gamma(c) = 1$
$\delta : X \to [0,1]$ *is defined as* $\delta(a) = 0.2;\ \delta(b) = 1;\ \delta(c) = 0.4.$

Then,

$$\begin{aligned} T_1 =& \{0,\lambda,\mu,\gamma,\lambda\vee\mu,\lambda\vee\gamma,\mu\vee\gamma,\lambda\wedge\mu,\lambda\wedge\gamma, \\ & \mu\wedge\gamma,\gamma\wedge[\lambda\vee\mu],\lambda\vee[\mu\wedge\gamma], \quad \mu\vee[\lambda\wedge\gamma],1\} \\ T_2 =& \{0,\lambda,\gamma,\delta,\lambda\vee\gamma,\lambda\vee\delta,\gamma\vee\delta,\lambda\wedge\gamma,\lambda\wedge\delta,\gamma\wedge\delta, \\ & \lambda\vee[\gamma\wedge\delta],\delta\vee[\lambda\wedge\gamma], \quad \gamma\wedge[\lambda\vee\delta],1\} \end{aligned}$$

are fuzzy topologies on X. On computation one can find that λ, γ, $\lambda\vee\mu$, $\lambda\vee\gamma$, $\mu\vee\gamma$, $\lambda\wedge\gamma$, $\gamma\wedge[\lambda\vee\mu]$, $\lambda\vee[\mu\wedge\gamma]$, $\mu\vee[\lambda\wedge\gamma]$, $\lambda\vee\delta$, $\gamma\vee\delta$, $\lambda\vee[\mu\wedge\delta]$, $\gamma\wedge[\lambda\vee\delta]$, $\delta\vee[\lambda\wedge\gamma]$, are pairwise fuzzy open sets in (X,T_1,T_2). Now, the fuzzy sets

$$\begin{aligned} \lambda =& \lambda\wedge[\lambda\vee\mu]\wedge[\lambda\vee\gamma]\wedge(\lambda\vee[\mu\wedge\delta]) \quad \text{and} \\ \lambda\wedge\gamma =& \gamma\wedge[\mu\vee\gamma]\wedge[\lambda\wedge\gamma]\wedge(\gamma\wedge[\lambda\vee\mu])\wedge(\mu\vee[\lambda\wedge\gamma] \end{aligned}$$

are pairwise fuzzy G_δ-sets in (X,T_1,T_2). Also $cl_{T_1}cl_{T_2}(\lambda) = cl_{T_2}cl_{T_1}(\lambda) = 1$ and $cl_{T_1}cl_{T_2}(\lambda\wedge\gamma) = 1$ and

$$cl_{T_2}cl_{T_1}(\lambda) = cl_{T_2}[1-(\mu\wedge\gamma)] = 1.$$

Hence λ and $\lambda\wedge\gamma$ are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X,T_1,T_2). Then, $1-\lambda$ and $1-[\lambda\wedge\gamma]$ are pairwise fuzzy first category sets in (X,T_1,T_2). Now for the fuzzy sets $\lambda\vee\mu,\mu\vee\gamma$ and $\mu\vee[\lambda\wedge\gamma]$ in (X,T_1,T_2) there are no pairwise fuzzy first category sets $1-\lambda$ and $1-[\lambda\wedge\gamma]$ in (X,T_1,T_2) such that

$$\begin{gathered} \lambda\vee\mu < 1-\lambda < 1; \lambda\vee\mu < 1-[\lambda\wedge\gamma] < 1 \\ \mu\vee\gamma < 1-\lambda < 1; \mu\vee\gamma < 1-[\lambda\wedge\gamma] < 1 \\ \mu\vee[\lambda\wedge\gamma] < 1-\lambda < 1; \quad \mu\vee[\lambda\wedge\gamma] < 1-[\lambda\wedge\gamma] < 1. \end{gathered}$$

Hence $\lambda\vee\mu,\mu\vee\gamma$ and $\mu\vee[\lambda\wedge\gamma]$, are the pairwise fuzzy Baire dense sets in (X,T_1,T_2).

Proposition 3.3. *If λ is an pairwise fuzzy Baire dense set in the fuzzy bitopological space (X,T_1,T_2), then for an non-zero pairwise fuzzy open set δ in (X,T_1,T_2) such that $\lambda < \delta < 1$, then δ is an pairwise fuzzy second category set in (X,T_1,T_2).*

Proof. Let λ be an pairwise fuzzy Baire dense set in (X,T_1,T_2). Then, there exists no pairwise fuzzy first category set μ in (X,T_1,T_2) such that

$$\lambda < \mu < 1. \tag{A}$$

Suppose that $\lambda < \delta$, where δ is an pairwise fuzzy open set in (X,T_1,T_2). Then, by (A), δ is not an pairwise fuzzy first category set in (X,T_1,T_2) and hence δ is an pairwise fuzzy second category set in (X,T_1,T_2). □

Proposition 3.4. *If $\lambda < \mu < 1$, where μ is an pairwise fuzzy first category set in (X,T_1,T_2), then λ is not an pairwise fuzzy Baire dense set in (X,T_1,T_2).*

