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Cartesian composition of $\Gamma-$ reset single valued neutrosophic automata

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Abstract
Cartesian composition of $\Gamma-$ reset single valued neutrosophic automata(SVNA) are introduce, prove that cartesian composition of $\Gamma-$ reset SVNA is $\Gamma-$ reset SVNA.

Keywords
Single Valued neutrosophic set, Single Valued neutrosophic automaton, $\Gamma-$ reset, Cartesian Composition.

AMS Subject Classification
03D05, 20M35, 18B20, 68Q45, 68Q70, 94A45.

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Contents

1. Introduction

The theory of neutrosophy and neutrosophic set was introduced by Florentin Smarandache in 1999 [2]. The fuzzy sets was introduced by Zadeh in 1965[5]. A neutrosophic set N is a set where each element of the universe has a degree of truth, indeterminacy and falsity respectively and which lies in the non standard unit interval $]0^-,1^+[$. Wang *et al*. [3] introduced the notion of single valued neutrosophic sets.

The fuzzy automaton was introduced Wee [4]. The concept of single valued neutrosophic finite state machine was introduced by Tahir Mahmood [1]. Cartesian composition of $\Gamma-$ reset single valued neutrosophic automata(SVNA) are introduce and studied their properties. Finally, prove that cartesian composition of $\Gamma-$ reset SVNA is $\Gamma-$ reset SVNA.

2. Preliminaries

Definition 2.1. *[1] A fuzzy automata is triple $F=(S,A,\alpha)$ where S,A are finite non empty sets called set of states and set of input alphabets and α is fuzzy transition function in $S\times A\times S\to[0,1]$.*

Definition 2.2. *[1] A fuzzy automata is triple $F=(S,A,\alpha)$ where S,A are finite non empty sets called set of states and set of input alphabets and α is fuzzy transition function in $S\times A\times S\to[0,1]$.*

Definition 2.3. *Neutrosophic Set [2] Let U be the universe of discourse. A neutrosophic set (NS) N in U is characterized by a truth membership function η_N, an indeterminacy membership function ζ_N and a falsity membership function ρ_N, where η_N,ζ_N, and ρ_N are real standard or non-standard subsets of $]0^-,1^+[$. That is*
$N=\{\langle x,(\eta_N(x),\zeta_N(x),\rho_N(x))\rangle, x\in U,$
$\eta_N,\zeta_N,\rho_N\in\,]0^-,1^+[\,\}$ *and with the condition*
$0^-\leq sup\ \eta_N(x)+sup\ \zeta_N(x)+sup\ \rho_N(x)\leq 3^+$.
we need to take the interval $[0,1]$ for technical applications instead of $]0^-,1^+[$.

Definition 2.4. *Single Valued Neutrosophic Set [2]*
Let U be the universe of discourse. A single valued neutrosophic set (NS) N in U is characterized by a truth membership function η_N, an indeterminacy membership function ζ_N and a

falsity membership function ρ_N.
$N = \{\langle x, (\eta_N(x), \zeta_N(x), \rho_N(x))\rangle, x \in U,\ \eta_N, \zeta_N, \rho_N \in [0,1].\}$

3. Single Valued Neutrosophic Automata

Definition 3.1. *[1]*
$F = (S, A, N)$ is called single valued neutrosophic automaton ($SVNA$ for short), where S and A are non-empty finite sets called the set of states and input symbols respectively, and $N = \{\langle \eta_N(x), \zeta_N(x), \rho_N(x)\rangle\}$ is an $SVNS$ in $S \times A \times S$. The set of all words of finite length of A is denoted by A^*. The empty word is denoted by ε, and the length of each $x \in A^*$ is denoted by $|x|$.

