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On some cutting number topological indices of nanostar Dendrimer $NS[n]$

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Abstract

Topological indices, are the numbers related to molecular graph to permit the quantitative structure-activity/property/toxicity relationships. In this Research paper, the Arithmetic and Geometric cutting number Index, Geometric Arithmetic cutting number Index, Sum connectivity cutting number Index, Reciprocal cutting number Index, ABC cutting number Index, Inverse sum cutting number Index, Augumented cutting number Index and cutting number Multiplicative indices of Nanostar Dendrimer are determined.

Keywords

Cutting number, Reciprocal Randic Index, Inverse sum Index, Augumented Zagreb Index, Nanostar Dendrimer, Topological Index.

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Contents

1. Introduction

All graphs in this paper are considered simple and finite undirected. Here we following Bondy and Murty [1] for the undefined notation and terminology. The first and second Zagreb indices are among the oldest and most famous topological indices, introduced by Gutman and Trinajstic [9] defined as follows

$$M_1(G) = \sum_{uv\in E(G)}(d(u)+d(v))$$
$$M_2(G) = \sum_{uv\in E(G)} d(u)d(v)$$

The Harmonic Index $H(G)$ of a graph G was first appeared in Fajtlowicz, $S.$ [7] and it is defined as

$$H(G) = \sum_{uv\in E(G)} \frac{2}{d(u)+d(v)}$$

where $d(v)$ denote the degree of vertex v in G.
In [13], Vukicevic et. al defined a new topological Index "Geometric Arithmetic Index" denoted by GA(G) and is defined by

$$GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{d(u)d(v)}}{d(u)+d(v)}.$$

In [11], Zhou and Trinajstic first introduced the Sum-connectivity Index in 2008 , and it is defined as

$$\chi(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d(u)+d(v)}}.$$

In [8], Gutman et.al introduced Reciprocal Randic Index, which is defined as $\mathrm{RR}(G) = \sum_{uv\in E(G)} \sqrt{d(u)d(v)}$. One of the well-known connectivity topological Index is Atom-Bond connectivity (ABC) Index introduced by Estrada et.al in [6]. The ABC Index is defined as

$$ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{d(u)+d(v)-2}{d(u)d(v)}}.$$

The selected Inverse Sum-Index is [1] is a significant predictor of total surface area of octane isomers. The external graphs obtained with the help of Mathematical Chemistry are simple and have elegant structure. The Inverse Sum- Index is defined as follow

$$I(G)=\sum_{uv\in E(G)}\frac{d(u)d(v)}{d(u)+d(v)}.$$

The following modified version of the *ABC* index (ie, Randic index) is defined by Furtula, et. al (2010) as

$$\mathrm{AZI}(G)=\sum_{uv\in E(G)}\left(\frac{d(u)d(v)}{d(u)+d(v)-2}\right)^3.$$

A vertex v of a graph G is called a cutvertex of G if its removal increases the number of components. The vertex connectivity or simply connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal from G results in a disconnected or trivial graph. A graph G is n-connected, $n\geq 1$ if $\kappa(G)\geq n$. A graph G is 2 -connected if and only if G is nontrivial, connected and contains no cut vertices. A cutting number [4] $c(v)$ of a vertex $v\in V(G)$ in a connected graph G is the number of pairs of vertices $\{v,w\}$ such that v and w are in different components of $G-v$. The cutting number based topological indices of graphs are introduced in this paper. For 2-connected graphs, cutting number of each vertex is zero. So, we define these indices, for graphs with cut vertices only.

We define the cutting number first and Second Zagreb indices as

$$M_{1C}(G)=\textstyle\sum_{uv\in E(G)}(c(u)+c(v))$$
$$M_{2C}(G)=\textstyle\sum_{uv\in E(G)}c(u)c(v).$$

We define the cutting number Harmonic Index is defined by us, as

$$H_C(G)=\sum_{uv\in E(G)}\left(\frac{2}{c(u)+c(v)}\right).$$

We define the cutting number Geometric Arithmetic Index as

$$GA_C(G)=\sum_{uv\in E(G)}\frac{2\sqrt{c(u)c(v)}}{c(u)+c(v)}.$$

We define the cutting number sum connectivity Index as

$$\chi_C(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{c(u)+c(v)}}.$$

We define the cutting number Reciprocal Randic Index as

$$\mathrm{RR}_C(G)=\sum_{uv\in E(G)}\sqrt{c(u)c(v)}.$$

We define the cutting number *ABC* Index as

$$ABC_C(G)=\sum_{uv\in E(G)}\sqrt{\frac{c(u)+c(v)-2}{c(u)c(v)}}.$$

We define the cutting number Inverse Sum Index as

$$IS_C(G)=\sum_{uv\in E(G)}\left[\frac{c(u)c(v)}{c(u)+c(v)}\right].$$

We define the cutting number Augmented Zagreb Index as

$$AZI_C(G)=\sum_{uv\in E(G)}\left[\frac{c(u)c(v)}{c(u)+c(v)-2}\right]^3,$$

where, $c(u)$ and $c(v)$ are the cutting numbers of u and v. We also define Multiplicative version of cutting number Topological Indices as follows:

Cutting number first and Second Multiplicative Zagreb indices are defined as

$$M_{1C}\pi(G)=\textstyle\sum_{uv\in E(G)}(c(u)+c(v))$$
$$M_{2C}\pi(G)=\textstyle\sum_{uv\in E(G)}c(u)c(v).$$

Cutting number Multiplicative Harmonic Index is defined as

$$H_C\pi(G)=\prod_{e=uv\in E(G)}\sqrt{\frac{2}{c(u)+c(v)}}.$$

Cutting number Multiplicative Geometric Arithmetic Index as

$$\mathrm{GA}_C\,\pi(G)=\prod_{e=uv\in E(G)}\left[\frac{2\sqrt{c(u)c(v)}}{c(u)+c(v)}\right].$$

Cutting number Multiplicative Sum Connectivity Index as

$$\chi_C\pi(G)=\prod_{e=uv\in E(G)}\left[\frac{1}{\sqrt{c(u)c(v)}}\right].$$

Cutting number Multiplicative Reciprocal Randic Connectivity Index as

$$\mathrm{RR}_c\,\pi(G)=\prod_{e=uv\in E(G)}\sqrt{c(u)c(v)}.$$

Cutting number Multiplicative Atom bond connectivity Index as

$$ABC_C\pi(G)=\prod_{e=uv\in E(G)}\sqrt{\frac{c(u)+c(v)-2}{c(u)+c(v)}}.$$

Cutting number Multiplicative Inverse Sum Index as

$$IS_C\pi(G)=\prod_{e=uv\in E(G)}\left[\frac{c(u)c(v)}{c(u)+c(v)}\right].$$

Cutting number Multiplicative Augmented Zagreb Index as

$$AZ_C\pi(G)=\prod_{e=uv\in E(G)}\left[\frac{c(u)c(v)}{c(u)+c(v)-2}\right]^3.$$

2. Cutting Number topological indices of Nanostar Dendrimer

Dendrimers are highly ordered branched macromolecules, which have attached more attention on both theoretical and

Table 1. The Nanostar Dendrimer $NS[n]$

Number of edges $\mathbf{e} = \mathbf{uv}$	Cutting number of end vertices $(\mathbf{c}(\mathbf{u}), \mathbf{c}(\mathbf{v}))$
$12 \times 2^{n-1}$	(0,0)
$6 \times 2^{n-1}$ $3 \times 2^{n-1}$ $6 \times 2^{n-1}$	[{ ((18 $\times 2^n$)-12)-6 } $\times$ 5,0] [{ ((18 $\times 2^n$)-12)-6 } $\times$5, { ((18 $\times 2^n$)-12)-7 } $\times$ 6] [{ ((18 $\times 2^n$)-12)-7 } $\times$6,0]
$6 \times 2^{n-2}$ $3 \times 2^{n-2}$ $6 \times 2^{n-2}$	[{ ((18 $\times 2^n$)-12)-18 } $\times$ 17,0] [{ ((18 $\times 2^n$)-12)-18 } $\times$ 17, { ((18 $\times 2^n$)-12)-19 } $\times$ 18] [{ ((18 $\times 2^n$)-12)-19 } $\times$ 18,0]
$\vdots$	
6×2^i 3×2^i 6×2^i	[{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-i}$)-6) } $\times$ ((6 $\times 2^{n-i}$)-7), 0] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-i}$)-6) } $\times$ ((6 $\times 2^{n-i}$)-7), { ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-i}$)-5) } $\times$ ((6 $\times 2^{n-i}$)-6)] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-i}$)-5) } $\times$ ((6 $\times 2^{n-i}$)-6), 0]
$\vdots$	
6×2^2 3×2^2 6×2^2	[{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-2}$)-6) } $\times$ ((6 $\times 2^{n-2}$)-7), 0] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-2}$)-6) } $\times$ ((6 $\times 2^{n-2}$)-7), { ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-2}$)-5) } $\times$ ((6 $\times 2^{n-2}$)-6)] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-2}$)-5) } $\times$ ((6 $\times 2^{n-2}$)-6), 0]
6×2 3×2 6×2	[{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-1}$)-6) } $\times$ ((6 $\times 2^{n-1}$)-7), 0] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-1}$)-6) } $\times$ ((6 $\times 2^{n-1}$)-7), { ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-1}$)-5) } $\times$ ((6 $\times 2^{n-1}$)-6)] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-1}$)-5) } $\times$ ((6 $\times 2^{n-1}$)-6), 0]
6×1 3×1 6×1	[{ ((18 $\times 2^n$)-12)- ((6 $\times 2^n$)-6) } $\times$ ((6 $\times 2^n$)-7), 0] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^n$)-6) } $\times$ ((6 $\times 2^n$)-7), { ((18 $\times 2^n$)-12)- ((6 $\times 2^n$)-5) } $\times$ ((6 $\times 2^n$)-6)] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^n$)-5) } $\times$ ((6 $\times 2^n$)-6), 0]
Total number of edges	$21 \times 2^n - 15$