Proof. Let λ be an fuzzy set defined on X such that $\lambda < \mu < 1$, where μ is an pairwise fuzzy first category set in (X,T_1,T_2). By the definition of pairwise fuzzy Baire denseness of fuzzy sets in fuzzy bitopological spaces, λ is not an pairwise fuzzy Baire dense set in (X,T_1,T_2). □

Proposition 3.5. *If $\lambda < \mu$, where μ is an pairwise fuzzy first category set in an pairwise fuzzy Baire space (X,T_1,T_2), then λ is not an pairwise fuzzy open and not an pairwise fuzzy Baire dense set in (X,T_1,T_2).*

Proof. Let λ be an fuzzy set defined on X such that $\lambda < \mu$, where μ is an pairwise fuzzy first category set in (X,T_1,T_2). Then, int $_{T_i}(\lambda) \leq \mathrm{int}_{T_i}(\mu)$ ($i = 1,2$). since (X,T_1,T_2) is an pairwise fuzzy Baire space, by the theorem 2.19, $int_{T_i}(\mu) = 0$, $(i = 1,2)$, for the pairwise fuzzy first category set μ in (X,T_1,T_2). Then, $int_{T_i}(\lambda) \leq 0$. That is, $\mathrm{int}_{T_i}(\lambda) = 0$, in (X,T_1,T_2) and then, $\mathrm{int}_{T_i}(\lambda) \neq \lambda$, in (X,T_1,T_2). Hence λ is not an pairwise fuzzy open and not an pairwise fuzzy Baire dense set in (X,T_1,T_2). □

Proposition 3.6. *If λ is an pairwise fuzzy Baire dense set in the fuzzy bitopological space (X,T_1,T_2), then*

$$cl_{T_i}cl_{T_j}(\lambda),\ (i,j = 1,2),$$

and $i \neq j$) is an pairwise fuzzy second category set in (X,T_1,T_2).

Proof. Let λ be an pairwise fuzzy Baire dense set in (X,T_1,T_2). Then, there exists no pairwise fuzzy first category set μ in (X,T_1,T_2) such that

$$\lambda < \mu < 1. \tag{A}$$

Since

$$\lambda < cl_{T_1}cl_{T_j}(\lambda),\ (i,j = 1,2 \text{ and } i \neq j),$$

here, $cl_{T_i}cl_{T_j}(\lambda)$ is not an pairwise fuzzy first category set in (X,T_1,T_2) and hence $clT_icl_{T_j}(\lambda)$ is an pairwise fuzzy second category set in (X,T_1,T_2). □

The following proposition ensures the existence of pairwise fuzzy second category sets sets in fuzzy bitopological spaces by means of pairwise fuzzy Baire dense sets.

Proposition 3.7. *If λ is an pairwise fuzzy Baire dense set in the fuzzy bitopological space (X,T_1,T_2), then there exists an pairwise fuzzy second category set δ such that $\lambda < \delta$, in (X,T_1,T_2).*

Proof. Let λ be an pairwise fuzzy Baire dense set in (X,T_1,T_2). Then, by the proposition 3.6, $cl_{T_i}cl_{T_j}(\lambda)$ is an pairwise fuzzy second category set in (X,T_1,T_2). Let $\delta = cl_{T_i}cl_{T_j}(\lambda)$, $(i,j = 1,2$ and $i \neq j)$. Then δ is an pairwise fuzzy second category set in (X,T_1,T_2). Since $\lambda < cl_{Ti}cl_{T_j}(\lambda)$, there exists an pairwise fuzzy second category set δ such that $\lambda < \delta$ in (X,T_1,T_2). □

Proposition 3.8. *If λ is an pairwise fuzzy Baire dense set in the fuzzy bitopological space (X,T_1,T_2), then there is no pairwise fuzzy residual set δ such that $1-\lambda > \delta$, in (X,T_1,T_2).*

Proof. Let λ be an pairwise fuzzy Baire dense set in (X,T_1,T_2). Then, there exists no pairwise fuzzy first category set μ in (X,T_1,T_2) such that

$$\lambda < \mu < 1. \tag{A}$$

Suppose that there exists an pairwise fuzzy residual set δ such that $1-\lambda > \delta$, in (X,T_1,T_2). Now, $1-\lambda > \delta$, implies that $1-\delta > \lambda$, in (X,T_1,T_2). Since δ is an pairwise fuzzy residual set, $1-\delta$ is an pairwise fuzzy first category set in (X,T_1,T_2) and thus there exists an pairwise fuzzy first category set $1-\delta$ in (X,T_1,T_2) such that $\lambda < 1-\delta < 1$, a contradiction to λ being an pairwise fuzzy Baire dense set in (X,T_1,T_2). Hence there is no pairwise fuzzy residual set δ such that $1-\lambda > \delta$, in (X,T_1,T_2). □

Proposition 3.9. *If λ is an pairwise fuzzy Baire dense set in the fuzzy bitopological space (X,T_1,T_2), then there is no pairwise fuzzy F_σ-set μ with*

$$\mathrm{int}_{T_i}\mathrm{int}_{T_j}(\mu) = 0,\ (i,j = 1,2 \text{ and } i \neq j),$$

such that $\lambda < \mu < 1$ in (X,T_1,T_2).

