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1-Harmonious coloring of triangular snakes

Akhlak Mansuri[1*], Rohit Mehta[2] and R. S. Chandel[3]

Abstract
In this article, we discuss the 1-harmonious coloring and investigate the 1-harmonious chromatic number of triangular snakes and alternate triangular snakes. We also find some relations between the 1-harmonious chromatic number of triangular snakes and alternate triangular snakes.

Keywords
1-harmonious coloring, triangular snakes, alternate triangular snakes.

AMS Subject Classification
05C15, 05C75.

[1] *Department of Mathematics, Government Girls College, Mandsaur (M.P.) India.*
[2] *Department of Mathematics, Radharaman Engineering College, Bhopal (M.P.) India.*
[3] *Department of Mathematics, Government Geetanjali Girls College, Bhopal (M.P.) India.*
*__Corresponding author__: [1]akhlaakmansuri@gmail.com; [2]rohitmehta1010@gmail.com; [3]rs_chandel2009@yahoo.co.in
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Contents

1. Introduction

Throughout this paper, we considered only finite and undirected graphs without any loops or multiple edges. A proper vertex coloring of a graph G is a function $c: V(G) \longrightarrow \{1,2,\ldots, k\}$ in which if $u, v \in V(G)$ are adjacent, then $c(u) \neq c(v)$ and if this coloring uses at most k colors is known as k-coloring. The minimum number of colors are required for this coloring is called its chromatic number, and is generally denoted by $\chi(G)$. The 1-harmonious coloring [4] is a kind of vertex coloring such that the color pairs of end vertices of every edge are different only for adjacent edges and a minimum number of colors are required for this coloring is called the 1-harmonious chromatic number, denoted by $h_1(G)$. A triangular snake [2, 5–7, 9, 10] is a triangular cactus whose block-cutpointgraph is a path (a triangular snake is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to a new vertex w_i for $i = 1, 2, \ldots, n-1$). A double triangular snake graph $D(T_n)$ consists of two triangular snakes that have a common path, a triple triangular snake consists of three triangular snakes with a common path and consequently k-triangular snake graph $k(T_n)$ consists of k triangular snakes with a common path. A double alternate triangular snake graph $D(AT_n)$ consists of two alternate triangular snakes with a common path, a triple alternate triangular snake consists of three alternate triangular snakes with a common path and consequently k-alternate triangular snake graph $k(AT_n)$ consists of k alternate triangular snakes with a common path. In this paper, we study the 1-harmonious coloring with the chromatic number of above mentioned triangular snakes and find some relations between the 1-harmonious chromatic number of these snakes.

2. Definitions

Definition 2.1 ([2, 5–7, 9, 10])**.** *A triangular snake T_n is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to new vertices v_i and adding edges v_i for $i = 1, 2, \ldots, n-1$. That is every edge of a path is replaced by a cycle C_3.*

Definition 2.2 ([2, 5–7, 9, 10])**.** *A double triangular snake $D(T_n)$ consists of two triangular snakes that have a common path. That is, a double triangular snake is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $i = 1, 2, \ldots, n-1$ and to a new vertex w_i for $i = 1, 2, \ldots, n-1$.*

Definition 2.3 ([2, 5, 9])**.** *A triple triangular snake $T(T_n)$ consists of three triangular snakes that have a common path.*

That is, a triple triangular snake is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $i = 1, 2, ..., n-1$, to a new vertex w_i for $i = 1, 2, ..., n-1$ and to a new vertex x_i for $i = 1, 2, ..., n-1$.

Definition 2.4 ([2, 3, 5–8])**.** *An alternate triangular snake AT_n is the graph obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i for $i = 1, 2, ..., n-1$ (that is, every alternate edge of a path is replaced by cycle C_3).*

Definition 2.5 ([2, 3, 5–8])**.** *A double alternate triangular snake $D(AT_n)$ is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} (alternatively) to new vertices v_i for $i = 1, 2, ..., n-1$ and w_i for $i = 1, 2, ..., n-1$.*

Definition 2.6 ([2, 5, 9])**.** *A triple alternate quadrilateral snake $T(AT_n)$ is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} (alternatively) to new vertices v_i for $i = 1, 2, ..., n-1$, w_i for $i = 1, 2, ..., n-1$ and x_i for $i = 1, 2, ..., n-1$.*

Throughout the paper we consider n as the number of vertices of path P_n.

