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Fuzzy TOPSIS method for group decision making problem using similarity measure

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Abstract
In this paper, another methodology for positioning of choices with fuzzy information for collective choice creation utilizing TOPSIS technique is proposed. Another likeness measure is acquainted all together with decide the best one among the other options. The proposed strategy is shown by a numerical model.

Keywords
Fuzzy Number, Triangular Fuzzy Number, Similarity Measure, TOPSIS Method, Group Decision Making.

AMS Subject Classification
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1. Introduction

In a down to earth choice circumstance, the utilization of traditional dynamic technique may see various requirements from the measures maybe containing imprecision in the data. Numerous standards dynamic was presented as a significant field of study in the mid 1970's. Various studies, C.A Bana e Costa [4] show the imperativeness of numerous strategy have been created. In 1965 Bellman and Zadeh [1] presented first the hypothesis of fluffy sets.

Later on, numerous specialists have been taking a shot at the way toward managing fluffy dynamic issues by applying fluffy set hypothesis. The idea of dynamic is, as the name recommends, the investigation of how choices are really caused a how they to can be made best or best. The dynamic has become an awkward assignment requiring models that can think about different numerous objectives, imperatives that are to some degree unsure and adaptable in nature. Fluffiness is inborn in choice information and collective choice creation forms.

Collective choice creation issue are broad in some genuine circumstances. A multi property dynamic issue is to look through a superior trade off arrangement from all conceivable achievable options surveyed on various characteristics both subjective and quantitative.

The various trait dynamic issue can be managed utilizing a few existing techniques to assess the presentation of options through the comparability with the perfect arrangement. The strategy for request of inclination by comparability to perfect arrangement (TOPSIS) is a multi-models choice investigation technique, which was initially evolved by Ching-Lai Hwang and Youn in 1981 [8] with further improvements in Youn in 1987, and Hwang, Lai and Liu in 1933 [10]. As per this procedure, the best option would be one that is nearest to the positive-perfect arrangement and most remote from the negative perfect arrangement. The positive perfect arrangement is augments the advantage models and limits the cost standards.

The Similarity proportion of fluffy numbers is assume vial job in many research fields in fuzzy condition. All the similitude estimates characterized for summed up fuzzy numbers faces some disadvantage and neglect to give precise outcomes sometimes.

In this paper, another likeness measure has been proposed for positioning of options with fluffy information for cooperative choice creation utilizing TOPSIS technique. The level of enrollment and level of non-participation work in intuitionistic fuzzy set is characterized by utilizing the aggregate of both the worth ought to be short of what one. Triangular intuitionstic fuzzy number is characterized by dang-fang-li [5].

2. Preliminaries

In this section, we recall some definitions and basic results which will be used throughout the paper.

Definition 2.1. *[Fuzzy Set] A fuzzy set $\bar{A}$ in X is characterized by a membership function $\mu_{\bar{A}}(x)$ which associates with each points X a real number in the interval $[0,1]$. A fuzzy set $\bar{A}$ of X is defined as $\bar{A} = \{x, \mu_{\bar{A}}(x) : x \in X\}$, where $\mu_{\bar{A}}(x)$ is called the membership function which maps each element of x to value between 0 and 1.*

Definition 2.2. *[Fuzzy Number] A fuzzy number A is a convex, normalized fuzzy subset A of the real line R such that*

1. *there exists exactly one $x_0 \in R$, $\mu_A(x_0) = 1$ (x_0 is called the mean value of A).*
2. *$\mu_A(x)$ is piecewise continuous.*

If A is convex, normalized, it's membership function is piecewise continuous and more than one element $x_0 \in R$ such that $\mu_A(x_0) = 1$ exists, then A is called flat fuzzy number.

Definition 2.3. *[Triangular Fuzzy Number] A fuzzy number $A = (a_1, a_2, a_3)$ is a triangular fuzzy number if it's membership function $\mu_A(x)$ is given by* $\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, \text{for } a_1 \le x \le a_2 \\ \frac{x-a_3}{a_2-a_3}, \text{for } a_2 \le x \le a_3 \\ 0, \text{otherwise} \end{cases}$

Definition 2.4. *[Triangular Intuitionistic Fuzzy Number] A triangular intuitionitic fuzzy number $\bar{A}^T$ is a subset of intuitionistic fuzzy set in R with membership function*

$$\mu_{\bar{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, \text{for } a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, \text{for } a_2 \le x \le a_3 \\ 0, \text{otherwise} \end{cases}$$

and non-membership function

$$y_{\bar{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1^1}, \text{for } a_1^1 \le x \le a_2^1 \\ \frac{x-a_2}{a_3^1-a_2}, \text{for } a_2 \le x \le a_3 \\ 0, \text{otherwise} \end{cases}$$

where $a_1^1 \le a_1 \le a_2 \le a_3 \le a_3^1$.