experimental part. The topological study on these macromolecules is a new field in Research [12]. Nanostar Dendrimer $NS[n]$, where n is the defining parameter is illustrated in Fig. 1&2. The node and edge cardinalities for graph of this Nanostar Dendrimer are $18 \times 2^n - 12$ and $21 \times 2^n - 15$ respectively.

To evaluate cutting topological Indices of this Nanostar, we require number of edges and cutting number of their end vertices. This is given in the Table 1.

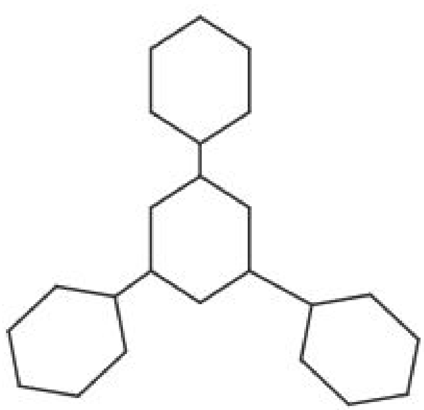

Figure 1. The Nanostar dendrimer $NS[1]$

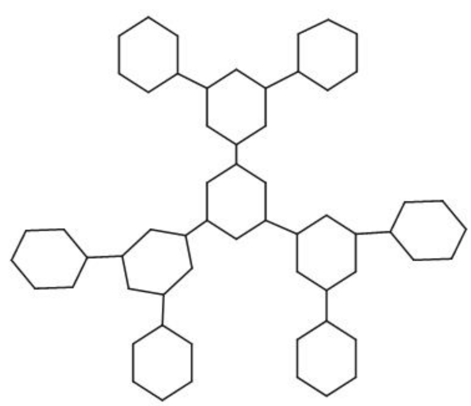

Figure 2. The Nanostar dendrimer $NS[2]$

Theorem 2.1. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$A_C[NS(n)] = 972 \times 2^{2n} - 1053 \times 2^{2n} - 324 \times 2^{2n+1} + 324 \times 2^{n+1} + 1431 \times 2^n - 378.$$

Proof.

$$A_C[NS(n)] = \sum_{uv \in E(NS[n])} \left(\frac{c(u) + c(v)}{2} \right) = \sum_{i=0}^{n-1} \left(6 \times 2^i \right)$$

$$\left[\frac{\left(\left[\left((18 \times 2^n) - 12 \right) - \left(6 \times 2^{n-i} - 6 \right) \right] \times \left(6 \times 2^{n-i} - 7 \right) \right) + 0}{2} \right]$$

$$+\sum_{i=0}^{n-1}\left(3\times 2^{i}\right)$$

$$\left[\frac{\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)}{2}\right]$$

$$+\sum_{i=0}^{n-1}\left(6\times 2^{i}\right)$$

$$\left[\frac{\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)+0}{2}\right]$$

$$=972\times 2^{2n}-1053\times 2^{2n}-324\times 2^{2n+1}$$
$$+324\times 2^{n+1}+1431\times 2^{n}-378.$$

□

Theorem 2.2. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$H_C[NS(n)]=6\sum_{i=0}^{n-1}2^{i}\left[\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}\right]$$
$$+3\sum_{i=0}^{n-1}2^{i}\left[\frac{2}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]$$
$$+6\sum_{i=0}^{n-1}2^{i}\left[\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}\right]$$

Proof.