3. 1-Harmonious Coloring of Triangular Snakes

Theorem 3.1. *For $n \geq 3$, triangular snake T_n, the 1-harmonious chromatic number, $h_1(T_n) = \triangle(T_n) + 1$.*

Proof. Let us consider the path graph P_n with n vertices $u_1, u_2, ..., u_n$ and T_n as the triangular snake with maximum degree, $\triangle(T_n) = 4$. Let the vertices of T_n, $V(T_n) = \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq n-1\}$ and the edges of T_n, $E(T_n) = \{u_i u_{i+1} : 1 \leq i \leq n\} \cup \{u_i v_i, v_i u_{i+1} : 1 \leq i \leq n-1\}$. The number of vertices and edges in T_n are $2n-1$ and $3n-3$ respectively. Now we split the proof into following three cases.

Case 1: Suppose $n = 3k$. Define coloring $c : V(T_n) \longrightarrow \{1,2,3,4,5\}$ for $n \geq 3$ by $c(u_i) = 1$ ($i = 1,4,7,...,n-2$), $c(u_i) = 2$ ($i = 2,5,8,...,n-1$), $c(u_i) = 3$ ($i = 3,6,9,...,n$). Two sub cases arise here for even n and odd n.

Sub case 1: If n is odd, $c(v_i) = 4$ ($i = 1,3,5,...,n-2$), $c(v_i) = 5$ ($i = 2,4,6,...,n-1$).

Sub case 2: If n is even, $c(v_i) = 4$ ($i = 1,3,5,...,n-1$), $c(v_i) = 5$ ($i = 2,4,6,...,n-2$).

Vertices $u_2, u_3, ..., u_{n-1}$ are of maximum degree 4 whereas the degree of u_1, u_n is 2, u_i is adjacent to u_{i+1} ($1 \leq i \leq n-1$) and vertices u_i ($1 \leq i \leq n$) are adjacent to v_j ($1 \leq j \leq n-1$). Therefore 5 colors are to be needed to color T_n. From figure 1, clearly we find that for each vertex, the adjacent vertices are colored with different color. Therefore, $h_1(T_n) = 5$.

Case 2: Suppose $n = 3k+1$. Define coloring $c : V(T_n) \longrightarrow \{1,2,3,4,5\}$ for $n \geq 3$ by $c(u_i) = 1$ ($i = 1,4,7,...,n$), $c(u_i) = 2$ ($i = 2,5,8,...,n-2$), $c(u_i) = 3$ ($i = 3,6,9,...,n-1$). Again two sub cases arises for even n and odd n, sub-cases and remaining procedure can be done as describe in case 1.

Case 3: Suppose $n = 3k+2$. Define coloring $c : V(T_n) \longrightarrow \{1,2,3,4,5\}$, for $n \geq 3$ by $c(u_i) = 1$ ($i = 1,4,7,...,n-1$),

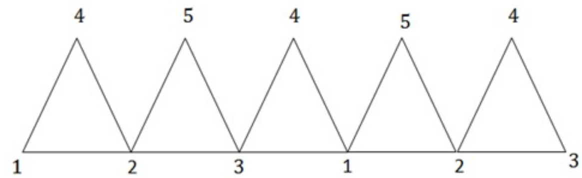

Figure 1. 1-harmonious coloring of T_6, $h_1(T_6) = 5$

$c(u_i) = 2$ ($i = 2,5,8,...,n$), $c(u_i) = 3$ ($i = 3,6,9,...,n-2$). Here again two sub cases arises for even n and odd n, for that we follow the procedure as described in case 1. In all three cases 1-harmonious chromatic number, $h_1(T_n) = 5$. Figure 2