Definition 2.5. *[Interval Valued Intuitionistic fuzzy set] Let X be a set, an interval valued intuitionistic fuzzy set (IVIFS) A in X is defined as $A = \langle x, \mu_A(x), V_A(x) \rangle | x \in X$ where $\mu_A(x)$ and $V_A(x)$ with the condition $0 \le sup(\mu_A(x) + V_A(x)) \le 1$, the intervals $\mu_A(x)$ and $V_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A.*

For every $x \in X$ and $\mu_A(x)$, $V_A(x)$ are closed intervals and their lower and upper end points are respectively denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$, $V_{AL}(x)$ and $V_{AU}(x)$. It is expressed by

$A = \langle x, [\mu_{AL}(x), \mu_{AU}(x)], [V_{AL}(x), V_{AU}(x)] \rangle | x \in X$

where $0 \le \mu_{AU}(x) + V_{AU}(x) \le 1$.

3. Similarity Measures

A generalized fuzzy number $A = (a,b,c)$ where $0 \le a \le b \le c \le 1$ is a fuzzy subset of the real line R with membership function μ_A which has the following properties

1. $\mu_A(x) = 0$ for all $x \in (-\infty, 0)$.
2. μ_A is strictly increasing on (a,b).
3. μ_A is strictly decreasing on (b,c).
4. $\mu_A(x) = 0$ for all $x \in (c, +\infty)$.

Wei and Chen [16] defined a new approach for similarity measures of generalized fuzzy numbers and compared with existing similarity measures.

3.1 Proposed Similarity Measure

Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be two generalized triangular fuzzy number. Then the degree of similarity $S(A,B)$ between the generalized triangular fuzzy numbers A and B can be calculated as below.

$$S(A,B) = 1 - \frac{\sum_{i=1}^{3} |a_i - b_i|}{e^{\sum_{i=1}^{3} |a_i - b_i|}} \times \frac{min(P(A), P(B))}{max(P(A), P(B))} \quad (3.1)$$

where,

$$P(A) = \sqrt{(a_1 - a_2)^2 + 1} + \sqrt{(a_2 - a_3)^2 + 1} + (a_3 - a_2) \quad (3.2)$$

and

$$P(B) = \sqrt{(b_1 - b_2)^2 + 1} + \sqrt{(b_2 - b_3)^2 + 1} + (b_3 - b_2) \quad (3.3)$$

are the perimeters of the generalized triangular fuzzy number A and B respectively. The larger value of $S(A,B)$, is the similarity between A and B.

3.2 Properties of Proposed Similarity Measures

Property 1

The similarity measure of the two generalized triangular fuzzy number must lie between 0 and 1. i.e., $0 \le S(A,B) \le 1$

Example

Consider $A = (0.1, 0.2, 0.3)$ and $B = (0.2, 0.3, 0.4)$. The similarity measure between A and B is,

$$S(A,B) = 0.7778$$

Property 2

Two generalized triangular fuzzy numbers A and B are identical if and only if $S(A,B) = 1$.

Example

Consider $A = (0.1, 0.2, 0.3)$ and $B = (0.1, 0.2, 0.3)$. Then,

$$P(A) = 2.110, P(B) = 2.110$$
$$S(A,B) = 1 - \frac{0}{e^0} \times \frac{2.110}{2.110} = 1$$

Property 3

The similarity measure of A and B is same as, the similarity measure of B and A. i.e., $S(A,B) = S(B,A)$

Example

Consider $A = (0.2, 0.4, 0.7)$ and $B = (0.1, 0.3, 0.4)$. Then,

$$S(A,B) = S(B,A)$$

Note 3.1. *Existing similarity measures of generalized triangular fuzzy numbers is*

$S(A,B) = 1 - \frac{\sum_{i=1}^{3}|a_i - b_i|}{3} \times \frac{min(P(A),P(B))}{max(P(A),P(B))}$

where $P(A)$ and $P(B)$, the perimeters of the generalized triangular fuzzy numbers A and B respectively are defined as follows.

$$P(A) = \sqrt{(a_1 - a_2)^2 + 1} + \sqrt{(a_2 - a_3)^2 + 1} + (a_3 - a_2)$$
$$P(B) = \sqrt{(b_1 - b_2)^2 + 1} + \sqrt{(b_2 - b_3)^2 + 1} + (b_3 - b_2)$$

The larger value of $S(A,B)$, is the similarity between A and B.

