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On λ_g^α-closed and λ_g^α-open sets in topological spaces

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Abstract

The purpose of this paper is to introduce a new form of generalized closed set called λ_g^α-closed set which employs the notions of λ-sets and α-open sets. Some fundamental properties and characterizations of such sets are analysed. Further λ_g^α-open set is defined and some of its properties are analysed. Moreover, the relationships between the newly defined sets and already existing sets are obtained with appropriate examples.

Keywords

Topological spaces, α-closed set, Λ-set, λ-closed set, λ_g^α-closed set, λ_g^α-open set.

AMS Subject Classification

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Contents

1. Introduction

The notion of α-sets in topological spaces was introduced by Njastad[12] and studied several fundamental properties. Levine[7] introduced the notion of generalized closed sets in topological spaces. Following this, many researchers introduced several variations of generalized closed sets and investigated some stronger and weaker forms of them. Mashhour et. al. [11] defined the complement of α-sets called α-closed sets by continuing the work of NJastad [12] and established its various properties.

Λ-set was introduced by Maki [8] , which is equal to its kemel(=saturated set), i.e. to the intersection of all open supersets of A. Arenas et. al. [6] introduced and investigated the notion of λ-closed sets and λ-open sets by involving Λ-sets and closed sets. Caldas et. al. [2] introduced the notion of λ-closure of a set by utilizing the notion of λ-closed sets defined in [6]. In this paper we introduced new classes of sets called λ_g^α-closed sets and λ_g^α-open sets in topological spaces. We presented the relationships between the newly defined set and the previously existing sets with corresponding examples. Further fundamental properties and characterizations of such sets are derived.

2. Preliminaries

Throughout this paper (X,τ) will always denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X,τ), $\mathrm{cl}(A)$ and A^c denote the closure and complement of A respectively.

Definition 2.1 ([12])**.** Let(X,τ) *be a topological space. A subset A of* (X,τ) *is called an* α*-open set if* $A\subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$*. The complement of an* α*-open set is called* α*-closed. The intersection of all* α*-closed sets containing A is called* α*-closure of A and is denoted by* $cl_\alpha(A)$*.*

Definition 2.2 ([6])**.** Let(X,τ) *be a topological space. A subset A of* (X,τ) *is called* λ*-closed if* $A=L\cap D$*, where L is a* Λ*-set and D is a closed set. The complement of* λ*-closed set is called* λ*-open.*

Definition 2.3 ([2])**.** *The* λ*-closure of a subset A of a topological space* (X,τ) *is the intersection of all* λ*-closed sets containing A and is denoted by* $cl_\lambda(A)$

Definition 2.4. *A subset A of a topological space* (X,τ) *is called* *(i)* *generalized closed (briefly g-closed) [7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)*

(ii) *generalized α-closed (briefly $g\alpha$-closed) [9] if $cl_\alpha(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X,τ)*

(iii) *α-generalized closed(briefly αg-closed) [10] if $cl_\alpha(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)*

(iv) *g^*-closed [14] if $\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X,τ).*

(v) *Λ-generalized closed (briefly Λ_g-closed) [3] if $\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is λ-open in (X,τ)*

(vi) *$\Lambda - g$-closed [2] if $cl_\lambda(A) \subseteq U$ whenever $A \subseteq U$ and U is λ-open in (X,τ).*

(vii) *$g\Lambda$-closed [3] if $cl_\lambda(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).*

(viii) *$g^{**}\Lambda$-closed [1] if $cl_\lambda(A) \subseteq U$ whenever $A \subseteq U$ and U is g^*-open in (X,τ).*

(ix) *$\Lambda_{g\alpha}$-closed [13] if $cl_\alpha(A) \subseteq U$ whenever $A \subseteq U$ and U is λ-open in (X,τ).*

The complements of the above-mentioned sets are called their respective open sets.

