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On fractional neutral Volterra-Fredholm integro-differential systems with non-instantaneous impulses in Banach space

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Abstract

The primary objective of this paper is to analyze the existence of PC-mild solution of fractional neutral Volterra-Fredholm integro-differential systems with non-instantaneous impulses in Banach spaces. Based on the Banach contraction principle, we develop the main results. An example is given to support the validation of the theoretical results achieved.

Keywords

Fractional neutral equations, mild solution, non-instantaneous impulses, fixed point theorem.

AMS Subject Classification

34K30, 35R12, 26A33.

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1. Introduction

In the past decades, research on fractional differential equations has been published. There has been a considerable improvement and new applications have been proposed since then. For example, in [3], where models of viscoplasticity are discussed, some examples of applications of fractional order equations can be identified. Due to its broad applications in physics, economics, population dynamics, control theory, etc., the study of differential equations with sudden and instantaneous impulses seems to be of great importance. The behavior of instantaneous impulses, however, does not characterize some physical processes. For this reason, Hernandez et al. [2] introduced the concept of non-instantaneous impulses.

Motivated by [1, 2, 4], in this paper, we consider a class of fractional neutral Volterra-Fredholm integro-differential systems with non-instantaneous impulses of the form

$$\begin{aligned}
&{}^cD^{\alpha}[x(t)+h(t,x(t))]\\
&\quad = A[x(t)+h(t,x(t))]\\
&\qquad + f\left(t,x(t),\int_0^t k(t,s,x(s))ds,\int_0^T \widetilde{k}(t,s,x(s))ds\right),\\
&\qquad t\in(s_i,t_{i+1}], i=0,1,2,\ldots,m\\
&x(t)=I_i(x(t_i))+g_i(t,x(t)),\quad t\in(t_i,s_i], i=1,2,\ldots,m \qquad (1.1)\\
&x(0)=x_0,
\end{aligned}$$

where ${}^cD^{\alpha}$ is the Caputo fractional derivative of order $\alpha\in(0,1)$ with the lower limit zero, $A: D(A)\subset X\to X$ is the generator of a C_0 -semigroup of bounded linear operators $\{T(t)\}_{t\geq 0}$ on a Banach space $(X,\|\cdot\|), x_0\in X, 0=t_0=s_0<t_1\leq s_1<t_2\leq s_2<\cdots<t_m\leq s_m<t_{m+1}=T$ are fixed numbers, $h:[0,T]\times X\to X$; $g_i\in C((t_i,s_i]\times X;X), I_i: X\to X$ for $i=1,2,\ldots,m$, $f:[0,T]\times X^3\to X$ is a nonlinear function and $k,\widetilde{k}:\Delta\times X\to X$, where $\Delta=\{(x,s): 0\leq s\leq x\leq\tau\}$ are given functions which satisfies assumptions to be specified later on. For our convenience, we denote $E_1x(t)=\int_0^t k(t,s,x(s))ds$ and $E_2x(t)=\int_0^T \widetilde{k}(t,s,x(s))ds$.

The impulses in problem (1.1) start abruptly at the points t_i and their action continues on the interval $[t_i,s_i]$. To be precise, the function x takes an abrupt impulse at t_i and follows different rules in the two subintervals $(t_i, s_i]$ and $(s_i, t_{i+1}]$ of the interval $(t_i, t_{i+1}]$. At the point s_i, the function x is continuous. The term $I_i(x(t_i))$ means that the impulses are also related to the value of $x(t_i) = x(t_i^-)$.

We remark that if $t_i = s_i$ and the second equation of (1) takes the form of $\Delta x(t_i) = I_i(x(t_i)) = x(t_i^+) - x(t_i^-)$ with $x(t_i^+) = \lim_{\varepsilon \to 0^+} x(t_i + \varepsilon), x(t_i^-) = \lim_{\varepsilon \to 0^-} x(t_i + \varepsilon)$ representing the right and left limits of $x(t)$ at $t = t_i$.