4. TOPSIS Method

In this section the proposed TOPSIS method and it's fuzzy extension is carried out as follows. Let us assume that the decision maker has to choose one of m possible alternatives described by n criteria. In the process of group decision making, the decision makers are asked to assess alternatives with respect to criteria.

Step (1)

Determination of the decision matrix consisting of m alternatives and n criteria, with the intersection of each alternative and criteria given as x_{ij}

$X = (x_{ij})_{m\times n}$ where $x_{ij} \in R$.

Step (2)

Calculation of the normalized decision matrix $R = (r_{ij})_{m\times n}$, using the normalization method

$$r_{ij} = \begin{cases} (\frac{a_{ij}}{max_j c_{ij}}, \frac{b_{ij}}{max_j c_{ij}}, \frac{c_{ij}}{max_j c_{ij}}), j \in B \\ \\ (\frac{min_j c_{ij}}{a_{ij}}, \frac{min_j c_{ij}}{b_{ij}}, \frac{min_j c_{ij}}{c_{ij}}), j \in C \end{cases}$$

Step (3)

Calculation of weighted normalized decision matrix $V = (v_{ij})_{m\times n}$. Using the vector of criteria weights $w = (w_1, w_2, \ldots, w_n)$, the weighted normalized fuzzy decision matrix is calculated for each decision matrix is calculated for each decision makings, so that $\sum_{i=1}^{n} w_i = 1$ where $v_{ij} = r_{ij}.w_{ij}$

Step (4)

Determination of the worst alternative and the best alternative by constitute the basis for the construction of the ranking of the alternatives and select the best one using fuzzy TOPSIS method. The positive ideal solution A^+ is calculated like

$$A^+ = \begin{bmatrix} v_{11}^+ & v_{12}^+ & \cdots & \cdots & v_{1n}^+ \\ v_{21}^+ & v_{22}^+ & \cdots & \cdots & v_{2n}^+ \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ v_{k1}^+ & v_{k2}^+ & \cdots & \cdots & v_{kn}^+ \end{bmatrix}$$

where $v_{kj}^+ = max_i v_{ij}$. The negative ideal solution A^- is calculated like

$$A^- = \begin{bmatrix} v_{11}^- & v_{12}^- & \cdots & \cdots & v_{1n}^- \\ v_{21}^- & v_{22}^- & \cdots & \cdots & v_{2n}^- \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ v_{k1}^- & v_{k2}^- & \cdots & \cdots & v_{kn}^- \end{bmatrix}$$

where $v_{kj}^- = min_i v_{ij}$

Step (5)

Instead of using distance measure here we should introduce the similarity measure, which are defined in section 3. The similarity of each alternative A_i represented in w from positive ideal solution is $S(A_i, A_i^+)$ and from negative ideal solution is $S(A_i, A_i^-)$ are calculated.

Step (6)

Calculation of the relative closeness of each alternative A_i to the positive ideal solution A_i^+

$$R_{C_i} = \frac{S(A_i, A_i^-)}{S(A_i, A_i^-) + S(A_i, A_i^+)}$$

Step (7)

Rank the alternatives according to the descending values of R_{C_i}, all alternatives are ordered by ranks and the best one is selected.

Remark 4.1. *Note that if in the proposed approach we use triangular fuzzy numbers and normalization process is done by new approach. Also instead of distance measure we introduce similarity measure idea.*

5. Numerical Example

In this section, we deal with numerical example using our proposed algorithm with similarity measures.

Consider a fuzzy Multiple Criteria Decision Making (MCDM) problem for group decision making, consisting of the set of feasible alternatives A_1,A_2,A_3 and their respective benefit criteria C_1,C_2,C_3 by the three group decision makers DM_1,DM_2,DM_3. Take the weights $w = 0.2, 0.3, 0.5$. The individual decisions matrices by the decision makers.

Table 1.

		C_1	C_2	C_3
DM_1	A_1	(3,4,9)	(4,6,11)	(2,10,12)
	A_2	(8,9,10)	(7,9,12)	(7,10,11)
	A_3	(6,8,12)	(8,9,11)	(5,8,13)
DM_2	A_1	(5,6,10)	(1,6,7)	(7,9,12)
	A_2	(8,9,11)	(5,10,11)	(14,6,11)
	A_3	(6,8,9)	(5,7,9)	(2,10,11)

