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An algorithm for multi objective dual hesitant fuzzy fractional transportation problem

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Abstract

In this paper a new algorithm is proposed to find the optimal solution of the multi objective dual hesitant fuzzy fractional transportation problem. The proposed algorithm is very simple and easy to understand. This algorithm gives the better solution in both crisp environment and fuzzy environment. The numerical example is solved to explain the algorithm. The proposed algorithm gives the better solution than the existing one.

Keywords

Transportation Problem, Multi Objective Transportation Problem, Fractional Transportation Problem, Hesitant Fuzzy set.

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Contents

1. Introduction

Transport plays a vital role in economic growth and globalization. The transportation problem is a distribution-type problem, the main goal of transportation problem is to decide how to transfer goods from various sending locations to various receiving locations with minimal costs or maximum profit. The transportation problem was first studied by Hitchcock in 1941 [1] .

The classical transportation problem involves only one objective at a time but in general there are many situations involving more objectives other that total cost. This leads to the concept of multi objective transportation problem (MOTP). To solve the multi objective transportation problem goal programming method was introduced by Lee(1973) [2]. Zeleny (1974) [3] solved the multi objective transportation problem by generating non dominated basic feasible solution. Diaz(1978) [4] developed the algorithm to obtain all non dominated solutions for MOTP. Also many authors, Aneja (1979)[5], Gupta(1983)[6] have developed the various solution procedures to solve the MOTP.

The generalization of linear programming problem is fractional programming problem (FPP) in which the objectives are ratio of two functions. The aim of this is to obtain optimization of the ratio of the cost functions. FPP is very applicable in many real life situations such as ratio between the profit and time, profit and cost , minimizing the inventory and sales etc. Several algorithms have been established by different authors, Charnes and Cooper(1962)[7], Borza(2012)[8], Chakraborty(2002)[9] solved the fractional transportation problem with multi objectives.

Due to shortage of information, insufficient data, lack of evidence, and so forth, the data for a transportation system such as availabilities, demands and conveyance capacities are not always exact but can be fuzzy or arbitrary or both. Fuzzy set was first introduced by Zadeh(1965)[10].

Torra and Narukawa have introduced the concept of a Hesitant Fuzzy Set (HFS) in 2009[11]. A proper definition of a HFS was given by Torra (2010)[12]. Zhu (2012)[13] proposed Dual-Hesitant Fuzzy Sets (DHFSs), which are an extension of HFSs that encompass fuzzy sets, intuitionistic fuzzy sets, HFSs and fuzzy multi-sets as special cases. Torra(2010)[12] and Zhu et al introduced the basic properties of DHFSs. Thereafter, they presented the concept of DHFSs in a group forecasting problem.

The fractional transportation problem was formulated with fuzzy parameters by Liu in 2016. Pop (2007)[14] have extended the goal programming algorithm for multi objective fractional transportation problem. Sheema(2017)[15] have developed the algorithm for solving multi objective transportation problem in fuzzy environment. Also Amit Kumar et. al (2020)[16] proposed the new ranking method for dual hesitant fuzzy element and Gurupada et. al (2019)[17] derived the arithmetic operations on dual hesitant fuzzy numbers.

In this paper a new algorithm is proposed to find the optimal solution of the multi objective fuzzy fractional transportation problem. The proposed algorithm is very simple and easy to understand. Section 2 presents preliminaries which have used in paper. The mathematical models of MODHFFTP and solution procedure for solving that problem are described in Section 3 and Section 4 respectively. The numerical problem is solved using proposed methodology in Section 5. Conclusion and references are provided in Section 6 and Section 7 respectively.

2. Preliminaries

Definition 2.1 (Hesitant fuzzy set). *A hesitant fuzzy set H on Y is defined in terms of a functions $h(y)$ that returns a subset of values in the interval $[0,1]$ once it is applied on Y i.e an element of power set of Y*

$$h : Y \to \rho([0,1])$$

Mathematically it can be stated that $H = \{ (y_i\ ,\ h(y_i))\ :\ y_i \in Y \}$ where $h(y_i)$ is a set of several values in $[0,1]$. In general each member of $h(y_i)$ is called a hesitant fuzzy element denoted by h_i.

Definition 2.2 (Dual hesitant fuzzy set). *A dual hesitant fuzzy set DH on Y is defined in terms of a functions $h(y)$ and $g(y)$ that returns a subset of values in the interval $[0,1]$ once it is applied on Y*

$$h\ :\ Y \to \rho([0,1]) \text{ and } g\ :\ Y \to \rho([0,1])$$

where $h(y)$ and $g(y)$ are mappings that takes set of values in $[0,1]$; they are denoted as the possible membership degree and non-membership degree of any element $y \in Y$.

Mathematically, $DH = \{ (y_i\ ,\ h(y_i),\ g(y_i)) :\ y_i \in Y \}$ where $h(y_i)$ is a set of several membership values in $[0,1]$ and $g(y_i)$ is a set of several possible non-membership values in $[0,1]$ with $0 = h(y_i) + g(y_i) = 1$.