We also study the nonlocal Cauchy problems for impulsive fractional evolution equations

$$\begin{aligned} &{}^cD^\alpha[x(t) + h(t, x(t))] \\ &\quad = A[x(t) + h(t, x(t))] \\ &\quad + f\left(t, x(t), \int_0^t k(t, s, x(s))ds, \int_0^T \widetilde{k}(t, s, x(s))ds\right), \\ &\qquad t \in (s_i, t_{i+1}], i = 0, 1, 2, \ldots, m \\ &x(t) = I_i(x(t_i)) + g_i(t, x(t)), \quad t \in (t_i, s_i], i = 1, 2, \ldots, m \qquad (1.2) \\ &x(0) = x_0 + b(x), \end{aligned}$$

where $A, f, h, I_i, g_i, k, \widetilde{k}$ are the same as above, b is a given function; this constitutes a nonlocal Cauchy problem. It is well known that the nonlocal condition has a better effect on the solution and is more precise for physical measurements than the classical initial condition alone.

The rest of the paper is organized as follows. In Section 2, we present the notations, definitions and preliminary results needed in the following sections. In Section 3 is concerned with the existence results of problems (1.1) and (1.2). An example is given in Section 4 to illustrate the results.

2. Preliminaries

Let us set $J = [0, T], J_0 = [0, t_1], J_1 = (t_1, t_2], \ldots, J_{m-1} = (t_{m-1}, t_m], J_m = (t_m, t_{m+1}]$ and introduce the space $PC(J, X) := \{u : J \to X \mid u \in C(J_k, X), k = 0, 1, 2, \ldots, m,$ and there exist $u(t_k^+)$ and $u(t_k^-), k = 1, 2, \ldots, m,$ with $u(t_k^-) = u(t_k)\}$. It is clear that $PC(J, X)$ is a Banach space with the norm $\|u\|_{PC} = \sup\{\|u(t)\| : t \in J\}$.

Definition 2.1. *[3] The Riemann-Liouville fractional integral of order q with the lower limit zero for a function f is defined as*

$$I^q f(t) = \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} f(s) ds, \quad q > 0$$

provided the integral exists, where $\Gamma(\cdot)$ is the gamma function.

Definition 2.2. *[3] The Riemann-Liouville derivative of order q with the lower limit zero for a function $f : [0, \infty) \to \mathbb{R}$ can be written as*

$${}^LD^q f(t) = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t (t-s)^{n-q-1} f(s) ds, \ n-1 < q < n, t > 0$$

Let us recall the following definition of mild solutions for fractional evolution equations involving the Caputo fractional derivative.

Definition 2.3. *[5, 6] A function $x \in C(J, X)$ is said to be a mild solution of the following problem:*

$$\begin{cases} {}^cD^\alpha x(t) = Ax(t) + y(t), & t \in (0, T] \\ x(0) = x_0 \end{cases}$$

if it satisfies the integral equation

$$x(t) = P_\alpha(t) x_0 + \int_0^t (t-s)^{\alpha-1} Q_\alpha(t-s) y(s) ds.$$

Here

$$P_\alpha(t) = \int_0^\infty \xi_\alpha(\theta) T(t^\alpha \theta) d\theta, \ Q_\alpha(t) = \alpha \int_0^\infty \theta \xi_\alpha(\theta) T(t^\alpha \theta) d\theta$$
$$\xi_\alpha(\theta) = \frac{1}{\alpha} \theta^{-1-\frac{1}{\alpha}} \sigma_\alpha\left(\theta^{-\frac{1}{\alpha}}\right) \geq 0$$

$$\varpi_\alpha(\theta) = \frac{1}{\pi} \sum_{n=1}^\infty (-1)^{n-1} \theta^{-n\alpha-1} \frac{\Gamma(n\alpha+1)}{n!} \sin(n\pi\alpha), \ \theta \in (0, \infty)$$

and ξ_α is a probability density function defined on $(0, \infty)$, that is,

$$\xi_\alpha(\theta) \geq 0, \theta \in (0, \infty), \quad \int_0^\infty \xi_\alpha(\theta) d\theta = 1$$

It is not difficult to verify that

$$\int_0^\infty \theta \xi_\alpha(\theta) d\theta = \frac{1}{\Gamma(1+\alpha)}.$$

We make the following assumption on A in the whole paper.

H(A) : The operator A generators a strongly continuous semigroup $\{T(t) : t \geq 0\}$ in X, and there is a constant $M_A \geq 1$ such that $\sup_{t \in [0, \infty)} \|T(t)\|_{L(X)} \leq M_A$. For any $t > 0, T(t)$ is compact.