Definition 3.2. *[1]*
$F = (S,\ A,\ N)$ be an $SVNA$. Define an $SVNS$ $N^* = \{\langle \eta_{N^*}(x), \zeta_{N^*}(x), \rho_{N^*}(x)\rangle\}$ in $S \times A^* \times S$ by

$$\eta_{N^*}(q_i,\ \varepsilon,\ q_j) = \begin{cases} 1 & \text{if } q_i = q_j \\ 0 & \text{if } q_i \neq q_j \end{cases}$$

$$\zeta_{N^*}(q_i,\ \varepsilon,\ q_j) = \begin{cases} 0 & \text{if } q_i = q_j \\ 1 & \text{if } q_i \neq q_j \end{cases}$$

$$\rho_{N^*}(q_i,\ \varepsilon,\ q_j) = \begin{cases} 0 & \text{if } q_i = q_j \\ 1 & \text{if } q_i \neq q_j \end{cases}$$

$\eta_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\eta_{N^*}(q_i, x, q_r) \wedge \eta_{N^*}(q_r, y, q_j)]$,
$\zeta_{N^*}(q_i, xy, q_j) = \wedge_{q_r \in Q}[\zeta_{N^*}(q_i, x, q_r) \vee \zeta_{N^*}(q_r, y, q_j)]$,
$\rho_{N^*}(q_i, xy, q_j) = \wedge_{q_r \in Q}[\rho_{N^*}(q_i, x, q_r) \vee \rho_{N^*}(q_r, y, q_j)]$,
$\forall q_i, q_j \in S$,
$x \in A^*$ *and* $y \in A$.

4. $\Gamma-$ Reset Single Valued Neutrosophic Automata

Definition 4.1. *Let $F = (S, A, N)$ be an SVNA. If F is said to be deterministic SVNA then for each $q_i \in Q$ and $x \in A$ there exists unique state q_j such that $\eta_{N^*}(q_i, x, q_j) > 0$. $\zeta_{N^*}(q_i, x, q_j) < 1$, $\rho_{N^*}(q_i, x, q_j) < 1$.*

Definition 4.2. *Let $F = (S, A, N)$ be an SVNA. If F is said to be connected SVNA if for all $q_i, q_j \in S$ there exists $x \in A$ such that $\eta_N(q_i, x, q_j) > 0$. $\zeta_{N^*}(q_i, u, q_j) < 1$, $\rho_{N^*}(q_i, u, q_j) < 1$. (or) $\eta_N(q_j, x, q_i) > 0$. $\zeta_{N^*}(q_j, x, q_i) < 1$, $\rho_{N^*}(q_j, x, q_i) < 1$.*

Definition 4.3. *Let $F = (S, A, N)$ be an SVNA. If F is said to be strongly connected SVNA if for all $q_i, q_j \in S$ there exists $u \in A^*$ such that $\eta_{N^*}(q_i, u, q_j) > 0$. $\zeta_{N^*}(q_i, u, q_j) < 1$, $\rho_{N^*}(q_i, u, q_j) < 1$. Equivalently, F is strongly connected if it has no proper subautomaton.*

Definition 4.4. *Let $\Theta = p_1, p_2, ..., p_z$ be a partition of the states set S such that if $\eta_{N^*}(q_i, x, q_j) > 0$. $\zeta_{N^*}(q_i, x, q_j) < 1$, $\rho_{N^*}(q_i, x, q_j) < 1$. for some $x \in A$ then $q_i \in p_s$ and $q_j \in p_{s+1}$. Then Θ will be called periodic partition of order $z \geq 2$. An SVNA F is periodic of period $z \geq 2$ if and only if $z = Maxcard(\Theta)$ where this maximum is taken over all periodic partitions Θ of F. If F has no periodic partition, then F is called aperiodic. Throughout this paper we consider aperiodic SVNA.*

Definition 4.5. *Let $F = (S, A, N)$ be an SVNA. We say that F is said to be $\Gamma-$ reset if there exists a word $w \in A^*$, $q_j \in S$ and a real number Γ with $\Gamma \in (0,1]$ such that $\eta_{N^*}(q_i, w, q_j) \geq \Gamma > 0$, $\zeta_{N^*}(q_i, w, q_j) \leq \Gamma < 1$, $\rho_{N^*}(q_i, w, q_j) \leq \Gamma < 1\ \ \forall q_i \in S$.*