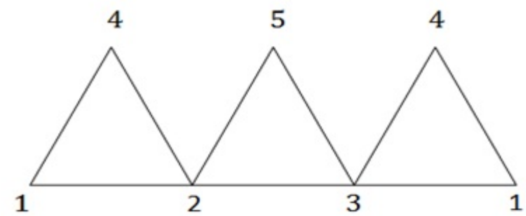

Figure 2. 1-harmonious coloring of T_4, $h_1(T_4) = 5$

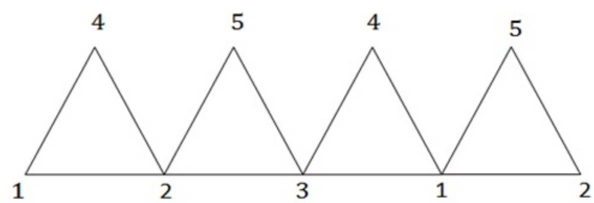

Figure 3. 1-harmonious coloring of T_5, $h_1(T_5) = 5$

and Figure 3 shows the coloring for case 2 and case 3. □

Theorem 3.2. *For $n \geq 3$, double triangular snake DT_n, the 1-harmonious chromatic number, $h_1(DT_n) = \triangle(DT_n) + 1$.*

Proof. Let us consider the path graph P_n with n vertices $u_1, u_2, ..., u_n$ and DT_n as the double triangular snake with maximum degree, $\triangle(DT_n) = 6$. Let the vertices of DT_n, $V(DT_n) = \{u_i : 1 \leq i \leq n\} \cup \{v_i, w_i : 1 \leq i \leq n-1\}$ and the edges of DT_n, $E(DT_n) = \{u_i u_{i+1} : 1 \leq i \leq n\} \cup \{u_i v_i, v_i u_{i+1}, u_i w_i, w_i u_{i+1} : 1 \leq i \leq n-1\}$. The number of vertices and edges in DT_n are $3n-2$ and $5n-5$ respectively. Now split the proof is into following three cases.

Case 1: Suppose $n = 3k$. Define coloring $c : V(DT_n) \longrightarrow \{1,2,3,4,5,6,7\}$ for $n \geq 3$ by $c(u_i) = 1$ ($i = 1,4,7,...,n-2$), $c(u_i) = 2$ ($i = 2,5,8,...,n-1$), $c(u_i) = 3$ ($i = 3,6,9,...,n$). Two sub cases arise here for even n and odd n.

Sub case 1: If n is odd, $c(v_i) = 4$ ($i = 1,3,5,...,n-2$), $c(v_i) = 5$ ($i = 2,4,6,...,n-1$), $c(w_i) = 6$ ($i = 1,3,5,...,n-2$), $c(w_i) = 7$ ($i = 2,4,6,...,n-1$).

Sub case 2: If n is even, $c(v_i) = 4$ ($i = 1,3,5,...,n-1$), $c(v_i) = 5$ ($i = 2,4,6,...,n-2$), $c(w_i) = 6$ for ($i = 1,3,5,...,n-1$), $c(w_i) = 7$ ($i = 2,4,6,...,n-2$).

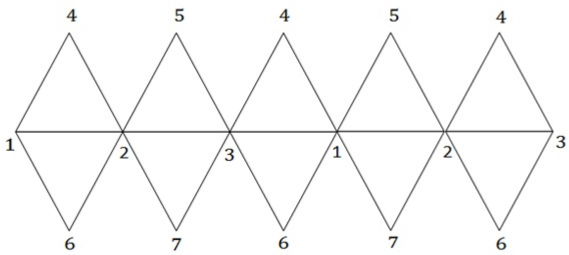

Figure 4. 1-harmonious coloring of DT_6, $h_1(DT_6) = 7$

Vertices $u_2, u_3, ..., u_{n-1}$ are of maximum degree 6 whereas the degree of u_1, u_n is 3, u_i is adjacent to u_{i+1} $(1 \leq i \leq n-1)$ and vertices u_i $(1 \leq i \leq n)$ are adjacent to v_j, w_j $(1 \leq j \leq n-1)$. Therefore 7 colors are to be needed to color DT_n. From figure 4, clearly we find that for each vertex, the adjacent vertices are colored with different color. Therefore, $h_1(DT_n) = 7$.