Lemma 2.4. *[5, 6] Let* H(A) *hold, then the operators P_α and Q_α have the following properties:*

(1) For any fixed $t \geq 0, P_\alpha(t)$ and $Q_\alpha(t)$ are linear and bounded operators, and for any $x \in X$,

$$\|P_\alpha(t) x\| \leq M_A \|x\|, \quad \|Q_\alpha(t) x\| \leq \tfrac{\alpha M_A}{\Gamma(1+\alpha)} \|x\|$$

(2) $\{P_\alpha(t), t \geq 0\}$ and $\{Q_\alpha(t), t \geq 0\}$ are strongly continuous;

(3) for every $t > 0, P_\alpha(t)$ and $Q_\alpha(t)$ are compact operators.

Next, by using the concept discussed in [1], we define the following definition of the mild solution for problem (1.1).

Definition 2.5. *A function $x \in PC(J, X)$ is said to be a PC -mild solution of problem (1.1) if it satisfies the following* *relation:*

$$x(t) = \begin{cases} P_\alpha(t)[x_0 + h(0,x(0))] - h(t,x(t)) + \int_0^t (t-s)^{\alpha-1} Q_\alpha(t-s) \\ f(s,x(s),E_1x(s),E_2x(s))ds, t \in [0,t_1] \\ I_1(x(t_1)) + g_1(t,x(t)), t \in (t_1,s_1] \\ P_\alpha(t-s_1)d_1 - h(t,x(t)) + \int_0^t (t-s)^{\alpha-1} Q_\alpha(t-s) \\ f(s,x(s),E_1x(s),E_2x(s))ds, t \in [s_1,t_2] \\ \ldots, \\ I_i(x(t_i)) + g_i(t,x(t)), t \in (t_i,s_i], i = 1,2,\ldots,m, \\ P_\alpha(t-s_i)d_i - h(t,x(t)) + \int_0^t (t-s)^{\alpha-1} Q_\alpha(t-s) \\ f(s,x(s),E_1x(s),E_2x(s))ds, t \in [s_i,t_{i+1}] \end{cases}$$

where, for $i = 1,2,\ldots,m$

$$\begin{aligned} d_i = & I_i(x(t_i)) + g_i(s_i,x(s_i)) + h(s_i,x(s_i)) \\ & - \int_0^{s_i} (s_i - s)^{\alpha-1} Q_\alpha(s_i - s) f(s,x(s),E_1x(s),E_2x(s))ds. \end{aligned} \tag{2.1}$$

3. Existence Results

In this section, we present and prove the existence and uniqueness of the system (1.1) under Banach contraction principle fixed point theorem.

From Definition 3.1, we define an operator $S : PC(J,X) \to PC(J,X)$ as

$$(Sx)(t) = \begin{cases} P_\alpha(t)[x_0 + h(0,x(0))] - h(t,x(t)) + \int_0^t (t-s)^{\alpha-1} Q_\alpha(t-s) \\ f(s,x(s),E_1x(s),E_2x(s))ds,\ t \in [0,t_1] \\ I_i(x(t_i)) + g_i(t,x(t)),\ t \in (t_i,s_i] \\ P_\alpha(t-s_i)d_i - h(t,x(t)) + \int_0^t (t-s)^{\alpha-1} Q_\alpha(t-s) \\ f(s,x(s),E_1x(s),E_2x(s))ds,\ t \in [s_i,t_{i+1}] \end{cases}$$

with $d_i, i = 1,2,\ldots,m$, defined by (2.1).

To prove our first existence result we introduce the following assumptions:

(H(f)) The function $f \in C(J \times X^3; X)$ and there exists $L_f \in L^{\frac{1}{\tau}}(J,\mathbb{R}^+)$ with $\tau \in (0,\alpha)$ such that

$$\begin{aligned} &\|f(t,x_1,x_2,x_3) - f(t,y_1,y_2,y_3)\| \\ &\quad \le L_f(t)[\|x_1-y_1\| + \|x_2-y_2\| + \|x_3-y_3\|] \end{aligned}$$

for all $(x_1,x_2,x_3),(y_1,y_2,y_3) \in X$ and every $t \in J$.