Remark 2.5 ([6]). *Every Λ-set is λ-closed.*

Remark 2.6 ([6]). *Every closed set and open set is λ-closed.*

Remark 2.7. *(i) In α-space, every α-closed subset of (X,τ) is closed in (X,τ).*[12]

(ii) *In $T_{1/2}$-space, every g-closed subset of (X,τ) is closed in (X,τ).*[7]

(iii) In $T_{1/2}$*-space, every subset of (X,τ) is λ-closed in (X,τ).* [6]

(iv) *In T_1-space, every Λ_g-closed subset of (X,τ) is closed in (X,τ).*[3]

(v) *In αT_b-space, every αg-closed subset of (X,τ) is closed in (X,τ).*[4]

(vi) *In door-space, every subset of (X,τ) is either open or closed in (X,τ).*[5]

(vii) In $T^*_{1/2}$*-space, every g^*-closed subset of (X,τ) is closed in (X,τ). [14]*

3. λ_g^α-Closed Sets

Definition 3.1. *Let (X,τ) be a topological space. A subset A of X is said to be a λ_g^α-closed set if $cl_\lambda(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.*

Proposition 3.2. *Every λ-closed set in (X,τ) is λ_g^α-closed but not conversely.*

Proof. Let A be a λ-closed set. Let U be any α-open set containing A in X. Since A is λ closed, $cl_\lambda(A) = A \subseteq U$. Therefore A is λ_g^α-closed. □

Example 3.3. *Let $X = \{a,b,c\}$ and $\tau = \{\phi, \{a,b\}, X\}$.Then the subset $\{a\}$ is λ_g^α-closed but not λ-closed.*

Proposition 3.4. *Every closed set in (X,τ) is λ_g^α-closed but not conversely.*

Proof. Obvious from Remark 2.6 and from Proposition 3.2. □

Example 3.5. *Let $X = \{a,b,c\}$ and $\tau = \{\phi, \{a\}, X\}$. Then the subset $\{a\}$ is λ_g^α-closed but not closed.*

Proposition 3.6. *Every open set in (X,τ) is λ_g^α-closed but not conversely.*

Proof. Obvious from Remark 2.6 and from Proposition 3.2 □

Example 3.7. *Let $X = \{a,b,c\}$ and $\tau = \{\phi, \{a\}, X\}$. Then the subset $\{b,c\}$ is λ_g^α-closed but not open.*

Proposition 3.8. *Every Λ-set in (X,τ) is λ_g^α-closed but not conversely.*

Proof. Obvious from Remark 2.5 and from Proposition 3.2. □

Example 3.9. *Let $X = \{a,b,c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a,b\}, X\}$. Then the subset $\{c\}$ is λ_g^α closed but not a Λ-set.*

Proposition 3.10. *Every λ_g^α-closed set in (X,τ) is $g\Lambda$-closed but not conversely.*

Proof. Let A be a λ_g^α-closed set and let U be any open set containing A in X. As every open set is α-open and A is λ_g^α-closed, we have $cl_\lambda(A) \subseteq U$. Hence A is $g\Lambda$-closed. □

Example 3.11. *Let $X = \{a,b,c,d\}$ and $\tau = \{\phi, \{a\}, \{a,b\}, X\}$. Then the subset $\{c\}$ is $g\Lambda$- closed but not λ_g^α-closed.*

Proposition 3.12. *Let A be an α-open subset of (X,τ). Then A is λ-closed if A is λ_g^α-closed.*

Proof. Suppose A is λ_g^α-closed. Since $A \subseteq A$ and A is α-open we have $\mathrm{cl}_\lambda(A) \subseteq A$. Hence from the fact that $A \subseteq cl_\lambda(A) \subseteq \mathrm{cl}(A)$, we have A is λ-closed. □

Remark 3.13. *The following example shows that α-closed sets and λ-closed sets are independent in general.*

Example 3.14. *Let $X=\{a,b,c\}$ and $\tau=\{\phi,\{a\},X\}$. Then the subset $\{c\}$ is α-closed but not λ- closed, also the subset $\{a\}$ is λ-closed but not α-closed.*

Remark 3.15. *g-closed (resp. α-closed, αg-closed , $g\alpha$-closed , Λ_g-closed, $\Lambda_{g\alpha}$-closed)sets and λ_g^α-closed sets are independent of each other as observed from the following example.*

Example 3.16. *Let $X=\{a,b,c\}$ and $\tau=\{\phi,\{a\},X\}$. Then the subset $\{a\}$ is λ_g^α-closed but not g-closed (resp. α-closed, αg-closed, $g\alpha$-closed, Λ_g-closed, $\Lambda_{g\alpha}$-closed) also the subset $\{b\}$ is g-closed (resp. α-closed, αg-closed, $g\alpha$-closed, Λ_g-closed, $\Lambda_{g\alpha}$-closed)but not λ_g^α-closed.*