Case 2: Suppose $n = 3k+1$ Define coloring $c : V(DT_n) \longrightarrow$

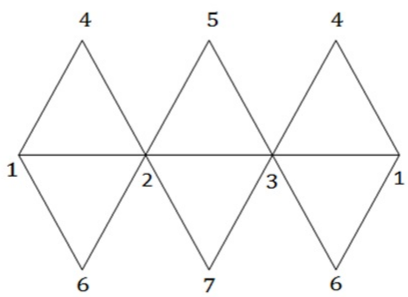

Figure 5. 1-harmonious coloring of DT_4, $h_1(DT_4) = 7$

$\{1,2,3,4,5,6,7\}$ for $n \geq 3$ by $c(u_i) = 1$ $(i = 1,4,7,...,n)$, $c(u_i) = 2$ $(i = 2,5,8,...,n-2)$, $c(u_i) = 3$ $(i = 3,6,9,...,n-1)$. Again two sub cases arises for even n and odd n, these subcases and remaining procedure can be done as described in case 1. Figure 5 shows the coloring for DT_4.

Case 3: Suppose $n = 3k+2$. Define coloring $c : V(DT_n) \longrightarrow$

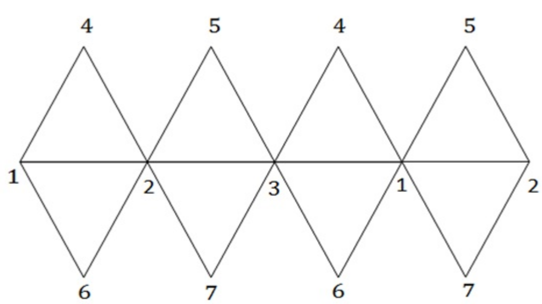

Figure 6. 1-harmonious coloring of DT_5, $h_1(DT_5) = 7$

$\{1,2,3,4,5,6,7\}$, for $n \geq 3$ by $c(u_i) = 1$ $(i = 1,4,7,...,n-1)$, $c(u_i) = 2$ $(i = 2,5,8,...,n)$, $c(u_i) = 3$ $(i = 3,6,9,...,n-2)$. Here again two sub cases arises for even n and odd n, for that we follow the procedure as described in case 1. In all three cases, 1-harmonious chromatic number, $h_1(DT_n) = 7$. Figure 6 shows the coloring for T_5. □

Theorem 3.3. *For $n \geq 3$, triple triangular snake TT_n, the 1-harmonious chromatic number, $h_1(TT_n) = \triangle(TT_n) + 1$.*

Proof. Let us consider the path graph P_n with n vertices $u_1, u_2, ..., u_n$ and TT_n as the triangular snake with maximum degree, $\triangle(TT_n) = 8$. Let the vertices of TT_n, $V(TT_n) = \{u_i : 1 \leq i \leq n\} \cup \{v_i, w_i, x_i : 1 \leq i \leq n-1\}$ the edges of TT_n $E(TT_n) = \{u_i u_{i+1} : 1 \leq i \leq n\} \cup \{u_i v_i, v_i u_{i+1}, u_i w_i, w_i u_{i+1}, u_i x_i, x_i u_{i+1} : 1 \leq i \leq n-1\}$. The number of vertices and edges in TT_n are $4n-3$ and $7n-7$ respectively. Now split the proof into following three cases.

Case 1: Suppose $n = 3k$. Define coloring $c : V(TT_n) \longrightarrow \{1,2,3,4,5,6,7,8,9\}$ for $n \geq 3$ by $c(u_i) = 1$ $(i = 1,4,7,...,n-2)$, $c(u_i) = 2$ $(i = 2,5,8,...,n-1)$, $c(u_i) = 3$ $(i = 3,6,9,...,n)$. Two sub cases arise here for even n and odd n.