$$H_C[NS(n)]=\sum_{uv\in E(NS[n])}\left(\frac{2}{c(u)+c(v)}\right)=\sum_{i=0}^{n-1}\left(6\times 2^{i}\right)$$
$$\left[\frac{2}{([((18\times 2^{n})-12)-(6\times 2^{n-i}-6)]\times(6\times 2^{n-i}-7))+0}\right]$$
$$+\sum_{i=0}^{n-1}\left(3\times 2^{i}\right)$$
$$\left[\frac{2}{\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)}\right]$$
$$+\sum_{i=0}^{n-1}\left(6\times 2^{i}\right)$$
$$\left[\frac{2}{([((18\times 2^{n})-12)-(6\times 2^{n-i}-5)]\times(6\times 2^{n-i}-6))+0}\right]$$
$$=6\sum_{i=0}^{n-1}2^{i}$$
$$\left[\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}\right]$$
$$+3\sum_{i=0}^{n-1}2^{i}$$
$$\left[\frac{2}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]$$
$$+6\sum_{i=0}^{n-1}2^{i}$$
$$\left[\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}\right]$$

□

Theorem 2.3. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$GA_C[NS(n)]=6\sum_{i=0}^{n-1}2^{i}$$
$$\left[\frac{\sqrt{\begin{array}{l}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}\\-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}\\-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{array}}}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]$$

Proof.

$$GA_C[NS(n)]=\sum_{uv\in E(NS[n])}\left(\frac{2\sqrt{c(u)c(v)}}{c(u)+c(v)}\right)=\sum_{i=0}^{n-1}\left(3\times 2^{i}\right)$$
$$\left[\frac{2\sqrt{\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)\times\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)}}{\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)}\right]$$
$$=6\sum_{i=0}^{n-1}2^{i}$$
$$\left[\frac{\sqrt{\begin{array}{l}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}\\-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}\\-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^{n}+1764\end{array}}}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^{n}+84}\right]$$

□

Theorem 2.4. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$GA_C[NS(n)]=6\sum_{i=0}^{n-1}2^{i}$$
$$\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^{n}+42}}\right]$$
$$+3\sum_{i=0}^{n-1}2^{i}\left[\frac{1}{\sqrt{216\times 2^{2n-i_{-7}2\times 2^{2n-2i}-234\times 2^{n}+84}}}\right]+6\sum_{i=0}^{n-1}2^{i}$$
$$\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^{n}+42}}\right]$$

Proof.

$$GA_C[NS(n)] = \sum_{uv\in E(NS[n])} \frac{1}{\sqrt{c(u)+c(v)}} = \sum_{i=0}^{n-1}\left(6\times 2^i\right)$$
$$\left[\frac{1}{\sqrt{([((18\times 2^n)-12)-(6\times 2^{n-i}-6)]\times(6\times 2^{n-i}-7))+0}}\right]$$
$$+\sum_{i=0}^{n-1}\left(3\times 2^i\right)$$
$$\left[\frac{1}{\sqrt{\begin{array}{c}\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\\ \left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)\end{array}}}\right]$$
$$\sum_{i=0}^{n-1}\left(6\times 2^i\right)$$
$$\left[\frac{1}{\sqrt{([((18\times 2^n)-12)-(6\times 2^{n-i}-5)]\times(6\times 2^{n-i}-6))+0}}\right]$$
$$=6\sum_{i=0}^{n-1}2^i\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42}}\right]$$
$$+3\sum_{i=0}^{n-1}2^i\left[\frac{1}{\sqrt{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}}\right]$$
$$+6\sum_{i=0}^{n-1}2^i\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42}}\right]$$

□

Theorem 2.5. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$GA_C(NS[n]) = 3\sum_{i=0}^{n-1}2^i$$
$$\sqrt{\begin{array}{c}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}\\ -25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}\\ -3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764\end{array}}$$

Proof.

$$GA_C(NS[n]) = \sum_{uv\in E(NS[n])}\sqrt{c(u)c(v)} = \sum_{i=0}^{n-1}\left(3\times 2^i\right)$$
$$\sqrt{\begin{array}{c}\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\\ \left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)\end{array}}$$
$$=3\sum_{i=0}^{n-1}2^i$$
$$\sqrt{\begin{array}{c}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}\\ -25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}\\ -3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764\end{array}}$$

□

Theorem 2.6. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$ABC_C(NS[n]) = 6\sum_{i=0}^{n-1}2^i$$
$$\sqrt{\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+40}{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42}}$$
$$+3\sum_{i=0}^{n-1}2^i\sqrt{\frac{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+82}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}}$$
$$+6\sum_{i=0}^{n-1}2^i$$
$$\sqrt{\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+40}{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42}}.$$

Proof.