Proof. Let λ be an pairwise fuzzy Baire dense set in the pairwise fuzzy strongly irresolvable space (X,T_1,T_2). Then, there exists no pairwise fuzzy first category set μ in (X,T_1,T_2) such that

$$\lambda < \mu < 1. \tag{A}$$

Suppose that there exists an pairwise fuzzy F_σ-set μ with

$$int_{T_i}\ int_{T_j}(\mu) = 0,\ (i,j = 1,2 \text{ and } i \neq j),$$

such that $\lambda < \mu$ in (X,T_1,T_2). Then, $1-\mu$ is an pairwise fuzzy G_δ-set in (X,T_1,T_2) such that $cl_{T_i}cl_{T_j}(1-\mu) = 1-\mathrm{int}_{T_i}\mathrm{int}_{T_j}(\mu) = 1-0 = 1$. Thus, $1-\mu$ is an pairwise fuzzy G_δ-set and pairwise fuzzy dense set in the fuzzy bitopological space (X,T_1,T_2). Then, by the theorem 2.20, $1-[1-\mu]$ is an pairwise fuzzy first category set in (X,T_1,T_2). Thus μ is an pairwise fuzzy first category set in (X,T_1,T_2) such that $\lambda < \mu < 1$, a contradiction to λ being an pairwise fuzzy Baire dense set in (X,T_1,T_2). Hence there is no pairwise fuzzy F_σ-set μ with

$$\mathrm{int}_{T_i}\ \mathrm{int}_{T_j}(\mu) = 0, (i,j = 1,2 \text{ and } i \neq j),$$

such that $\lambda < \mu < 1$, in (X,T_1,T_2). □

Proposition 3.10. *If λ is an pairwise fuzzy Baire dense set in the fuzzy bitopological space (X, T_1, T_2), then there is no pairwise fuzzy σ-nowhere dense set μ such that $\lambda < \mu < 1$ in (X, T_1, T_2).*

Proof. The proof follows from the definition 2.11 and the proposition 3.9. □

Proposition 3.11. *If λ is an pairwise fuzzy Baire dense set in the fuzzy bitopological space (X, T_1, T_2) and μ is an pairwise fuzzy residual set in (X, T_1, T_2), then $1 - \mu \leq \lambda$, in (X, T_1, T_2).*

Proof. Let λ be an pairwise fuzzy Baire dense set in (X, T_1, T_2) and μ be an pairwise fuzzy residual set in (X, T_1, T_2). Suppose that $1 - \mu > \lambda$, in (X, T_1, T_2). Then, for the pairwise fuzzy Baire dense set λ in (X, T_1, T_2), there exists an pairwise fuzzy residual set μ in (X, T_1, T_2) such that $1 - \lambda > \mu$, a contradiction by the proposition 3.6. Hence, for the pairwise fuzzy Baire dense set $\lambda, 1 - \mu \leq \lambda$, in (X, T_1, T_2) □

Proposition 3.12. *If λ is an pairwise fuzzy Baire dense set in an pairwise fuzzy Baire space (X, T_1, T_2), then $cl_{T_j}(\lambda)$, $(j = 1, 2)$, is not an pairwise fuzzy nowhere dense set in (X, T_1, T_2).*

Proof. Let λ be an pairwise fuzzy Baire dense set in (X, T_1, T_2). Then, by the proposition

$$\mathrm{cl}_{T_i} cl_{T_j}(\lambda),\ (i, j = 1, 2 \text{ and } i \neq j),$$

is an pairwise fuzzy second category set in (X, T_1, T_2) and hence is not an pairwise fuzzy first category set in (X, T_1, T_2). Since (X, T_1, T_2) is an pairwise fuzzy Baire space, by the theorem 2.19,

$$\mathrm{int}_{T_i}\left(cl_{T_i} cl_{T_j}(\lambda)\right) \neq 0,\ (i, j = 1, 2 \text{ and } i \neq j),$$

in (X, T_1, T_2) and thus $\mathrm{cl}_{T_j}(\lambda))\,[j = 1, 2]$, is not an pairwise fuzzy nowhere dense set in (X, T_1, T_2). □

Proposition 3.13. *If λ is an pairwise fuzzy Baire dense set and δ is an pairwise fuzzy G_δ-set in the pairwise fuzzy P -space (X, T_1, T_2) such that $\lambda < \delta$ then the pairwise fuzzy G_δ-set is an pairwise fuzzy second category set in (X, T_1, T_2).*

Proof. Let λ be an pairwise fuzzy Baire dense set and δ is an pairwise fuzzy G_δ-set such that $\lambda < \delta$, in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy P-space, the pairwise fuzzy G_δ-set δ is an pairwise fuzzy open set in (X, T_1, T_2). Then, by the proposition 3.3, δ is an pairwise fuzzy second category set in (X, T_1, T_2). □

Proposition 3.14. *If λ is an pairwise fuzzy Baire dense set and μ is an pairwise fuzzy residual set in the pairwise fuzzy submaximal space (X, T_1, T_2), then $1 - \mu$ is an pairwise fuzzy F_σ-set such that $1 - \mu \leq \lambda$ in (X, T_1, T_2).*