5. Cartesian Composition of $\Gamma-$ Reset Single Valued Neutrosophic Automata

5.1 Definition

Let $F_i = (S_i, A_i, N_i)$ be SVNA, $i = 1, 2$ and let $A_1 \cap A_2 = \phi$. Let $F_1 \circ F_2 = (S_1 \times S_2, A_1 \cup A_2, N_1 \circ N_2)$, where

$$(\eta_{N_1} \circ \eta_{N_2})((q_i, q_j), x, (q_k, q_l)) = \begin{cases} \eta_{N_1}(q_i, x, q_k) \\ \text{if } x \in A_1,\ q_j = q_l \\ \eta_{N_2}(q_j, x, q_l) \\ \text{if } x \in A_2,\ q_i = q_k \\ 0 \\ \text{otherwise} \end{cases}$$

$$(\zeta_{N_1} \circ \zeta_{N_2})((q_i, q_j), x, (q_k, q_l)) = \begin{cases} \zeta_{N_1}(q_i, x, q_k) \\ \text{if } x \in A_1,\ q_j = q_l \\ \zeta_{N_2}(q_j, x, q_l) \\ \text{if } x \in A_2,\ q_i = q_k \\ 0 \\ \text{otherwise} \end{cases}$$

$$(\rho_{N_1} \circ \rho_{N_2})((q_i, q_j), x, (q_k, q_l)) = \begin{cases} \rho_{N_1}(q_i, x, q_k) \\ \text{if } x \in A_1,\ q_j = q_l \\ \rho_{N_2}(q_j, x, q_l) \\ \text{if } x \in A_2,\ q_i = q_k \\ 0 \\ \text{otherwise} \end{cases}$$

$\forall\ (q_i, q_j), (q_k, q_l) \in S_1 \times S_2,\ x \in A_1 \cup A_2$. Then $F_1 \circ F_2$ is called the Cartesian composition of $F_1 \circ F_2$.

5.2 Definition

Let $F_i = (S_i, A_i, N_i)$ be SVNA, $i = 1, 2$ and let $A_1 \cap A_2 = \phi$. Let $F_1 \circ F_2 = (S_1 \times S_2, A_1 \cup A_2, N_1 \circ N_2)$, be the cartesian composition of F_1 and F_2. Then $\forall w \in A_1^* \cup A_2^*, w \neq \varepsilon$.

$$(\eta_{N_1^*} \circ \eta_{N_2^*})((q_i,q_j),w,(q_k,q_l)) = \begin{cases} \eta_{N_1^*}(q_i,w,q_k) \\ \text{if } w \in A_1^*,\ q_j = q_l \\ \eta_{N_2^*}(q_j,w,q_l) \\ \text{if } w \in A_2^*,\ q_i = q_k \\ 0 \\ \text{otherwise} \end{cases}$$

$$(\zeta_{N_1^*} \circ \zeta_{N_2^*})((q_i,q_j),w,(q_k,q_l)) = \begin{cases} \zeta_{N_1^*}(q_i,w,q_k) \\ \text{if } w \in A_1^*,\ q_j = q_l \\ \zeta_{N_2^*}(q_j,w,q_l) \\ \text{if } w \in A_2^*,\ q_i = q_k \\ 0 \\ \text{otherwise} \end{cases}$$

$$(\rho_{N_1^*} \circ \rho_{N_2^*})((q_i,q_j),w,(q_k,q_l)) = \begin{cases} \rho_{N_1^*}(q_i,w,q_k) \\ \text{if } w \in A_1^*,\ q_j = q_l \\ \rho_{N_2^*}(q_j,w,q_l) \\ \text{if } w \in A_2^*,\ q_i = q_k \\ 0 \\ \text{otherwise} \end{cases}$$

$\forall\ (q_i,q_j),(q_k,q_l) \in S_1 \times S_2,\ w \in A_1^* \cup A_2^*$. Then $F_1 \circ F_2$ is called the Cartesian composition of $F_1 \circ F_2$.

Theorem 5.1. *Let $F_i = (S_i, A_i, N_i)$ be SVNA, $i = 1,2$ and let $A_1 \cap A_2 = \phi$.*
Let $F_1 \circ F_2 = (S_1 \times S_2, A_1 \cup A_2, N_1 \circ N_2)$, be the cartesian composition of F_1 and F_2.
Then $\forall w_1 \in A_1^*, w_2 \in A_2^*$
$(\eta_{N_1^*} \circ \eta_{N_2^*})((q_i,q_j),w_1w_2,(q_k,q_l)) = \eta_{N_1^*}(q_i,w_1,q_k) \wedge \eta_{N_2^*}(q_j,w_2,q_l)$
$(\zeta_{N_1^*} \circ \zeta_{N_2^*})((q_i,q_j),w_1w_2,(q_k,q_l)) = \zeta_{N_1^*}(q_i,w_1,q_k) \vee \zeta_{N_2^*}(q_j,w_2,q_l)$
$(\rho_{N_1^*} \circ \rho_{N_2^*})((q_i,q_j),w_1w_2,(q_k,q_l)) = \rho_{N_1^*}(q_i,w_1,q_k) \vee \rho_{N_2^*}(q_j,w_2,q_l)$