Sub case 1: If n is odd, $c(v_i) = 4$ $(i = 1,3,5,...,n-2)$, $c(v_i) = 5$ $(i = 2,4,6,...,n-1)$, $c(w_i) = 6$ $(i = 1,3,5,...,n-2)$, $c(w_i) = 7$ $(i = 2,4,6,...,n-1)$, $c(x_i) = 8$ $(i = 1,3,5,...,n-2))$, $c(x_i) = 9$ $(i = 2,4,6,...,n-1)$.

Sub case 2: If n is even, $c(v_i) = 4$ $(i = 1,3,5,...,n-1)$, $c(v_i) = 5$ $(i = 2,4,6,...,n-2)$, $c(w_i) = 6$ $(i = 1,3,5,...,n-1)$, $c(w_i) = 7$ $(i = 2,4,6,...,n-2)$,$c(x_i) = 8$ $(i = 1,3,5,...,n-1)$, $c(x_i) = 9$ $(i = 2,4,6,...,n-2)$.

Vertices $u_2, u_3, ..., u_{n-1}$ are of maximum degree 8 whereas

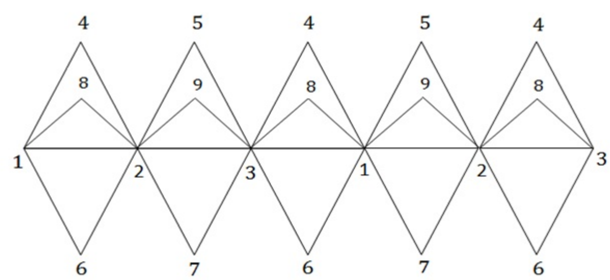

Figure 7. 1-harmonious coloring of TT_6, $h_1(TT_6) = 9$

the degree of u_1, u_n is 4, u_i is adjacent to u_{i+1} $(1 \leq i \leq n-1)$, vertices u_i $(1 \leq i \leq n)$ are adjacent to v_j,w_j and x_i $(1 \leq j \leq n-1)$. Therefore we need 9 colors to color TT_n, as shown in figure 7. Therefore 9 colors are to be needed to color TT_n .From figure 7, clearly we find that for each vertex, the adjacent vertices are colored with different color. Therefore, $h_1(TT_n) = 9$.

Case 2: Suppose $n = 3k+1$ Define coloring $c : V(TT_n) \longrightarrow \{1,2,3,4,5,6,7\}$ for $n \geq 3$ by $c(u_i) = 1$ $(i = 1,4,7,...,n)$, $c(u_i) = 2$ $(i = 2,5,8,...,n-2)$, $c(u_i) = 3$ $(i = 3,6,9,...,n-1)$. Again two sub cases arises for even n and odd n, these subcases and remaining procedure can be done as described in case 1. Figure 8 shows the coloring for TT_4.

Case 3: Suppose $n = 3k+2$. Define coloring $c : V(TT_n) \longrightarrow \{1,2,3,4,5,6,7,8,9\}$, for $n \geq 3$ by $c(u_i) = 1$ $(i = 1,4,7,...,n-1)$, $c(u_i) = 2$ $(i = 2,5,8,...,n)$, $c(u_i) = 3$ $(i = 3,6,9,...,n-2)$. Here again two sub cases arises for even n and odd n, for that we follow the procedure as describe in case 1. In all three

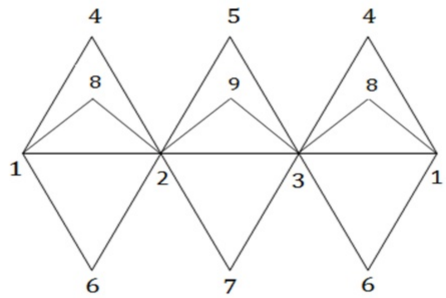

Figure 8. 1-harmonious coloring of TT_4, $h_1(TT_4) = 9$

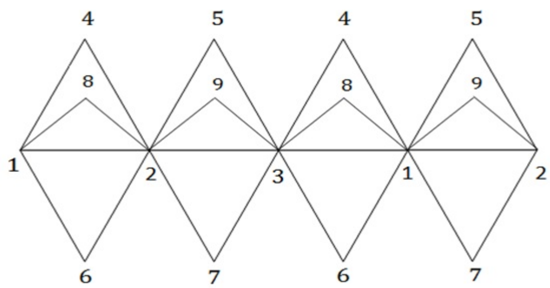

Figure 9. 1-harmonious coloring of TT_5, $h_1(TT_5) = 9$

cases, 1-harmonious chromatic number, $h_1(TT_n) = 9$. Figure 9 shows the coloring for TT_5. □