Proof. Let λ be an pairwise fuzzy Baire dense set and μ is an pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy submaximal space, by the theorem 2.24, μ is an pairwise fuzzy G_δ-set in (X, T_1, T_2). Then, $1 - \mu$ is an pairwise fuzzy F_σ-set in (X, T_1, T_2). By the proposition 3.11, $1 - \mu \leq \lambda$, in (X, T_1, T_2). Thus, $1 - \mu$ is an pairwise fuzzy F_σ-set such that $1 - \mu \leq \lambda$, in (X, T_1, T_2). □

Proposition 3.15. *If λ is an pairwise fuzzy Baire dense set and μ is an pairwise fuzzy residual set in the pairwise fuzzy submaximal space and pairwise fuzzy P-space (X, T_1, T_2), then there exists an pairwise fuzzy first category set and pairwise fuzzy closed set $1 - \mu$ such that $1 - \mu \leq \lambda$, in (X, T_1, T_2).*

Proof. Let λ be an pairwise fuzzy Baire dense set and μ is an pairwise fuzzy residual set in (X, T_1, T_2). Then, by the proposition 3.11, $1 - \mu \leq \lambda$ in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy submaximal and pairwise fuzzy P-space, by the theorem 2.25, the pairwise fuzzy residual set μ is an pairwise fuzzy open set in (X, T_1, T_2) and then $1 - \mu$ is an pairwise fuzzy closed set and pairwise fuzzy first category set in (X, T_1, T_2). Hence, for the pairwise fuzzy Baire dense set λ, there exists an pairwise first category and pairwise fuzzy closed set $1 - \mu$ such that $1 - \mu \leq \lambda$, in (X, T_1, T_2). □

Proposition 3.16. *If λ is an pairwise fuzzy Baire dense set and μ is an pairwise fuzzy residual set in the pairwise fuzzy globally disconnected space (X, T_1, T_2), then $1 - \mu$ is an pairwise fuzzy F_σ-set such that $1 - \mu \leq \lambda$, in (X, T_1, T_2).*

Proof. Let λ be an pairwise fuzzy Baire dense set and μ is an pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy globally disconnected space, by the theorem 2.26, μ is an pairwise fuzzy G_δ-set in (X, T_1, T_2). Then, $1 - \mu$ is an pairwise fuzzy F_σ-set in (X, T_1, T_2). By the proposition 3.11, $1 - \mu \leq \lambda$, in (X, T_1, T_2). Thus, $1 - \mu$ is an pairwise fuzzy F_σ-set such that $1 - \mu \leq \lambda$, in (X, T_1, T_2). □

Proposition 3.17. *If λ is an pairwise fuzzy Baire dense set and μ is an pairwise fuzzy residual set in the pairwise fuzzy globally disconnected and pairwise fuzzy ∂-space (X, T_1, T_2), then $1 - \mu$ is an pairwise fuzzy simply open set in (X, T_1, T_2) such that $1 - \mu \leq \lambda$.*

Proof. Let λ be an pairwise fuzzy Baire dense set and μ is an pairwise fuzzy residual set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy globally disconnected space and pairwise fuzzy ∂-space, by the theorem 2.27, the pairwise fuzzy residual set μ is an pairwise fuzzy simply open set in (X, T_1, T_2). By the proposition 3.11, $1 - \mu \leq \lambda$, in (X, T_1, T_2). Thus, for the pairwise fuzzy Baire dense set λ, $1 - \mu \leq \lambda$, where μ is an pairwise fuzzy simply open set in (X, T_1, T_2). □

Proposition 3.18. *If $(\lambda_k)'s$, $(k = 1$ to $\infty)$ are pairwise fuzzy nowhere dense sets and λ is an pairwise fuzzy Baire dense set in the fuzzy bitopological space (X, T_1, T_2), then $\lambda_k < \lambda$, for all k.*

Proof. Let $(\lambda_k)'s$, $(k = 1$ to $\infty)$ be the pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Suppose that $\lambda \leq \lambda_k$ in (X, T_1, T_2). Then $\lambda \leq V_{k=1}^{\infty}(\lambda_k)$. Now $V_{k=1}^{\infty}(\lambda_k)$ is an pairwise fuzzy first category set in (X,T_1,T_2). Let $\mu = V_{k=1}^{\infty}(\lambda_k)$ and then μ is an pairwise fuzzy first category set in (X,T_1,T_2) Thus, for the pairwise fuzzy Baire dense set λ, there is an pairwise fuzzy first category set μ in (X,T_1,T_2) such that $\lambda < \mu$. But this is a contradiction by the definition of pairwise fuzzy Baire denseness of fuzzy sets in fuzzy bitopological spaces and hence $\lambda_k < \lambda$, for all k in (X,T_1,T_2). □

The following propositions give condition for an pairwise fuzzy Baire dense set to become an pairwise fuzzy dense set in an bitopological space.