(H(h)) The function $h \in C([0,T] \times X; X)$ and there exists a positive constant $L_h > 0$ in a ways that

$$\|h(t,x) - h(t,y)\| \le L_h\|x-y\|, \quad x,y \in X \text{ and } t \in J.$$

$(H(k,\widetilde{k}))$ The functions $k,\widetilde{k} : \Delta \times X \to X$ are continuous and there exist constants $L_k, L_{\widetilde{k}} > 0$ such that

$$\left\|\int_0^t [k(t,s,x(s)) - k(t,s,y(s))]ds\right\| \le L_k\|x-y\|,$$

for all, $x,y \in X$; and

$$\left\|\int_0^T [\widetilde{k}(t,s,x(s)) - \widetilde{k}(t,s,y(s))]ds\right\| \le L_{\widetilde{k}}\|x-y\|,$$

for all, $x,y \in X$;

(H(I)) For $i = 1,2,\ldots,m, I_i \in C(X,X)$ and there is a constant $L_I > 0$ such that $\|I_i(x) - I_i(y)\| \le L_I\|x-y\|$ for all $x,y \in X$.

(H(g)) For $i = 1,2,\ldots,m$, the functions $g_i \in C([t_i,s_i] \times X; X)$ and there exists $L_g \in C(J,\mathbb{R}^+)$ such that

$$\|g_i(t,x) - g_i(t,y)\| \le L_g(t)\|x-y\|$$

for all $x,y \in X$ and $t \in [t_i,s_i]$.

Theorem 3.1. *Assume* H(f)$, H(h),$ H(I)$, H(k,\widetilde{k})$ *and* H(g) *are satisfied and*

$$\begin{aligned} &\left[M_A\left(L_I + \|L_g\|_{C(J)}\right) + (1+M_A)\left\{L_h + \frac{\alpha M_A}{\Gamma(\alpha+1)}\left(\frac{1-\tau}{\alpha-\tau}\right)^{1-\tau}\right.\right. \\ &\left.\left. T^{\alpha-\tau}\|L_f\|_{L^{\frac{1}{\tau}}(J)}[1 + L_k + L_{\widetilde{k}}]\right\}\right] < 1. \end{aligned} \tag{3.1}$$

Then there exists a unique PC -mild solution of problem (1.1).

Proof. Proof From the assumptions it is easy to show that the operator S is well defined on $PC(J,X)$ Let $x,y \in PC(J,X)$. For $t \in [0,t_1]$, from Lemma 2.1, we have

$$\begin{aligned} &\|(Sx)(t) - (Sy)(t)\| \\ &\le \|h(t,x(t)) - h(t,y(t))\| + \int_0^t (t-s)^{\alpha-1}\|Q_\alpha(t-s) \\ &(f(s,x(s),E_1x(s),E_2x(s)) - f(s,y(s),E_1y(s),E_2y(s)))\|ds \\ &\le \left[L_h + \frac{\alpha M_A}{\Gamma(\alpha+1)}\left(\frac{1-\tau}{\alpha-\tau}\right)^{1-\tau} t_1^{\alpha-\tau}\|L_f\|_{L^{\frac{1}{\tau}}([0,t_1])}[1 + L_k + L_{\widetilde{k}}]\right] \\ &\quad \|x-y\|_{PC}. \end{aligned}$$

Similarly, we have, for $t \in (t_i,s_i], i = 1,2,\ldots,m$

$$\begin{aligned} \|(Sx)(t) - (Sy)(t)\| &\le \|I_i(x(t_i)) - I_i(y(t_i))\| \\ &\quad + \|g_i(t,x(t)) - g_i(t,y(t))\| \\ &\le \left(L_I + \|L_g\|_{C(J)}\right)\|x-y\|_{PC} \end{aligned}$$