Remark 3.17. *g^*-closed sets and λ_g^α-closed sets are independent of each other as observed from the following example.*

Example 3.18. *Let $X=\{a,b,c\}$ and $\tau=\{\phi,\{a\},\{a,b\},X\}$. Then the subset $\{a\}$ is λ_g^α-closed but not g^*-closed also the subset $\{a,c\}$ is g^*-closed but not λ_g^α-closed.*

Remark 3.19. *$g^{**}\Lambda$-closed sets and λ_g^α-closed sets are independent of each other as observed from the following example.*

Example 3.20. *Let $X=\{a,b,c,d,e\}$ and $\tau=\{\phi,\{a\},\{b,c\},\{a,b,c\},\{b,c,d,e\},X\}$. Then the subset $\{b\}$ is λ_g^α-closed but not $g^{**}\Lambda$-closed also the subset $\{b,d\}$ is $g^{**}\Lambda$-closed but not λ_g^α closed.*

Remark 3.21. *In any topological space, the following example shows that the union of any two λ_g^α-closed sets need not be λ_g^α-closed.*

Example 3.22. *Let $X=\{a,b,c\}$ and $\tau=\{\phi,\{a\},\{a,b\},X\}$. Here the subsets $\{a\}$ and $\{b\}$ are λ_g^α-closed sets but their union $\{a,c\}$ is not a λ_g^α-closed set.*

Remark 3.23. *λ_g^α-closed sets will not form a topology, since it does not satisfy the condition that union of λ_g^α-closed sets is a λ_g^α-closed set.*

Remark 3.24. *In any topological space, the following example shows that the difference of two λ_g^α-closed sets need not be λ_g^α-closed.*

Example 3.25. *Let $A=\{a,b,c,d,e\}$ and $\tau=\{\phi,\{a\},\{b,c\},\{a,b,c\},\{b,c,d,e\},X\}$. Here the subsets $\{a,c,d\}$ and $\{a,c,e\}$ are λ_g^α-closed sets but their difference $\{d\}$ is not a λ_g^α-closed set.*

Proposition 3.26. *If (X,τ) is an α-space, then every α-closed set is λ_g^α-closed.*

Proof. Follows from Remark 2.7 (i) and Proposition 3.4. □

Proposition 3.27. *If (X,τ) is a $T_{1/2}$-space, then every g-closed set is λ_g^α-closed.*

Proof. Follows from Remark 2.7 (ii) and Proposition 3.4 . □

Proposition 3.28. *If (X,τ) is a $T_{1/2}$-space, then every subset is λ_g^α-closed.*

Proof. Follows from Remark 2.7 (iii) and Proposition 3.2. □

Proposition 3.29. *If (X,τ) is a T_1-space, then every Λ_g-closed set is λ_g^α-closed.*

Proof. Follows from Remark 2.7 (iv) and Proposition 3.4 □

Proposition 3.30. *If (X,τ) is an αT_b-space, then every αg-closed set is λ_g^α-closed.*

Proof. Follows from Remark 2.7 (v) and Proposition 3.4. □

Proposition 3.31. *If (X,τ) is a door space, then every subset is λ_g^α-closed.*

Proof. Follows from Remark 2.7 (vi) and Proposition 3.4 and 3.5. □

Proposition 3.32. *If (X,τ) is a $T^*_{1/2}$-space, then every g^*-closed set is λ_g^α-closed.*

Proof. Follows from Remark 2.7 (vii) and Proposition 3.4. □

Definition 3.33. *A partition space is a space where every open set is closed.*

Theorem 3.34. *Let (X,τ) be an α-space. If X is a partition space, then every subset of X is a λ_g^α-closed set.*

Proof. Let A be any subset of (X,τ) such that $A\subseteq U$ and U is α-open. Since (X,τ) is an α- space, U is open. Since X is a partition space, U is closed. As every closed set is λ-closed, U is λ-closed. Hence $cl_\lambda(A)\subseteq cl_\lambda(U)=U$. Hence every subset of X is λ_g^α-closed. □

Theorem 3.35. *In a partition space, every λ_g^α-closed set is g-closed.*

Proof. Let A be a λ_g^α-closed set and $A\subseteq U$ and U is open. Since every open set is an α-open and A is λ_g^α-closed we have $cl_\lambda(A)\subseteq U$. Since (X,τ) is a partition space, the class of λ closed sets coincide with the class of closed sets. Therefore, we have $\mathrm{cl}(A)=cl_\lambda(A)\subseteq U$. Hence A is g-closed. □