Proposition 3.19. *If μ is an pairwise fuzzy residual set in the fuzzy bitopological space (X,T_1,T_2) such that $\mathrm{int}_{T_i}\ \mathrm{int}_{T_j}(\mu) = 0$, $[i,j=1,2$ and $i \neq j]$, then the pairwise fuzzy Baire dense set λ in (X,T_1,T_2), is an pairwise fuzzy dense set in (X,T_1,T_2).*

Proof. Let λ be an pairwise fuzzy Baire dense set and μ is an pairwise fuzzy residual set in (X,T_1,T_2). Then, by the proposition 3.11, $1-\mu \leq \lambda$, in (X,T_1,T_2). By hypothesis $int_{T_i}\ \mathrm{int}_{T_j}(\mu) = 0$, $[i,j=1,2$ and $i \neq j]$, in (X,T_1,T_2). Now $cl_{T_i}cl_{T_j}(1-\mu) = 1-\mathrm{int}_{T_i}\mathrm{int}_{T_j}(\mu) = 1-0 = 1$ and then $cl_{T_i}cl_{T_j}(1-\mu) \leq cl_{T_i}cl_{T_j}(\lambda)$, implies that $1 \leq cl_{T_i}cl_{T_j}(\lambda)$. That is, $cl_{T_i}cl_{T_j}(\lambda) = 1$, in (X,T_1,T_2).
Hence the pairwise fuzzy Baire dense set λ is an pairwise fuzzy dense set in (X,T_1,T_2). □

Proposition 3.20. *If there exists an pairwise fuzzy residual set μ in the pairwise fuzzy extraresolvable space (X,T_1,T_2), then the pairwise fuzzy Baire dense set λ in (X,T_1,T_2), is an pairwise fuzzy dense set in (X,T_1,T_2).*

Proof. Let λ be an pairwise fuzzy Baire dense set and μ be an pairwise fuzzy residual set in (X,T_1,T_2). Then, from the proposition 3.11, $1-\lambda \leq \mu$, in (X,T_1,T_2). Since (X,T_1,T_2) is an pairwise fuzzy extraresolvable space, by the theorem 2.30, $\mathrm{int}_{T_1}\mathrm{int}_{T_2}(1-\lambda) = 0$ and $\mathrm{int}_{T_2}\mathrm{int}_{T_1}(1-\lambda) = 0$ in (X,T_1,T_2). Then, $1-cl_{T_1}cl_{T_2}(\lambda) = 0$ and $1-cl_{T_2}cl_{T_1}(\lambda) = 0$ in (X,T_1,T_2). Hence λ is an pairwise fuzzy dense set in (X,T_1,T_2). □

Remark 3.21. *The pairwise fuzzy dense set in an fuzzy bitopological space need not be an pairwise fuzzy Baire dense set. For, cosider the following example.*

Example 3.22. *Let $X = \{a,b,c\}$. Consider the fuzzy sets $\lambda,\mu,\gamma\beta$, and α are defined on X as follows:*

$\lambda : X \to [0,1]$ *is defined as* $\lambda(a) = 0.8; \lambda(b) = 0.6; \lambda(c) = 0.7$
$\mu : X \to [0,1]$ *is defined as* $\mu(a) = 0.6; \mu(b) = 0.9; \mu(c) = 0.8$
$\delta : X \to [0,1]$ *is defined as* $\delta(a) = 0.7; \delta(b) = 0.5; \delta(c) = 0.9$
$\beta : X \to [0,1]$ *is defined as* $\beta(a) = 0.7; \beta(b) = 0.4; \beta(c) = 0.9$
$\alpha : X \to [0,1]$ *is defined as* $\alpha(a) = 0.3; \alpha(b) = 0.5; \alpha(c) = 0.4$.

Then,

$$\begin{aligned}T_1 = &\{0,\lambda,\mu,\delta,\lambda\vee\mu,\lambda\vee\delta,\mu\vee\delta,\lambda\wedge\mu,\lambda\wedge\delta,\\ &\mu\wedge\delta,(\lambda\vee[\mu\wedge\delta]),(\mu\vee[\lambda\wedge\delta]),(\delta\vee[\lambda\wedge\mu]),\\ &(\lambda\wedge[\mu\vee\delta]),(\mu\wedge[\lambda\vee\delta])\\ &(\delta\wedge[\lambda\vee\mu]),[\lambda\vee\mu\vee\delta],[\lambda\wedge\mu\wedge\delta],1\}\\ T_2 = &\{0,\lambda,\delta,\beta,\lambda\vee\delta,\lambda\vee\beta,\delta\vee\beta,\lambda\wedge\delta,\lambda\wedge\beta,\delta\wedge\beta,\\ &(\lambda\vee[\delta\wedge\beta]),(\delta\vee[\lambda\wedge\beta]),(\beta\vee[\lambda\wedge\delta]),(\lambda\wedge[\delta\vee\beta]),\\ &(\delta\wedge[\lambda\vee\beta]),(\beta\wedge[\lambda v\delta]),[\lambda\vee\delta\vee\beta],[\lambda\wedge\delta\wedge\beta],1\},\end{aligned}$$

are fuzzy topologies on X. On computation $\lambda\wedge\delta$ is an pairwise fuzzy dense and pairwise fuzzy $G\delta$-set in (X,T_1,T_2). Then, $1-[\lambda\wedge\delta]$ is an pairwise fuzzy first category set in (X,T_1,T_2). Clearly $\alpha > 1-[\lambda\wedge\delta]$ and hence α is an pairwise fuzzy Baire dense set in (X,T_1,T_2). The fuzzy set λ is an pairwise fuzzy dense set in (X,T_1,T_2), but is not an pairwise fuzzy Baire dense set in (X,T_1,T_2).

4. Pairwise Fuzzy Baire Dense Sets and Pairwise Fuzzy Baire Spaces

The following propositions give conditions for an fuzzy bitopological space to become an pairwise fuzzy Baire space.