and, for $t \in [s_i,t_{i+1}], i = 1,2,\ldots,m$

$$
\begin{aligned}
&\|(Sx)(t)-(Sy)(t)\| \\
&\leq \Bigg\| P_\alpha(t-s_i)\Big[I_i(x(t_i))-I_i(y(t_i))+g_i(s_i,x(s_i)) \\
&-g_i(s_i,y(s_i))+h(s_i,x(s_i))-h(s_i,y(s_i)) \\
&-\int_0^{s_i}(s_i-s)^{\alpha-1}Q_\alpha(s_i-s)(f(s,x(s),E_1x(s),E_2x(s)) \\
&-f(s,y(s),E_1y(s),E_2y(s)))ds\Big]\Bigg\| \\
&+\|h(t,x(t))-h(t,y(t))\|+\int_0^t(t-s)^{\alpha-1} \\
&\|Q_\alpha(t-s)(f(s,x(s),E_1x(s),E_2x(s)) \\
&-f(s,y(s),E_1y(s),E_2y(s)))\|ds \\
&\leq M_A\left(L_I+\|L_g\|_{C(J)}+L_h+\frac{\alpha M_A}{\Gamma(\alpha+1)}\left(\frac{1-\tau}{\alpha-\tau}\right)^{1-\tau}\right. \\
&\left. s_i^{\alpha-\tau}\|L_f\|_{L^{\frac{1}{\tau}}([0,s_i])}[1+L_k+L_{\widetilde{k}}]\right)\|x-y\|_{PC} \\
&+L_h+\frac{\alpha M_A}{\Gamma(\alpha+1)}\left(\frac{1-\tau}{\alpha-\tau}\right)^{1-\tau}t_{i+1}^{\alpha-\tau}\|L_f\|_{L^{\frac{1}{\tau}}([0,t_{i+1}])} \\
&[1+L_k+L_{\widetilde{k}}]\|x-y\|_{PC}.
\end{aligned}
$$

From the above we can deduce that (since $M_A \geq 1$)

$$
\begin{aligned}
&\|(Sx)(t)-(Sy)(t)\|_{PC} \\
&\leq \Big[M_A\left(L_I+\|L_g\|_{C(J)}\right)+(1+M_A) \\
&\left\{L_h+\tfrac{\alpha M_A}{\Gamma(\alpha+1)}\left(\tfrac{1-\tau}{\alpha-\tau}\right)^{1-\tau}T^{\alpha-\tau}\|L_f\|_{L^{\frac{1}{\tau}}(J)}[1+L_k+L_{\widetilde{k}}]\right\}\Big] \\
&\|x-y\|_{PC}.
\end{aligned}
$$

Then it follows from condition (3.1) that S is a contraction on the space $PC(J,X)$. Hence by the Banach contraction mapping principle, S has a unique fixed point $x \in PC(J,X)$ which is just the unique PC-mild solution of problem (1.1). The proof is now complete. □

Definition 3.2. *A function $x \in PC(J,X)$ is said to be a PC -mild solution of problem (1.2) if it satisfies the following relation:*

$$
x(t) = \begin{cases}
P_\alpha(t)(x_0+b(x)+h(0,x(0)))-h(t,x(t)) \\
\quad +\int_0^t(t-s)^{\alpha-1}Q_\alpha(t-s)f(s,x(s),E_1x(s),E_2x(s))ds, \\
\qquad t\in[0,t_1] \\
I_i(x(t_i))+g_i(t,x(t)), t\in(t_i,s_i], i=1,2,\ldots,m \\
P_\alpha(t-s_i)d_i+\int_0^t(t-s)^{\alpha-1}Q_\alpha(t-s) \\
f(s,x(s),E_1x(s),E_2x(s))ds, t\in[s_i,t_{i+1}]
\end{cases}
$$

with $d_i, i=1,2,\ldots,m$, defined by (2.1).

H(b) : $b: PC(J,X)\to X$ and there exist a constant $L_b>0$ and for $x,y\in PC(J,X)$,

$$\|b(x)-b(y)\| \leq L_b\|x-y\|_{PC}.$$

Theorem 3.3. *Assume* H(f), $H(h)$, H(I), $H(k,\widetilde{k})$ *and* H(g) *are satisfied and*

$$
\begin{aligned}
&\Bigg[M_A\left(L_I+\|L_g\|_{C(J)}+L_b\right)+(1+M_A)\Bigg\{L_h+\frac{\alpha M_A}{\Gamma(\alpha+1)} \\
&\left(\frac{1-\tau}{\alpha-\tau}\right)^{1-\tau}T^{\alpha-\tau}\|L_f\|_{L^{\frac{1}{\tau}}(J)}[1+L_k+L_{\widetilde{k}}]\Bigg\}\Bigg]<1.
\end{aligned}
\tag{3.2}
$$

Then there exists a unique PC -mild solution of problem (1.2).

Proof. The proof of this theorem is very similar to Theorem 3.1, so we omit it. □

4. Application

A simple example is given in this section to illustrate the result.