Theorem 3.4. *For $n \geq 3$, k-triangular snake kT_n, the 1- harmonious chromatic number, $h_1(kT_n) = \triangle(kT_n) + 1$.*

Proof. Consequently, it is obvious from above theorems. □

4. 1-Harmonious Coloring of Alternate Triangular Snakes

Theorem 4.1. *For $n \geq 4$, alternate triangular snake AT_n, the 1-harmonious chromatic number, $h_1(AT_n) = \triangle(AT_n) + 1$.*

Proof. Let us consider the path graph P_n with n vertices $u_1, u_2, ..., u_n$ and AT_n as the alternate triangular snake with maximum degree, $\triangle(AT_n) = 3$.

Let the vertices of AT_n, $V(AT_n) = \{u_i : 1 \leq i \leq n\} \cup \{v_i : 1 \leq i \leq \frac{n}{2}\}$ and the edges of AT_n, $E(AT_n) = \{u_iu_{i+1} : 1 \leq i \leq n\} \cup \{u_iv_i, v_iu_{i+1} : 1 \leq i \leq n-1\}$. The number of vertices and edges in AT_n are $\frac{3n}{2}$ and $2n-1$ respectively. Define coloring $c : V(AT_n)) \longrightarrow \{1,2,3,4\}$. Three case are arises here; for $n = 6k$, $n = 6k-2$ and $n = 6k+2$. Remaining proof and coloring process may be followed as discussed in the section 3.

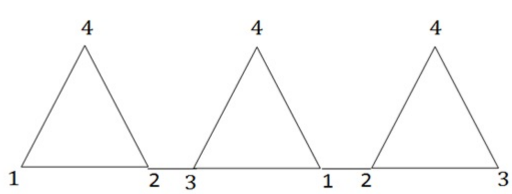

Figure 10. 1-harmonious coloring of AT_6, $h_1(AT_6) = 4$

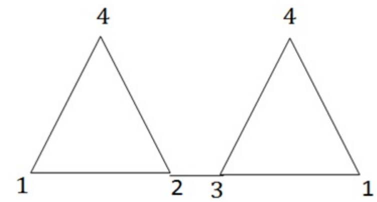

Figure 11. 1-harmonious coloring of AT_4, $h_1(AT_4) = 4$

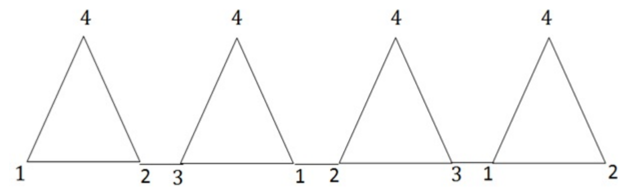

Figure 12. 1-harmonious coloring of AT_8, $h_1(AT_8) = 4$

Figure 10, 11 and 12 shows the coloring for $n = 6k$, $n = 6k-2$ and $n = 6k+2$ respectively. Hence the result. □

Theorem 4.2. *For $n \geq 4$, double alternate triangular snake $D(AT_n)$, the 1-harmonious chromatic number, $h_1(D(AT_n)) = \triangle(D(AT_n)) + 1$.*