$$ABC_C(NS[n]) = \sum_{uv\in E(NS[n])}\sqrt{\frac{c(u)+c(v)-2}{c(u)+c(v)}} = \sum_{i=0}^{n-1}\left(6\times 2^i\right)$$
$$\sqrt{\frac{([((18\times 2^n)-12)-(6\times 2^{n-i}-6)]\times(6\times 2^{n-i}-7))+0-2}{([((18\times 2^n)-12)-(6\times 2^{n-i}-6)]\times(6\times 2^{n-i}-7))+0}}$$
$$+\sum_{i=0}^{n-1}\left(3\times 2^i\right)$$
$$\sqrt{\frac{\begin{array}{c}\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\\ \left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)-2\end{array}}{\begin{array}{c}\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\\ \left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)\end{array}}}$$
$$+\sum_{i=0}^{n-1}\left(6\times 2^i\right)$$
$$\sqrt{\frac{([((18\times 2^n)-12)-(6\times 2^{n-i}-5)]\times(6\times 2^{n-i}-6))+0-2}{([((18\times 2^n)-12)-(6\times 2^{n-i}-5)]\times(6\times 2^{n-i}-6))+0}}$$
$$=6\sum_{i=0}^{n-1}2^i\sqrt{\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+40}{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42}}$$
$$+3\sum_{i=0}^{n-1}2^i\sqrt{\frac{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+82}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}}$$
$$+6\sum_{i=0}^{n-1}2^i\sqrt{\frac{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+40}{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42}}$$

□

Theorem 2.7. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$IS_C(NS[n]) = 3\sum_{i=0}^{n-1}2^i$$
$$\left[\frac{\begin{array}{c}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}\\ -25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}\\ -3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764\end{array}}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}\right]$$

Proof.

$$\mathrm{ISc}(NS[n]) = \sum_{uv\in E(NS[n])}\left[\frac{c(u)c(v)}{c(u)+c(v)}\right] = \sum_{i=0}^{n-1}\left(3\times 2^i\right)$$

$$\left[\frac{\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)\times\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)}{\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)}\right]$$

$$= 3\sum_{i=0}^{n-1}2^i \left[\frac{11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}\right]$$

□

Theorem 2.8. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$AZ_C(NS[n]) = 3\sum_{i=0}^{n-1}2^i \left[\frac{11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}\right]^3$$

Proof.

$$AZ_C(NS[n]) = \sum_{uv\in E(NS[n])}\left[\frac{c(u)c(v)}{c(u)+c(v)-2}\right]^3 = \sum_{i=0}^{n-1}\left(3\times 2^i\right)$$

$$\left[\frac{\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)\times\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)}{\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)-2}\right]^3$$

$$= 3\sum_{i=0}^{n-1}2^i \left[\frac{11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}\right]^3$$

□

3. Multiplicative Version of cutting number Topological indices of Nanostar

Theorem 3.1. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$AC_\pi(NS[n]) = 108\{(54n\times 2^{2n}-63\times 2^{2n}-18\times 2^{2n+1}+18\times 2^{n+1}+3n\times 2^n+84\times 2^n-21)(108n\times 2^{2n}-117\times 2^{2n}-36\times 2^{2n+1}+36\times 2^{n+1}+159\times 2^n-42)(54n\times 2^{2n}-54\times 2^{2n}-18\times 2^{2n+1}+18\times 2^{n+1}-3n\times 2^n+75\times 2^n-21)\}$$

Proof.

$$\mathrm{Ac}_\pi(NS[n]) = \prod_{e=uv\in E(NS[n])}\left(\frac{c(u)+c(v)}{2}\right) = \prod_{i=0}^{n-1}\left(6\times 2^i\right)\left[\frac{\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+0}{2}\right]$$

$$\times\prod_{i=0}^{n-1}\left(3\times 2^i\right)\left[\frac{\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)}{2}\right]$$

$$\times\prod_{i=0}^{n-1}\left(6\times 2^i\right)\left[\frac{\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)+0}{2}\right]$$

$$= 108\{(54n\times 2^{2n}-63\times 2^{2n}-18\times 2^{2n+1}+18\times 2^{n+1}+3n\times 2^n+84\times 2^n-21)(108n\times 2^{2n}-117\times 2^{2n}-36\times 2^{2n+1}+36\times 2^{n+1}+159\times 2^n-42)(54n\times 2^{2n}-54\times 2^{2n}-18\times 2^{2n+1}+18\times 2^{n+1}-3n\times 2^n+75\times 2^n-21)\}$$