Table 2. Normalized Decision Matrix

		C_1	C_2	C_3
DM_1	A_1	(0.2500,0.3333,0.7500)	(0.3333,0.5000,0.9167)	(0.1667,0.8333,1.0000)
	A_2	(0.6667,0.7500,0.8333)	(0.5833,0.7500,1.0000)	(0.5833,0.8333,0.9167)
	A_3	(0.4615,0.6154,0.9231)	(0.6154,0.6923,0.8462)	(0.3846,0.6154,1.0000)
DM_2	A_1	(0.4164,0.5000,0.8333)	(0.0833,0.5000,0.5833)	(0.5833,0.7500,1.0000)
	A_2	(0.7273,0.8182,1.0000)	(0.4545,0.9091,1.0000)	(0.3636,0.5455,1.0000)
	A_3	(0.5455,0.7273,0.8182)	(0.4545,0.6364,0.8182)	(0.1818,0.9091,1.0000)

Table 3. Weighted Normalized Decision Matrices

		C_1	C_2	C_3
DM_1	A_1	(0.0500,0.0667,0.1500)	(0.1000,0.1500,0.2750)	(0.0834,0.4167,0.5000)
	A_2	(0.1333,0.1500,0.1667)	(0.1750,0.2250,0.3000)	(0.2917,0.4167,0.4584)
	A_3	(0.0923,0.1233,0.1846)	(0.1846,0.2077,0.2538)	(0.1923,0.3077,0.5000)
DM_2	A_1	(0.0833,0.1000,0.1667)	(0.0250,0.1500,0.1750)	(0.2917,0.3750,0.5000)
	A_2	(0.1455,0.1636,0.2000)	(0.1364,0.2727,0.3000)	(0.1818,0.2727,0.5000)
	A_3	(0.1091,0.1455,0.1636)	(0.1364,0.1909,0.2455)	(0.0909,0.4545,0.5000)

Table 4. Weighted Normalized Decision Matrices for the alternatives

		C_1	C_2	C_3
A_1	DM_1	(0.0500,0.0667,0.1500)	(0.1000,0.1500,0.2750)	(0.0834,0.4167,0.5000)
	DM_2	(0.0833,0.1000,0.1667)	(0.0250,0.1500,0.1750)	(0.2917,0.3750,0.5000)
A_2	DM_1	(0.1330,0.1500,0.1667)	(0.1750,0.2250,0.3000)	(0.2917,0.4167,0.4584)
	DM_2	(0.1455,0.1636,0.2000)	(0.1364,0.2727,0.3000)	(0.1018,0.2727,0.5000)
A_1	DM_1	(0.0923,0.1233,0.1846)	(0.1846,0.2077,0.2538)	(0.1923,0.3077,0.5000)
	DM_2	(0.1091,0.1455,0.1636)	(0.1364,0.1909,0.2455)	(0.0909,0.4545,0.5000)

Table 5. Positive Ideal Solution

		C_1	C_2	C_3
A_1	DM_1	0.8108	0.8287	0.8158
	DM_2	0.8716	0.7641	0.9413
A_2	DM_1	0.9516	0.9752	0.8926
	DM_2	1.0000	1.0000	0.7180
A_1	DM_1	0.9524	0.9631	0.8634
	DM_2	0.9088	0.8722	0.8045

Table 6. Negative Ideal Solution

		C_1	C_2	C_3
A_1	DM_1	1.0000	1.0000	0.8880
	DM_2	0.9753	1.0000	0.7067
A_2	DM_1	0.8192	0.8461	0.7564
	DM_2	0.8667	0.7641	0.9190
A_1	DM_1	0..8521	0.8349	0.8249
	DM_2	0.9310	0.8337	0.7883

Table 7. The relative closeness co-efficient and the ranking order

	$S(A_i,A_i^+)$	$S(A_i^I,\bar{A})$	RC_i	**Rank**
A_1	0.7641	0.7067	0.4805	3
A_2	0.7180	0.7564	0.5130	1
A_2	0.8045	0.7883	0.4949	2

Now the preference can be ranked according to the order R. Therefore the best alternative is $A_2 \leq A_3 \leq A_1$.

6. Significance of the proposed comparability measure

The proposed approach (comparability measure) is again littler capacity of a, b and c which incorporates all the size of summed up fluffy number by barring reiteration. So the proposed measure clearly decreases the running time of the program. Despite the fact that the proposed strategy introduced in this paper is delineated by a model, it tends to be applied to issues, for example, numerous other administration choice issue.

7. Conclusion

In this paper, another likeness measure between summed up fluffy numbers has been characterized by changed strategy for Wei and Chen. It will decrease the unpredictability and a few downsides of the leaving similitude measure. The numerical model has indicated that the proposed approach, as contrasted and different strategies it gives the choice of the best one. Likewise it is applied in the genuine issues.

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