Proof. Let us consider the path graph P_n with n vertices $u_1, u_2, ..., u_n$ and $D(AT_n)$ as the double alternate triangular snake with maximum degree, $\triangle(D(AT_n)) = 4$. Let the vertices of $D(AT_n)$, $V(D(AT_n)) = \{u_i : 1 \leq i \leq n\} \cup \{v_i, w_i : 1 \leq i \leq \frac{n}{2}\}$ and the edges of $D(AT_n)$, $E(D(AT_n)) = \{u_iu_{i+1} : 1 \leq i \leq n\} \cup \{u_iv_i, v_iu_{i+1}, u_iw_i, w_iu_{i+1} : 1 \leq i \leq n-1\}$.
The number of vertices and edges in $D(AT_n))$ are $2n$ and $3n-1$

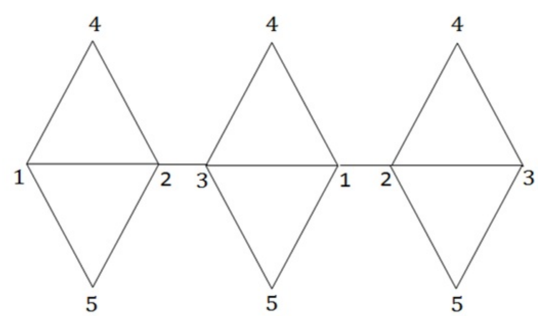

Figure 13. 1-harmonious coloring of $D(AT_6)$, $h_1(D(AT_6)) = 5$

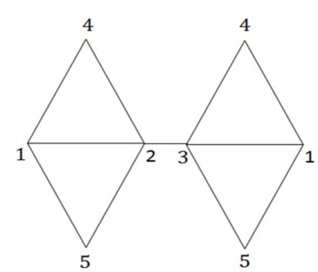

Figure 14. 1-harmonious coloring of $D(AT_4)$, $h_1(D(AT_4)) = 5$

respectively. Define coloring $c : V(D(AT_n)) \longrightarrow \{1,2,3,4,5\}$. Three case are arises here; for $n = 3k$,$n = 6k-2$ and $n = 6k+2$. Remaining proof and coloring process may be followed as discussed in the section 3. Figure 13, 14 and 15

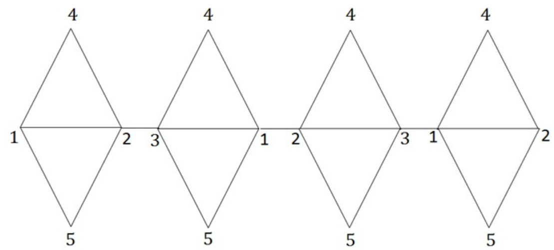

Figure 15. 1-harmonious coloring of $D(AT_8)$, $h_1(D(AT_8)) = 5$.

shows the coloring for $n = 3k$, $n = 6k-2$ and $n = 6k+2$ respectively. Hence the result. □

Theorem 4.3. *For $n \geq 4$, triple alternate triangular snake $T(AT_n)$, the 1-harmonious chromatic number, $h_1(T(AT_n)) = \triangle(T(AT_n)) + 1$.*

Proof. Let P_n Let us consider the path graph P_n with n vertices $u_1, u_2, \ldots, u_n$ and $T(AT_n)$ as the triple alternate triangular snake with maximum degree, $\triangle(T(AT_n)) = 5$.