Proposition 4.1. *If there exists an pairwise fuzzy Baire dense set λ in the fuzzy bitopological space (X,T_1,T_2) such that $\lambda < \mu$, for each pairwise fuzzy open set μ in (X,T_1,T_2), then (X,T_1,T_2) is an pairwise fuzzy Baire space.*

Proof. Let λ be an pairwise fuzzy Baire dense set such that $\lambda < \mu$, where μ is an pairwise fuzzy open set in (X,T_1,T_2). Since λ is an pairwise fuzzy Baire dense set in (X,T_1,T_2), there exists no pairwise fuzzy first category set δ in (X,T_1,T_2) such that

$$\lambda < \delta < 1. \tag{A}$$

Since $\lambda < \mu$, μ cannot be an pairwise fuzzy first category set in (X,T_1,T_2) and hence the pairwise fuzzy open set μ must be an pairwise fuzzy second category set in (X,T_1,T_2). This implies that the pairwise fuzzy open sets in (X,T_1,T_2) are pairwise fuzzy second category sets in (X,T_1,T_2). Hence by the theorem 2.21, (X,T_1,T_2) is an pairwise fuzzy Baire space. □

Proposition 4.2. *If λ is an pairwise fuzzy Baire dense set in the fuzzy bitopological space (X,T_1,T_2) such that if $\mathrm{int}_{T_i}(\lambda) = 0$, then (X,T_1,T_2) is an pairwise fuzzy Baire space.*

Proof. Let $(\lambda_k)'s$, $(k = 1$ to $\infty)$ be pairwise fuzzy nowhere dense sets and λ is an pairwise fuzzy Baire dense set in (X,T_1,T_2). Then, by the proposition 3.18, $\lambda_k < \lambda$, for all k. This implies that $V_{k=1}^{\infty}(\lambda_k) \leq \lambda$ in (X,T_1,T_2) and then $\mathrm{int}_{T_i}\ V_{k=1}^{\infty}(\lambda_k) \leq \mathrm{int}_{T_i}(\lambda)$, $[i=1,2]$. By hypothesis, $\mathrm{int}_{T_i}(\lambda) = 0$ and hence $\mathrm{int}_{T_i}\ V_{k=1}^{\infty}(\lambda_k) = 0$, in (X,T_1,T_2). Thus, (X,T_1,T_2) is an pairwise fuzzy Baire space. □

Proposition 4.3. *If there exists an pairwise fuzzy Baire dense set λ in the fuzzy bitopological space (X,T_1,T_2) such that $\lambda < \mu$, for each pairwise fuzzy open set μ in (X,T_1,T_2), then (X,T_1,T_2) is an pairwise fuzzy second category space.*

Proof. Let λ be an pairwise fuzzy Baire dense set such that $\lambda < \mu$, where μ is an pairwise fuzzy open set in (X,T_1,T_2). Then, by the proposition 4.1, (X,T_1,T_2) is an pairwise fuzzy Baire space. By the theorem 2.22, the pairwise fuzzy Baire space (X,T_1,T_2) is an fuzzy second category space. □

Proposition 4.4. *If there exists an pairwise fuzzy Baire dense set λ in the fuzzy bitopological space (X,T_1,T_2) such that $\lambda < \mu$, for each pairwise fuzzy open set μ in (X,T_1,T_2), then (X,T_1,T_2) is an pairwise fuzzy almost irresolvable space.*

Proof. Let λ be an pairwise fuzzy Baire dense set such that $\lambda < \mu$, where μ is an pairwise fuzzy open set in (X,T_1,T_2). Then , by the proposition 4.1, (X,T_1,T_2) is an pairwise fuzzy Baire space. By the theorem 2.28, the pairwise fuzzy Baire space (X,T_1,T_2) is an pairwise fuzzy almost irresolvable space. □

Proposition 4.5. *If λ is an pairwise fuzzy Baire dense set in the fuzzy bitopological space (X,T_1,T_2) such that* $\text{int}_{T_i}(\lambda) = 0,\ [i = 1,2]$ *then (X,T_1,T_2) is an pairwise fuzzy second category space.*

Proof. Let λ_k's, ($k = 1$ to ∞) be pairwise fuzzy nowhere dense sets and λ is an pairwise fuzzy Baire dense set in (X,T_1,T_2). By hypothesis, $int_{T_i}(\lambda) = 0$, in (X,T_1,T_2) and then by the proposition 4.2, (X,T_1,T_2) is an pairwise fuzzy Baire space. By the theorem 2.22, the pairwise fuzzy Baire space (X,T_1,T_2) is an pairwise fuzzy second category space. □

Proposition 4.6. *Let λ be an pairwise fuzzy Baire dense set and μ be an pairwise fuzzy residual set in the fuzzy bitopological space (X,T_1,T_2). If*

(i) $\text{int}_{T_i}\text{int}_{T_j}(\mu) = 0,\ [i, j = 1,2$ *and* $i \neq j]$

(ii) $\text{int}_{T_i}\text{int}_{T_j}(\lambda) = 0,\ [i, j = 1,2$ *and* $i \neq j]$,

then (X,T_1,T_2) is an pairwise fuzzy resolvable space.