□

Theorem 3.2. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$H_C\pi(NS[n]) = 108\prod_{i=0}^{n-1}2^{3i}\left\{\sqrt{\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42}}\times\sqrt{\frac{2}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}}\times\sqrt{\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42}}\right\}$$

Proof.

$$H_C\pi(NS[n]) = \prod_{e=uv\in E(NS[n])}\sqrt{\frac{2}{c(u)+c(v)}} = \prod_{i=0}^{n-1}(6\times 2^i)\sqrt{\frac{2}{([((18\times 2^n)-12)-(6\times 2^{n-i}-6)]x(6\times 2^{n-i}-7))+0}}$$

$$\times\prod_{i=0}^{n-1}(3\times 2^i)$$

$$\sqrt{\frac{2}{\begin{matrix}\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right] x\left(6\times 2^{n-i}-7\right)\right)+\\ \left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right] x\left(6\times 2^{n-i}-6\right)\right)\end{matrix}}}$$

$$\times\prod_{i=0}^{n-1}\left(6\times 2^i\right)$$

$$\sqrt{\frac{2}{([((18\times 2^n)-12)-(6\times 2^{n-i}-6)]\times(6\times 2^{n-i}-7))+0}}$$

$$=108\prod_{i=0}^{n-1}2^{3i}$$

$$\left\{\sqrt{\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42}}\right.$$

$$\times\sqrt{\frac{2}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}}$$

$$\left.\times\sqrt{\frac{2}{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42}}\right\}$$

□

Theorem 3.3. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$GA_C\pi(NS[n])=6\prod_{i=0}^{n-1}2^i$$

$$\left[\frac{\sqrt{\begin{matrix}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}\\ -25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}\\ -3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764\end{matrix}}}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}\right]$$

Proof.

$$GA_C\pi[NS(n)]=\prod_{e=uv\in E(NS[n])}\left(\frac{2\sqrt{c(u)c(v)}}{c(u)+c(v)}\right)=\prod_{i=0}^{n-1}\left(3\times 2^i\right)$$

$$\left[\frac{2\sqrt{\begin{matrix}\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)\times\\ \left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)\end{matrix}}}{\begin{matrix}\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\\ \left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)\end{matrix}}\right]$$

$$=6\prod_{i=0}^{n-1}2^i$$

$$\left[\frac{\sqrt{\begin{matrix}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}\\ -25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}\\ -3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764\end{matrix}}}{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}\right]$$

□

Theorem 3.4. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$GA_C\pi[NS(n)]=108\prod_{i=0}^{n-1}2^{3i}$$

$$\left\{\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42}}\right]\right.$$

$$\left[\frac{1}{\sqrt{216\times 2^{2n-i_-7}2\times 2^{2n-2i}-234\times 2^n+84}}\right]$$

$$\left.\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42}}\right]\right\}$$

Proof.

$$GA_C\pi[NS(n)]=\prod_{e=uv\in E(NS[n])}\frac{1}{\sqrt{c(u)+c(v)}}=\prod_{i=0}^{n-1}\left(6\times 2^i\right)$$

$$\left[\frac{1}{\sqrt{([((18\times 2^n)-12)-(6\times 2^{n-i}-6)]\times(6\times 2^{n-i}-7))+0}}\right]$$

$$\times\prod_{i=0}^{n-1}\left(3\times 2^i\right)$$

$$\left[\frac{1}{\sqrt{\begin{matrix}\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\\ \left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)\end{matrix}}}\right]$$

$$\times\prod_{i=0}^{n-1}\left(6\times 2^i\right)$$

$$\left[\frac{1}{\sqrt{([((18\times 2^n)-12)-(6\times 2^{n-i}-5)]\times(6\times 2^{n-i}-6))+0}}\right]$$

$$=108\prod_{i=0}^{n-1}2^{3i}$$

$$\left\{\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42}}\right]\right.$$

$$\left[\frac{1}{\sqrt{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}}\right]$$

$$\left.\left[\frac{1}{\sqrt{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42}}\right]\right\}$$

□

Theorem 3.5. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$GA_C\pi(NS[n])=3\prod_{i=0}^{n-1}2^i$$

$$\sqrt{\begin{matrix}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}\\ -25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}\\ -3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764\end{matrix}}$$

Proof.

$$GA_C\pi(NS[n])=\prod_{e=uv\in E(NS[n])}\sqrt{c(u)c(v)}=\prod_{i=0}^{n-1}\left(3\times 2^i\right)$$

$$\sqrt{\begin{matrix}\left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+\\ \left(\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)\end{matrix}}$$

$$= 3\prod_{i=0}^{n-1} 2^{i} \sqrt{\begin{array}{c} 11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} \\ -25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} \\ -3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764 \end{array}}$$