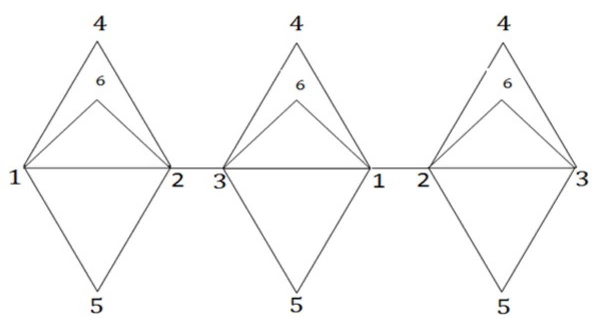

Figure 16. 1-harmonious coloring of $T(AT_6)$, $h_1(T(AT_6)) = 6$

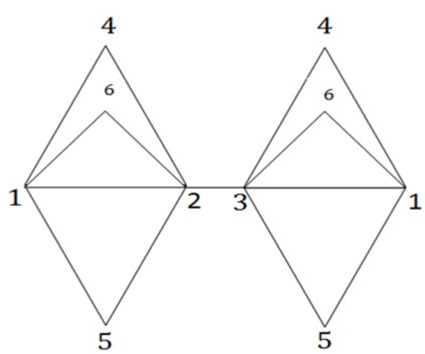

Figure 17. 1-harmonious coloring of $T(AT_4)$, $h_1(T(AT_4)) = 6$

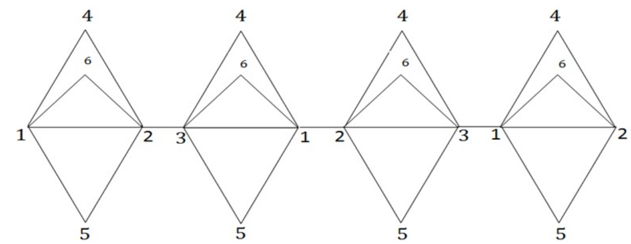

Figure 18. 1-harmonious coloring of $T(AT_8)$, $h_1(T(AT_8)) = 6$

Let the vertices of $T(AT_n)$,

$$V(T(AT_n)) = \{u_i : 1 \leq i \leq n\} \cup \{v_i, w_i, x_i : 1 \leq i \leq \frac{n}{2}\}$$

and the edges of $T(AT_n)$, $E(T(AT_n)) = \{u_iu_{i+1} : 1 \leq i \leq n\} \cup \{u_iv_i, v_iu_{i+1}, w_iu_{i+1}, u_ix_i, x_iu_{i+1} : 1 \leq i \leq n-1\}$. The number of vertices and edges in $T(AT_n)$ are $\frac{5n}{2}$ and $4n-1$ respectively. Define coloring $c : V(T(AT_n)) \longrightarrow \{1,2,3,4,5,6\}$ Three case are arises here; for $n = 3k$,$n = 6k-2$ and $n = 6k+2$. Remaining proof and coloring process may be followed as discussed in the section 3. Figure 16, 17 and 18 shows the coloring for $n = 3k$, $n = 6k-2$ and $n = 6k+2$ respectively. Hence the result. □

Theorem 4.4. *For $n \geq 4$, k-alternate triangular snake kAT_n, the 1-harmonious chromatic number, $h_1(kAT_n) = \triangle(kT_n) + 1$.*

Proof. Consequently, it is obvious from above theorems. □

5. Relations Between the 1-Harmonious Chromatic Number of Triangular and Alternate Triangular Snakes

From section 3 and 4, we observed the following relations between the 1-harmonious chromatic number of these triangular and alternate triangular snakes;

- $h_1(T_n) = h_1(AT_n) + 1$.
- $h_1(DT_n) = h_1(D(AT_n)) + 2$.
- $h_1(TT_n) = h_1(T(AT_n)) + 3$ and so on.... consequently,
- $h_1(kT_n) = h_1(kAT_n) + k$.

6. Conclusions

In this article, we discuss the 1-harmonious coloring and find the 1-harmonious chromatic number of triangular and alternate triangular snakes *i.e.*

$$\begin{aligned} h_1(T_n) &= \triangle(T_n)+1, \\ h_1(DT_n) &= \triangle(DT_n)+1, \\ h_1(TT_n) &= \triangle(TT_n)+1, \\ h_1(kT_n) &= 2k+3, \\ h_1(AT_n) &= \triangle(AT_n)+1, \\ h_1(D(AT_n)) &= \triangle(D(AT_n))+1, \\ h_1(T(AT_n)) &= \triangle(T(AT_n))+1 \\ h_1(kAT_n) &= \triangle(kT_n)+1. \end{aligned}$$

We also find the relations between 1-harmonious chromatic number of these snakes *i.e.* $h_1(T_n)=h_1(AT_n)+1$, $h_1(DT_n)=h_1(D(AT_n))+2$, $h_1(TT_n)=h_1(T(AT_n))+3$ and so on.... consequently, $h_1(kT_n)=h_1(kAT_n)+k$.

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