Proof. : Let λ be an pairwise fuzzy Baire dense set and μ be an pairwise fuzzy residual set in the fuzzy bitopological space (X,T_1,T_2). By hypothesis, $int_{T_i}\ int_{T_j}(\mu) = 0,\ [i, j = 1,2$ and $i \neq j]$ and then, by the proposition 3.19, the pairwise fuzzy Baire dense set λ is an pairwise fuzzy dense set in (X,T_1,T_2). Also $cl_{T_i}cl_{T_j}(1-\lambda) = 1-\text{int}_{T_i}\text{int}_{T_j}(\lambda) = 1-0 = 1$ and then $cl_{T_i}cl_{T_j}(1-\lambda) = 1$, in (X,T_1,T_2) and hence $(1-\lambda)$ is also an pairwise fuzzy dense set in (X,T_1,T_2). Thus, there exists an pairwise fuzzy dense set λ such that $(1-\lambda)$ is also an pairwise fuzzy dense set in (X,T_1,T_2). Hence (X,T_1,T_2) is an pairwise fuzzy resolvable space. □

Proposition 4.7. *Let λ be an pairwise fuzzy Baire dense set and μ be an pairwise fuzzy residual set in the fuzzy bitopological space (X,T_1,T_2). If*

(i) $\text{int}_{T_i}\text{int}_{T_j}(\mu) = 0,\ [i, j = 1,2$ *and* $i \neq j]$

(ii) $\text{int}_{T_i}\text{int}_{T_j}(\lambda) = 0,\ [i, j = 1,2$ *and* $i \neq j]$,

then the pairwise fuzzy resolvable space (X,T_1,T_2) is not an pairwise fuzzy submaximal space.

Proof. Let λ be an pairwise fuzzy Baire dense set and μ be an pairwise fuzzy residual set in the fuzzy bitopological space (X,T_1,T_2). By hypothesis, $\text{int}_{T_i}\text{int}_{T_j}(\mu) = 0,\ [i, j = 1,2$ and $i \neq j]$ and then, by the proposition 3.19, the pairwise fuzzy Baire dense set λ is an pairwise fuzzy dense set in (X,T_1,T_2). Since $\text{int}_{T_i}\text{int}_{T_j}(\lambda) = 0,\ [i, j = 1,2$ and $i \neq j]$, λ is not an pairwise fuzzy open set in (X,T_1,T_2). Hence (X,T_1,T_2) is not an pairwise fuzzy submaximal space. By proposition 4.7, (X,T_1,T_2) is an pairwise fuzzy resolvable space. Thus, the pairwise fuzzy resolvable space (X,T_1,T_2) is not an pairwise fuzzy submaximal space. □

Proposition 4.8. *Let λ be an pairwise fuzzy Baire dense set and μ be an pairwise fuzzy residual set in the fuzzy bitopological space (X,T_1,T_2). If*

(i) $\text{int}_{T_i}\text{int}\,T_j(\mu) = 0,\ [i, j = 1,2$ *and* $i \neq j]$

(ii) $\text{int}_{T_i}\text{int}_{T_j}(\lambda) = 0,\ [i, j = 1,2$ *and* $i \neq j]$,

then (X,T_1,T_2) is an pairwise fuzzy almost resolvable space.

Proof. The proof follows from the proposition 4.6 and the theorem 2.29. □

The following propositions give conditions for an pairwise fuzzy extraresolvable space to become an pairwise fuzzy resolvable space.

Proposition 4.9. *If λ is an pairwise fuzzy Baire dense set in the pairwise fuzzy extraresolvable space (X,T_1,T_2) such that* $\text{int}_{T_i}\text{int}_{T_j}(\lambda) = 0,\ [i, j = 1,2$ *and* $i \neq j]$, *then (X,T_1,T_2) is an pairwise fuzzy resolvable space.*

Proof. Let λ be an pairwise fuzzy Baire dense set and μ be an pairwise fuzzy reidual set in (X,T_1,T_2). Since (X,T_1,T_2) is an pairwise fuzzy extraresolvable space, by the proposition 3.20, the pairwise fuzzy Baire dense set λ is an pairwise fuzzy dense set in (X,T_1,T_2). Also $cl_{T_i}cl_{T_j}(1-\lambda) = 1-int_{T_i}\text{int}_{T_j}(\lambda) = 1-0 = 1$ and then $\text{cl}_{T_i}\,cl_{T_j}(1-\lambda) = 1$, in (X,T_1,T_2) and hence $(1-\lambda)$ is also an pairwise fuzzy dense set in (X,T_1,T_2). Thus, there exists an pairwise fuzzy dense set λ such that $(1-\lambda)$ is also an pairwise fuzzy dense set in (X,T_1,T_2). Hence (X,T_1,T_2) is an pairwise fuzzy resolvable space. □

Proposition 4.10. *If there exists an pairwise fuzzy Baire dense set λ in the fuzzy bitopological space (X,T_1,T_2) such that $\lambda < \mu$, for each pairwise fuzzy open set μ in (X,T_1,T_2), then (X,T_1,T_2) is not an pairwise fuzzy extraresolvable space.*

Proof. The proof follows from the proposition 4.1 and the theorem 2.31. □

Proposition 4.11. *If λ is an pairwise fuzzy Baire dense set in the fuzzy bitopological space (X, T_1, T_2) such that $\text{int}_{T_i}(\lambda) = 0$, then (X, T_1, T_2) is not an pairwise fuzzy extraresolvable space.*