□

Theorem 3.6. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$ABC_{C}\pi(NS[n]) = 108\prod_{i=0}^{n-1} 2^{3i}$$
$$\left\{\sqrt{\frac{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 40}{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42}}\right.$$
$$\sqrt{\frac{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 82}{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}}$$
$$\left.\sqrt{\frac{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 40}{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42}}\right\}.$$

Proof.

$$ABC_{C}\pi(NS[n]) = \prod_{e=uv\in E(NS[n])}\sqrt{\frac{c(u)+c(v)-2}{c(u)+c(v)}} = \prod_{i=0}^{n-1}\left(6\times 2^{i}\right)$$
$$\sqrt{\frac{([((18\times 2^{n})-12)-(6\times 2^{n-i}-6)]\times(6\times 2^{n-i}-7))+0-2}{([((18\times 2^{n})-12)-(6\times 2^{n-i}-6)]\times(6\times 2^{n-i}-7))+0}}$$
$$\times\prod_{i=0}^{n-1}\left(3\times 2^{i}\right)$$
$$\sqrt{\frac{\begin{array}{c}\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+ \\ \left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)-2\end{array}}{\begin{array}{c}\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+ \\ \left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)\end{array}}}$$
$$\times\prod_{i=0}^{n-1}\left(6\times 2^{i}\right)$$
$$\sqrt{\frac{([((18\times 2^{n})-12)-(6\times 2^{n-i}-5)]\times(6\times 2^{n-i}-6))+0-2}{([((18\times 2^{n})-12)-(6\times 2^{n-i}-5)]\times(6\times 2^{n-i}-6))+0}}$$
$$= 108\prod_{i=0}^{n-1} 2^{3i}$$
$$\left\{\sqrt{\frac{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 40}{108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-i} - 126\times 2^{n} + 42}}\right.$$
$$\sqrt{\frac{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 82}{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}}$$
$$\left.\sqrt{\frac{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 40}{108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^{n} + 42}}\right\}$$

□

Theorem 3.7. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$IS_{C}\pi(NS[n]) = 3\prod_{i=0}^{n-1} 2^{i}$$
$$\left[\frac{\begin{array}{c} 11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} \\ -25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} \\ -3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764 \end{array}}{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}\right]$$

Proof.

$$\mathrm{IS_C}\,\pi(NS[n]) = \prod_{e=uv\in E(NS[n])}\left[\frac{c(u)c(v)}{c(u)+c(v)}\right] = \prod_{i=0}^{n-1}\left(3\times 2^{i}\right)$$
$$\left[\frac{\begin{array}{c}\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)\times \\ \left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)\end{array}}{\begin{array}{c}\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+ \\ \left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)\end{array}}\right]$$
$$= 3\prod_{i=0}^{n-1} 2^{i}$$
$$\left[\frac{\begin{array}{c} 11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} \\ -25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} \\ -3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764 \end{array}}{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 84}\right]$$

□

Theorem 3.8. *Let $NS[n]$ be a Nanostar Dendrimer. Then,*

$$AZ_{C}\pi(NS[n]) = 3\prod_{i=0}^{n-1} 2^{i}$$
$$\left[\frac{\begin{array}{c} 11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} \\ -25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} \\ -3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764 \end{array}}{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 82}\right]^{3}$$

Proof.

$$AZ_{C}\pi(NS[n]) = \prod_{e=uv\in E(NS[n])}\left[\frac{c(u)c(v)}{c(u)+c(v)-2}\right]^{3} = \prod_{i=0}^{n-1}\left(3\times 2^{i}\right)$$
$$\left[\frac{\begin{array}{c}\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)\times \\ \left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)\end{array}}{\begin{array}{c}\left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right)+ \\ \left(\left[((18\times 2^{n})-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right)-2\end{array}}\right]^{3}$$

$$= 3\prod_{i=0}^{n-1} 2^i \left[\frac{\begin{array}{c} 11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i} \\ -25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i} \\ -3060\times 2^{2n-2i} + 13608\times 2^{2n} - 9828\times 2^{n} + 1764 \end{array}}{216\times 2^{2n-i} - 72\times 2^{2n-2i} - 234\times 2^{n} + 82} \right]^3$$

□

Example 3.9. *For the Nanostar Dendrimer NS[1], we shall compute the indices. (Refer Fig. 1.)*