Proof. Let$w_1 \in A_1^*,\ w_2 \in A_2^*$, $(q_i,q_j),(q_k,q_l) \in S_1 \times S_2$.
If $w_1 = \varepsilon = w_2$, then $w_1w_2 = \varepsilon$. Suppose $(q_i,q_j) = (q_k,q_l)$. Then $q_i = q_k$ and $q_j = q_l$. Hence,
$(\eta_{N_1^*} \circ \eta_{N_2^*})((q_i,q_j),w_1w_2,(q_k,q_l)) = 1 = \eta_{N_1^*}(q_i,w_1,q_k) \wedge \eta_{N_2^*}(q_j,w_2,q_l)$
$(\zeta_{N_1^*} \circ \zeta_{N_2^*})((q_i,q_j),w_1w_2,(q_k,q_l)) = 0 = \zeta_{N_1^*}(q_i,w_1,q_k) \vee \zeta_{N_2^*}(q_j,w_2,q_l)$
$(\zeta_{N_1^*} \circ \zeta_{N_2^*})((q_i,q_j),w_1w_2,(q_k,q_l)) = 0 = \zeta_{N_1^*}(q_i,w_1,q_k) \vee \zeta_{N_2^*}(q_j,w_2,q_l)$
Suppose $(q_i,q_j) \neq (q_k,q_l)$. Then either $q_i \neq q_k$ or $q_j \neq q_l$. Then
$\eta_{N_1^*}(q_i,w_1,q_k) \wedge \eta_{N_2^*}(q_j,w_2,q_l) = 0$,
$\zeta_{N_1^*}(q_i,w_1,q_k) \vee \zeta_{N_2^*}(q_j,w_2,q_l) = 1$ and
$\rho_{N_1^*}(q_i,w_1,q_k) \vee \rho_{N_2^*}(q_j,w_2,q_l) = 1$.
If $x = \varepsilon$ and $y \neq \varepsilon$ or $x \neq \varepsilon$ and $y = \varepsilon$ then the result is holds. Now,

$$\begin{aligned}&(\eta_{N_1^*} \circ \eta_{N_2^*})((q_i,q_j),w_1w_2,(q_k,q_l)) = \\ &\vee_{(q_r,q_s)\in S_1\times S_2}\{(\eta_{N_1^*} \circ \eta_{N_2^*})((q_i,q_j),w_1,(q_r,q_s)) \wedge \\ &(\eta_{N_1^*} \circ \eta_{N_2^*})((q_r,q_s),w_2,(q_k,q_l))\} \\ &= \vee_{q_r \in S_1}\{(\eta_{N_1^*} \circ \eta_{N_2^*})((q_i,q_j),w_1w_2,(q_r,q_l)) \wedge \\ &(\eta_{N_1^*} \circ \eta_{N_2^*})((q_r,q_l),w_2,(q_k,q_l))\} \\ &= \eta_{N_1^*}(q_i,w_1,q_k) \wedge \eta_{N_2^*}(q_j,w_2,q_l).\end{aligned}$$

$$\begin{aligned}&(\zeta_{N_1^*} \circ \zeta_{N_2^*})((q_i,q_j),w_1w_2,(q_k,q_l)) = \\ &\wedge_{(q_r,q_s)\in S_1\times S_2}\{(\zeta_{N_1^*} \circ \zeta_{N_2^*})((q_i,q_j),w_1,(q_r,q_s)) \vee \\ &(\zeta_{N_1^*} \circ \zeta_{N_2^*})((q_r,q_s),w_2,(q_k,q_l))\} \\ &= \wedge_{q_r \in S_1}\{(\zeta_{N_1^*} \circ \zeta_{N_2^*})((q_i,q_j),w_1w_2,(q_r,q_l)) \vee \\ &(\zeta_{N_1^*} \circ \zeta_{N_2^*})((q_r,q_l),w_2,(q_k,q_l))\} \\ &= \zeta_{N_1^*}(q_i,w_1,q_k) \vee \zeta_{N_2^*}(q_j,w_2,q_l).\end{aligned}$$