Proof. The proof follows from the proposition 4.2 and the theorem 2.31. □

Proposition 4.12. *If there exists an pairwise fuzzy Baire dense set λ in the fuzzy bitopological space (X, T_1, T_2) such that $\lambda < \mu$, for each pairwise fuzzy open set μ in (X, T_1, T_2) and if each pairwise fuzzy first category set is an pairwise fuzzy closed set in (X, T_1, T_2) then (X, T_1, T_2) is an pairwise fuzzy weakly Volterra space.*

Proof. Let λ be an pairwise fuzzy Baire dense set such that $\lambda < \mu$, where μ is an pairwise fuzzy open set in (X, T_1, T_2). Then , by the proposition 4.3, (X, T_1, T_2) is an pairwise fuzzy second category space. By hypothesis, each pairwise fuzzy first category set is an pairwise fuzzy closed set in (X, T_1, T_2). Then, by the theorem 2.32, (X, T_1, T_2) is an pairwise fuzzy weakly Volterra space. □

Proposition 4.13. *If there exists an pairwise fuzzy Baire dense set λ in the fuzzy bitopological space (X, T_1, T_2) such that $\lambda < \mu$, for each pairwise fuzzy open set μ in (X, T_1, T_2) and if each pairwise fuzzy first category set is an pairwise fuzzy closed set in (X, T_1, T_2) then (X, T_1, T_2) is not an pairwise fuzzy σ-Baire space.*

Proof. The proof follows from the proposition 4.12 and the theorem 2.34. □

5. Conclusion

In this paper, a new type of fuzzy sets, namely pairwise fuzzy Baire dense sets in fuzzy bitopological spaces is introduced by means of pairwise fuzzy first category sets . Several characterizations of pairwise fuzzy Baire dense sets are established. The existence of pairwise fuzzy second category sets in fuzzy bitopological spaces is ensured by the existence of pairwise fuzzy Baire dense sets. The conditions under which pairwise fuzzy Baire dense sets become pairwise fuzzy Baire dense sets in fuzzy bitopological spaces, are also obtained. The conditions under which fuzzy bitopological spaces become pairwise fuzzy Baire spaces, pairwise fuzzy second category spaces and pairwise fuzzy resolvable space, are obtained. . The conditions by which pairwise fuzzy extraresolvable space becomes pairwise fuzzy resolvable space, are also established by means of pairwise fuzzy Baire dense sets.

References

[1] K. K. Azad, On Fuzzy semi continuity, Fuzzy almost continuity and Fuzzy Weakly continuity, *J. Math. Anal. Appl,* 82(1981), 14–32.

[2] C. L. Chang, Fuzzy topological spaces, *J. Math. Anal. Appl,* 24(1968), 182–190.

[3] A. Kandil and M. E. El-Shafee, Biproximities and fuzzy bitopological spaces, *Simen Stevin*, 63(1989), 45–66.

[4] G. Thangaraj, On pairwise fuzzy resolvable and fuzzy irresolvable spaces, *Bull. Cal. Math. Soc*, 102(1)(2010), 59–68.

[5] G. Thangaraj and V. Chandiran, On pairwise fuzzy Volterra spaces, *Ann. Fuzzy Math. Inform*, 7(6)(2014), 1005–1012.

[6] G. Thangaraj and V. Chandiran, Some Remarks on Pairwise Fuzzy Weakly Volterra Spaces, *Inter. J. Fuzzy Math. and Sys*, 4(2)(2014), 227–235.

[7] G. Thangaraj and V. Chandiran, A note on pairwise fuzzy Volterra spaces, *Ann. Fuzzy Math. Inform*, 9(3)(2015), 365–372.

[8] G. Thangaraj and S. Sethuraman, On pairwise fuzzy Baire bitopological spaces, *Gen. Math. Notes*, 20(2)(2014), 12–21.

[9] G. Thangaraj and S. Sethuraman, A note on pairwise fuzzy Baire spaces, *Ann Fuzzy Math. Inform*, 8(5)(2014), 729–737.

[10] G. Thangaraj and P. Vivakanandan, Pairwise Fuzzy Almost Resolvable Spaces, *Inter. J. Fuzzy Math. and Sys*, 4(2)(2014), 245–253.

[11] G. Thangaraj and P. Vivakanandan, Pairwise Fuzzy Extraresolvable Spaces, *Adv. Fuzzy Math*, 11(1)(2016), 25–37.

[12] G. Thangaraj and A. Vinothkumar, Pairwise fuzzy globally disconnected spaces and pairwise fuzzy σ-Baire spaces, *Global J . Math. Sciences*, 9(2)(2017), 117–125.

[13] G. Thangaraj and E. Roseline Gladis, On pairwise fuzzy *P*-Spaces, *Adv. Fuzzy Math*, 12(5)(2017), 1075–1085.

[14] G. Thangaraj and E. Roseline Gladis, On pairwise fuzzy ∂-spaces, *J. Fuzzy Math*, 27(3)(2019), 671–686.

[15] L. A. Zadeh, Fuzzy Sets, *Inform. and Control*, 8(1965), 338–353.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