Table 2

Number of edges $e = uv$	Cutting number of end vertices $(c(u), c(v))$
$12\times 2^{n-1}$	$(0,0)$
$6\times 2^{n-1}$	$[\{((18\times 2^n)-12)-6\}\times 5, 0]$
$3\times 2^{n-1}$	$[\{((18\times 2^n)-12)-6\}\times 5,$ $\{((18\times 2^n)-12)-7\}\times 6]$
$6\times 2^{n-1}$	$[\{((18\times 2^n)-12)-7\}\times 6, 0]$

$$\begin{aligned} \mathrm{Ac}[NS(n)] &= \sum_{uv\in E(NS[n])} \left(\frac{c(u)+c(v)}{2}\right) \\ &= \sum_{i=0}^{n-1} \left(6\times 2^i\right) \left[\frac{\{((18\times 2^n)-12)-6\}\times 5+0}{2}\right] \\ &\quad + \sum_{i=0}^{n-1} \left(3\times 2^i\right) \left[\frac{\{((18\times 2^n)-12)-6\}\times 5}{2} + \frac{\{((18\times 2^n)-12)-7\}\times 6}{2}\right] \\ &\quad + \sum_{i=0}^{n-1} \left(6\times 2^i\right) \left[\frac{\{((18\times 2^n)-12)-7\}\times 6+0}{2}\right] \\ &= \left(6\times 2^0\right)[45] + \left(3\times 2^0\right)[96] + \left(6\times 2^0\right)[51] = 864. \end{aligned}$$

Similarly

$$H_c[NS(n)] = 2303/8160$$

$$\mathrm{GAc}[NS(n)] = \sqrt{9}180/32.$$

4. Conclusion

In this paper, we worked on a chemical structure Nanostar dendrimer and studied their cutting number topological indices and Multiplicative cutting number topological indices. We have determined sum and Multiplicative Arithmetic cutting number index, Harmonic cutting number index, Geometric Arithmetic cutting number index, Sum connectivity cutting number index, Reciprocal Randic cutting number index, ABC cutting number index, Inverse sum and Augmented Zagreb cutting number Index for Nanostar dendrimer. It would be interesting to investigate other topological indices of these Nanostar dendrimer in future.

References

[1] Ashrafi, M. Saheli, The eccentric connectivity index of a new class of Nanostar Dendrimers, Optoelectron, *Adv. Mater. Rapid Commun.,* 4(6)(2010), 898–899.

[2] A. T. Balaban, Highly discriminating distance-based topological Index, *Chem. Phys. Lett.,* 89(1982), 399-404.

[3] M. Bhanumathi and K. Easu Julia Rani, On Multiplicative sum connectivity Index, Multiplicative Randic Index and Multiplicative Harmonic Index of some Nanostar Dendrimers, *International journal of Engineering Science, Advanced Computing and Bio –Tech.,* 9, 52-67.

[4] J. A. Bondy, U. S. Murty, *Graph Theory and its Applications,* The Macmillan Press, London, 1976.

[5] Buckly and F. Harary, *Distance in Graphs*, Addison-Wesly, Reading.

[6] M. V. Diudea, G. Katona, In:dendritic Macromol, *G. A. Ed. Advan,* 4(135)(1999).

[7] E. Estrada, L. Torres, L. Rodriguez, I. Gutman, An atom-bond connectivity index: Modelling the enthalpy of formation of alkanes, *Indian J. Chem.,* 37A(849)(1998).

[8] S. Fajtlowicz, On conjectures of Graffitil II, *Congr. Number.,* 60(1987), 187-197.

[9] I. Gutman, B. Furtula, C. Elphick, Three New/old vertex degree based topological Indices, *MATCH Commun. Math. Comput. Chem.*, T2(2014), 617-632.

[10] I. Gutman, N. Trinajstic, Graph theory and molecular orbitals, Total-electron energy of alternant hydrocarbons, *Chemical Physics Letters,* 17(1972), 535-538.

[11] V. R. Kulli, Multiplicative connectivity Indices of $TU(4c8[m,n]$ and $TUC4[m,n]$ Nanotubes, *Journal of computer and Mathematical sciences*, 7(11)(2016), 599-605.

[12] B. LuCic, N. Trinajstic, B. Zhou, Comparision between the sum-connectivity Index and product-connectivity Index for benzenoid hydrocarbons, *Chem. Phys. Lett,* 475(2009), 146-148.

[13] D. A. Tomalia, Aldrichimica Acta, Birth of a new macromolecular Architecture, *Dendrimers as Quantized Building Block for Nanoscale Synthetic Polymer Chemistry*, 37(39)(2004).

[14] D. Vukicevic, B. Furtula, Topological Index based on the ratios of geometrical and arithmetical means of end vertex degrees of edges, *J. Math. Chem.*, 46(2009), 1369-1376.

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