$$\begin{aligned}&(\rho_{N_1^*} \circ \rho_{N_2^*})((q_i,q_j),w_1w_2,(q_k,q_l)) = \\ &\wedge_{(q_r,q_s)\in S_1\times S_2}\{(\rho_{N_1^*} \circ \rho_{N_2^*})((q_i,q_j),w_1,(q_r,q_s)) \vee \\ &(\rho_{N_1^*} \circ \rho_{N_2^*})((q_r,q_s),w_2,(q_k,q_l))\} \\ &= \wedge_{q_r \in S_1}\{(\rho_{N_1^*} \circ \rho_{N_2^*})((q_i,q_j),w_1w_2,(q_r,q_l)) \vee \\ &(\rho_{N_1^*} \circ \rho_{N_2^*})((q_r,q_l),w_2,(q_k,q_l))\} \\ &= \rho_{N_1^*}(q_i,w_1,q_k) \vee \rho_{N_2^*}(q_j,w_2,q_l).\end{aligned}$$

□

Theorem 5.2. *Let $F_i = (S_i, A_i, N_i)$ be SVNA, $i = 1,2$ and let $A_1 \cap A_2 = \phi$. If F_1 and F_2 are $\Gamma-$ reset SVNA then cartesian composition of $F_1 \circ F_2$ is $\Gamma-$ reset SVNA.*

Proof. Let $F_i = (S_i, A_i, N_i),\ i = 1,2$ be $\Gamma-$ reset SVNA. Then there exists a word $w_1 \in A^*,\ q_j \in S_1$ and $w_2 \in A^*,\ q_l \in S_2$a real number Γ with $\Gamma \in (0,1]$such that
$\eta_{N_1^*}(q_i,w_1,q_k) \geq \Gamma > 0,\ \zeta_{N_1^*}(q_i,w_1,q_k) \leq \Gamma < 1$,
$\rho_{N_1^*}(q_i,w,q_k) \leq \Gamma < 1\ \ \forall q_i \in S_1$.
$\eta_{N_2^*}(q_j,w_2,q_l) \geq \Gamma > 0,\ \zeta_{N_2^*}(q_j,w_2,q_l) \leq \Gamma < 1$,
$\rho_{N_2^*}(q_j,w_2,q_l) \leq \Gamma < 1\ \ \forall q_j \in S_2$.

Now,

$$(\eta_{N_1^*} \circ \eta_{N_2^*})((q_i,q_j),w,(q_k,q_l)) =$$
$$(\eta_{N_1^*} \circ \eta_{N_2^*})((q_i,q_j),w_1w_2,(q_k,q_l)), w = w_1w_2$$
$$= \vee_{(q_r,q_s)\in S_1\times S_2}\{(\eta_{N_1^*} \circ \eta_{N_2^*})((q_i,q_j),w_1,(q_r,q_s))\wedge$$
$$(\eta_{N_1^*} \circ \eta_{N_2^*})((q_r,q_s),w_2,(q_k,q_l))\}$$
$$= \vee_{q_r\,\in\, S_1}\{(\eta_{N_1^*} \circ \eta_{N_2^*})((q_i,q_j),w_1,(q_r,q_j))\wedge$$
$$(\eta_{N_1^*} \circ \eta_{N_2^*})((q_r,q_j),w_2,(q_k,q_l))\}$$
$$= \eta_{N_1^*}(q_i,w_1,q_k)\ \wedge\ \eta_{N_2^*}(q_j,w_2,q_l).$$

Hence, the cartesian composition of $F_1 \circ F_2$ is $\Gamma-$reset SVNA. □

6. Conclusion

Cartesian composition of $\Gamma-$ reset single valued neutrosophic automata(SVNA) are introduce, prove that cartesian composition of $\Gamma-$ reset SVNA is $\Gamma-$ reset SVNA.

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