Let $X=L^2([0,\pi])$. Define an operator $A: D(A)\subseteq X\to X$ by $Ax=x''$ with $D(A)=\{x\in X : x''\in X, x(0)=x(\pi)=0\}$. It is well known that A is the infinitesimal generator of a strongly continuous semigroup $\{T(t): t\geq 0\}$ in X. Moreover, $T(t)$ is compact for $t>0$ and $\|T(t)\|_{L(X)}\leq e^{-t}\leq 1=M_A, t\geq 0$.

Consider the following impulsive problem:

$$
\begin{aligned}
&{}^{c}D_t^{\frac{3}{4}}[u(t,y)+h(t,u(t,y))] \\
&\quad=\frac{\partial^2}{\partial y^2}[u(t,y)+h(t,u(t,y))] \\
&\quad+f(t,u(t,y),E_1u(t,y),E_2u(t,y)), \\
&\qquad t\in\left[0,\frac{1}{2}\right]\cup\left(\frac{2}{3},1\right], y\in[0,\pi] \\
&u(t,y)=I\left(u\left(t_{\frac{1}{2}},y\right)\right)+g(t,u(t,y)), \\
&\qquad t\in\left(\frac{1}{2},\frac{2}{3}\right], y\in[0,\pi] \\
&u(t,y)=u_0(y), \quad y\in[0,\pi] \\
&u(t,0)=u(t,\pi)=0, \quad t\in[0,1]
\end{aligned}
\tag{4.1}
$$

Here ${}^{c}D_t^{\frac{1}{2}}$ means that the Caputo fractional derivative is taken for the time variable t with the lower limit zero.

Let

$$
\begin{aligned}
&f(t,u(t,y),E_1u(t,y),E_2u(t,y)) \\
&\quad=\frac{e^{-t}u(t,y)}{(9+e^t)(1+|u(t,y)|)}+\frac{1}{10}\int_0^t e^{-\frac{1}{2}u(s,y)}ds \\
&\quad+\frac{1}{10}\int_0^t e^{-\frac{1}{4}u(s,y)}ds
\end{aligned}
$$

$$h(t,u(t,y))=\frac{1}{10}u(t,y), \quad I(u(t,y))=\frac{|u(t,y)|}{9+|u(t,y)|},$$

$$g(t,u(t,y))=\frac{1}{16}\sin u(t,y)+e^t$$

Define $x(t)(y) = u(t,y), (t,y) \in [0,1] \times [0,\pi]$. Then f, I, and g can be rewritten as

$$f(t,x(t),E_1x(t),E_2x(t)) = \frac{e^{-t}x(t)}{(9+e^t)(1+|x(t)|)} + \frac{1}{10}\int_0^t e^{-\frac{1}{2}x(s)}ds + \frac{1}{10}\int_0^t e^{-\frac{1}{4}x(s)}ds$$

$$h(t,x(t)) = \frac{1}{10}x(t), \quad I(x(t)) = \frac{|x(t)|}{9+|x(t)|},$$

$$g(t,x(t)) = \frac{1}{16}\sin x(t) + e^t$$

We can verify that H(f), $H(h)$, H(I), $H(k,\widetilde{k})$ and H(g) hold by putting $L_f(t) = \frac{1}{10}, L_h = \frac{1}{10}, L_I = \frac{1}{9}, L_k = \frac{1}{4}, L_{\widetilde{k}} = \frac{1}{8}$ and $L_g(t) = \frac{1}{16}$.

Moreover, since $\alpha = \frac{3}{4}$, let $\tau = \frac{1}{2}$, we have

$$M_A\left(L_I + \|L_g\|_{C(J)}\right) + (1+M_A)\left\{L_h + \frac{\alpha M_A}{\Gamma(\alpha+1)}\left(\frac{1-\tau}{\alpha-\tau}\right)^{1-\tau} T^{\alpha-\tau}\|L_f\|_{L^{\frac{1}{\tau}}(J)}[1+L_k+L_{\widetilde{k}}]\right\}$$
$$\leq \frac{1}{9} + \frac{1}{16} + 2\{0.1 + 0.8160 \times 1.4142 \times \frac{1}{10}\left[1+\frac{1}{4}+\frac{1}{8}\right]\right\} = 0.52 < 1.$$

Therefore by Theorem 3.1, we deduce that problem (4.1) has a unique PC-mild solution on $[0,1]$.

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