4. Application of λ_g^α-Closed Sets

Theorem 4.1. *If a subset A is λ_g^α-closed, then $cl_\lambda(A)\backslash A$ does not contain any non-empty closed set in X.*

Proof. Let A be a λ_g^α-closed set in (X,τ). Suppose F is a non-empty closed set contained in $cl_\lambda(A)\backslash A$, which implies $A\subseteq F^c$, where F^c is open. Since A is λ_g^α-closed and as every open set is α-open, we have $cl_\lambda(A)\subseteq F^c$. Hence $F\subseteq X\backslash cl_\lambda(A)$. Also, we have $F\subseteq cl_\lambda(A)$. Therefore $F\subseteq [X\backslash cl_\lambda(A)]\cap cl_\lambda(A)=\phi$. Hence $cl_\lambda(A)\backslash A$ does not contain any non- empty closed set. □

Remark 4.2. *Converse of the above theorem need not be true as seen from the following example.*

Example 4.3. *Let $X = \{a,b,c\}$ and $\tau = \{\phi,\{a\},X\}$. If $A = \{b\}$, then $cl_\lambda(A) = \{b,c\}$ and $cl_\lambda(A)\backslash A = \{c\}$, which does not contain any non-empty closed set, but A is not λ_g^α-closed.*

Theorem 4.4. *If a subset A is λ_g^α-closed, then $cl_\lambda(A)\backslash A$ does not contain any non-empty α- closed set.*

Proof. Let A be a λ_g^α-closed set in (X,τ). Suppose F is an α-closed set contained in $cl_\lambda(A)\backslash A$, which implies $A \subseteq F^c$, where F^c is α-open. Since, A is λ_g^α-closed, $cl_\lambda(A) \subseteq F^c$. Hence $F \subseteq X\backslash cl_\lambda(A)$. Also, we have $F \subseteq cl_\lambda(A)$. Therefore $F \subseteq [X\backslash cl_\lambda(A)] \cap cl_\lambda(A) = \phi$. Hence $cl_\lambda(A)\backslash A$ does not contain any non-empty α-closed set. □

Remark 4.5. *Converse of the above theorem need not be true as seen from the following example.*

Example 4.6. *Let $A = \{a,b,c,d,e\}$ and $\tau = \{\phi,\{a\},\{b,c\}, \{a,b,c\},\{b,c,d,e\},X\}$. If $A = \{e\}$, then $cl_\lambda(A) = \{d,e\}$ and $cl_\lambda(A)\backslash A = \{d\}$, which does not contain any non-empty $\alpha-$closed set, but A is not λ_g^α-closed.*

Theorem 4.7. *In a topological space (X,τ), for each $x \in X$, $\{x\}$ is α-closed or λ_g^α-open.*

Proof. Suppose $\{x\}$ is not α-closed then $X\backslash\{x\}$ is not α-open, then the only α-open set containing $X\backslash\{x\}$ is X. That is $[X\backslash\{x\}] \subseteq X$. Obviously $cl_\lambda(X\backslash\{x\}) \subseteq X$. Hence $X\backslash\{x\}$ is λ_g^α-closed and $\{x\}$ is λ_g^α-open. □

Theorem 4.8. *Let A be a λ_g^α-closed set in a topological space (X,τ). Then A is λ-closed if and only if $cl_\lambda(A)\backslash A$ is closed.*

Proof. **(Necessity)** Suppose A is λ_g^α-closed and λ-closed in a topological space (X,τ). A is λ- closed implies $cl_\lambda(A) = A$. Hence $cl_\lambda(A)\backslash A = \phi$, which is a closed set.
Sufficiency: Suppose A is λ_g^α-closed and $cl_\lambda(A)\backslash A$ is closed. Then by Theorem 4.1 $cl_\lambda(A)\backslash A$ contains no non-empty closed subsets. Hence $cl_\lambda(A)\backslash A = \phi$, which implies $cl_\lambda(A) = A$. Therefore A is λ-closed. □

Theorem 4.9. *If every λ_g^α-closed set is λ-closed then for each $x \in (X,\tau)$ either $\{x\}$ is α-closed or λ-open.*

Proof. Suppose $\{x\}$ is not α-closed, then $X\backslash\{x\}$ is not α-open. Hence, we have X is the only α-open set containing $X\backslash\{x\}$. Obviously $cl_\lambda(X\backslash\{x\}) \subseteq X$. Therefore $X\backslash\{x\}$ is λ_g^α-closed. By hypothesis $X\backslash\{x\}$ is λ-closed. Hence $\{x\}$ is λ-open. □

Theorem 4.10. *Let A be α-open and λ_g^α-closed in a topological space (X,τ). If F is λ-closed then $A \cap F$ is λ_g^α-closed.*

Proof. By Proposition 3.12 we have if a set A is both α-open and λ_g^α-closed then A is λ- closed. Since F is λ-closed, $A \cap F$ is λ-closed as the intersection of λ-closed sets is a λ-closed set. Hence by Proposition 3.2 $A \cap F$ is λ_g^α-closed. □

Theorem 4.11. *If A is λ_g^α-closed then $cl_\alpha(\{x\}) \cap A \neq \phi$ for every $x \in cl_\lambda(A)$*

Proof. Let A be a λ_g^α-closed set. Suppose $cl_\alpha(\{x\}) \cap A = \phi$ for some $x \in cl_\lambda(A)$. Then $X\backslash cl_\alpha(\{x\})$ is an α-open set containing A. Further $x \in cl_\lambda(A)$ and $x \notin cl_\alpha(\{x\})$ implies $cl_\lambda(A) \nsubseteq X\backslash cl_\alpha(\{x\})$ is a contradiction to A is a λ_g^α-closed set. Therefore $cl_\alpha(\{x\}) \cap A \neq \phi$ for every $x \in \mathrm{cl}_\lambda(A)$ □

Theorem 4.12. *In a topological space (X,τ) the following are equivalent: 1. Every α-open set is λ-closed. 2. Every subset is λ_g^α-closed.*

Proof. $1 \Rightarrow 2$: Let A be any subset of (X,τ) such that $A \subseteq U$, where U is α-open. By the result that "if $A \subseteq B$, then $cl_\lambda(A) \subseteq cl_\lambda(B)$ ", we have $A \subseteq U \Longrightarrow cl_\lambda(A) \subseteq cl_\lambda(U)$. By hypothesis U is λ-closed, then $cl_\lambda(U) = U \Rightarrow cl_\lambda(A) \subseteq U$. Hence A is λ_g^α-closed. $2 \Rightarrow 1$: Let A be an α-open set. By hypothesis A is λ_g^α-closed. Then we have $\mathrm{cl}_\lambda(A) \subseteq A$ Therefore A is λ-closed. Hence every α-open set is λ-closed. □

Definition 4.13. *Let A be a subset of a topological space (X,τ). Then the α-kernel of the set A denoted by $\alpha - \ker(A)$ is the intersection of all α-open supersets of A.*

Theorem 4.14. *A subset A of a topological space (X,τ) is λ_g^α-closed if and only if $cl_\lambda(A) \subseteq \alpha - \ker(A)$*

Proof. **(Necessity)** Suppose A is λ_g^α-closed in X. Let $x \in cl_\lambda(A)$ but $x \notin \alpha - \ker(A)$. Then $\exists$ an α-open set $H \supseteq A$, such that $x \notin H$. Since A is $\lambda_g^\alpha -$ closed ,$cl_\lambda(A) \subseteq H$ and H is an α-open set containing A. So, we have $x \in cl_\lambda(A)$ and $x \notin H$ which is a contradiction. Therefore $cl_\lambda(A) \subseteq \alpha - \ker(A)$
Sufficiency: Let $cl_\lambda(A) \subseteq \alpha - \ker(A)$ and let U be an α-open set containing A. Then $\alpha - \ker(A) \subseteq U$, implies $cl_\lambda(A) \subseteq U$. Thus A is λ_g^α-closed. □

Theorem 4.15. *Let $A \subseteq B \subseteq cl_\lambda(A)$. If A is λ_g^α-closed then B is λ_g^α-closed.*

Proof. Let A be a λ_g^α-closed set in X and $B \subseteq U$, where U is α-open in X. Then $A \subseteq U$, where U is α-open in X. Since A is λ_g^α-closed we have $cl_\lambda(A) \subseteq U$. By hypothesis $B \subseteq cl_\lambda(A)$ implies $cl_\lambda(B) \subseteq cl_\lambda(cl_\lambda(A)) = cl_\lambda(A)$, that is $cl_\lambda(B) \subseteq cl_\lambda(A) \subseteq U$. Hence B is λ_g^α-closed. □

5. λ_g^α-Open Sets

Definition 5.1. *Let (X,τ) be a topological space. A subset A of X is said to be a λ_g^α-open set if $int_\lambda(A) \supseteq U$ whenever $A \supseteq U$, where U is α-closed in X and $int_\lambda(A)$ is the union of all α- open sets contained in A. Equivalently, a subset A of a topological space (X,τ) is said to be λ_g^α-open if its complement A^c is λ_g^α-closed.*

Theorem 5.2. *A subset A of a topological space (X,τ) is λ_g^α-open if and only if $F \subseteq \mathrm{int}_\lambda(A)$ whenever F is α-closed in X and $F \subseteq A$.*

Proof. **(Necessity)** Let F be an α-closed set contained in A and let A be λ_g^α-open. By definition A^c is λ_g^α-closed ,$A^c \subseteq F^c$, where F^c is α-open. Since A^c is λ_g^α-closed, $cl_\lambda(A^c) \subseteq F^c$ implies $F \subseteq (X \backslash cl_\lambda(A^c)) = \text{int}_\lambda(X \backslash A^c) = \text{int}_\lambda(A)$
Sufficiency: Let $F \subseteq \text{int}_\lambda(A)$, where F is an α-closed set contained in A. We have $A^c \subseteq F^c$ and $(X \backslash \text{int}_\lambda(A)) \subseteq (X \backslash F)$ implies $cl_\lambda(A^c) \subseteq F^c$. Thus, by definition A^c is λ_g^α-closed and hence A is λ_g^α-open. □

Proposition 5.3. *Every λ-open (resp. closed,open, Λ-) set is λ_g^α-open in (X,τ). Every λ_g^α-open set is $g\Lambda$-open.*

Theorem 5.4. *Let A be an α-closed subset of a topological space (X,τ). If A is λ_g^α-open then A is λ-open.*

Proof. Let A be λ_g^α-open and α-closed. Since $A \subseteq A$ and A is λ_g^α-open we have $A \subseteq \text{int}_\lambda(A)$. Then we get $X \backslash \text{int}_\lambda(A) \subseteq X \backslash A$. By the fact that $X \backslash \text{int}_\lambda(A) = cl_\lambda(X \backslash A)$, we have $cl_\lambda(X \backslash A) \subseteq X \backslash A$. Hence $X \backslash A$ is λ-closed and hence A is λ-open. □

Theorem 5.5. *If int $_\lambda(A) \subseteq B \subseteq A$ and A is λ_g^α-open, then B is λ_g^α-open.*

Proof. $int_\lambda(A) \subseteq B \subseteq A$ implies $A^c \subseteq B^c \subseteq cl_\lambda(A^c)$.Since A is λ_g^α-open ,A^c is λ_g^α-closed. By Theorem 4.15 B^c is λ_g^α-closed. Therefore B is λ_g^α-open. □

Theorem 5.6. *If a subset A of a topological space X is λ_g^α-open in X then $H = X$, whenever H is α-open and $\text{int}_\lambda(A) \cup A^c \subseteq H$*

Proof. Let A be λ_g^α-open, H be α-open and $\text{int}_\lambda(A) \cup A^c \subseteq H$. This gives $H^c \subseteq (X \backslash \text{int}_\lambda(A)) \cap A = cl_\lambda(A^c) \cap A = cl_\lambda(A^c) \backslash A^c$. Since H^c is α-closed ,A^c is λ_g^α-closed and by Theorem 4.4 we have $H^c = \phi$. Hence $H = X$. □

Theorem 5.7. *If a subset A of a topological space X is λ_g^α-closed, then $cl_\lambda(A) \backslash A$ is λ_g^α-open.*

Proof. Let $A \subseteq X$ be λ_g^α-closed. Let F be α-closed such that $F \subseteq (cl_\lambda(A) \backslash A)$. Then by Theorem 4.4 $F = \phi$. So $\phi = F \subseteq \text{int}_\lambda(cl_\lambda(A) \backslash A)$. This shows that $cl_\lambda(A) \backslash A$ is λ_g^α-open. □

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