Definition 2.3 (Arithmetic Operations of dual hesitant fuzzy elements). *Let $DH1 = \{ (y_i\ ,\ h_1(y_i),\ g_1(y_i)) :\ y_i \in Y \}$ and Let $DH2 = \{ (y_i\ ,\ h_2(y_i),\ g_2(y_i)) :\ y_i \in Y \}$ be two dual hesitant fuzzy elements. Then addition, subtraction and multiplication of the elements are defined as follows:*

1. *Addition:*

$$DH1 \oplus DH2 = \cup_{\gamma h1\ \in h_1\ ,\ \gamma h2 \in h_2,\ \delta h1 \in h_1\ ,\ \delta h2 \in h_2} \{ \{ \gamma h1 + \gamma h2 - \gamma h1\ \gamma h2 \},\ \{\delta h1.\delta h2\}\}$$

2. *Subtraction:*

$$DH1 \ominus DH2 = \cup_{\gamma h1\ \in h_1\ ,\ \gamma h2 \in h_2,\ \delta h1 \in h_1\ ,\ \delta h2 \in h_2} \{ \{ \delta h1.\ \delta h2 \},\ \{\delta h1 + \delta h2 - dh1.dh2\}\}$$

3. *Multiplication:*

$$DH1 \otimes DH2 = \cup_{\gamma h1\ \in h_1\ ,\ \gamma h2 \in h_2,\ \delta h1 \in h_1\ ,\ \delta h2 \in h_2} \{ \{ \gamma h1.\ \gamma h2 \},\ \{\delta h1.\delta h2\}\}$$

Definition 2.4 (Ranking function of dual hesitant fuzzy sets). *Let $DH = \{ (y_i\ ,\ h(y_i),\ g(y_i)) :\ y_i \in Y \}$ be a dual hesitant fuzzy set where $\{y_1,\ y_2, \ldots\ldots y_n\}$ and $d = (h_d,\ g_d)$ be dual hesitant fuzzy element. The score function s_d of dual hesitant fuzzy set is defined as*

$$s_d(y_i) = y_i + \frac{1}{k}\sum_{i=1}^{k} h_d(y_i) - \frac{1}{k}\sum_{i=1}^{k} g_d(y_i)$$

Let d_1 and d_2 be any two dual hesitant fuzzy sets. Then using the score function the order relations are defined as follows :

1. *If $s_{d_1} > s_{d_2}$, then d_1 is said to be superior to d_2 and it is denoted by $d_1 > d_2$*
2. *If $s_{d_1} < s_{d_2}$, then d_1 is said to be inferior to d_2 and it is denoted by $d_1 < d_2$*
3. *If $s_{d_1} = s_{d_2}$, then d_1 is said to be equivalent to d_2 and it is denoted by $d_1 = d_2$*

3. Mathematical Formulation of Multi-objective Dual Hesitant Fractional Fuzzy Transportation Problem (MODHFFTP)

Notations

x	:	Number of Sources
y	:	Number of destinations
$\widetilde{s_{lpr}}$	:	Dual hesitant fuzzy numerator cost coefficients of the r-th objective from the l-th source to p-th destination
$\widetilde{u_{lpr}}$	:	Dual hesitant fuzzy denominator cost coefficient of the r-th objective from the l-th source to p-th destination
t_{lp}	:	Number of units transported from l-th source to p-th destination
$\tilde{a}_l$	:	Dual hesitant fuzzy availability in l-th source
$\widetilde{b_p}$	:	Dual hesitant fuzzy demand in p-th destination

Mathematical formulation of multi-objective fractional dual hesitant fuzzy transportation problem is defined as follows:

$$\min\,(\max)\, q^1 = \frac{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{s_{lp1}}t_{lp}}{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{u_{lp1}}t_{lp}}$$

$$\min\,(\max)\, q^2 = \frac{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{s_{lp2}}t_{lp}}{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{u_{ij2}}t_{lp}}$$

$$\vdots$$

$$\min\,(\max)\, q^r = \frac{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{s_{lpr}}t_{lp}}{\sum_{l=1}^{x}\sum_{p=1}^{y}\widetilde{u_{lpr}}t_{lp}}$$

Subject to

$$\sum_{p=1}^{y} t_{lp} = \tilde{a}_l$$

$$\sum_{l=1}^{x} t_{lp} = \widetilde{b_p};$$

$$t_{lp} = 0$$

for every $l = 1$ to x , $p = 1\ to\ y$ and $k = 1\ to\ r$

4. Procedure to solve Multi-objective Dual Hesitant Fractional Fuzzy Transportation Problem

1. Find the score value for every dual hesitant fuzzy number using ranking function defined in Section 2.
2. Find the fractional values of the each cell.
3. If all the objectives are maximization then it can be converted into minimization type by subtracting the greatest element from all the fractional values.
4. Find the maximum ratio of the each row γ_{lk} and each column δ_{pk} and fix as given in Table 4.1.
5. Choose L = max { γ_{lr}, δ_{pr} } for every $l = 1\ to\ x$, $p = 1\ to\ y$ and $k = 1\ to\ r$.
6. Select the call having L as one of its ratio. Suppose there are more than one cell choose the cell which has maximum ration for other fractional objectives.
7. Choose the cell containing $\min\left\{\sum_{l=1}^{x}\frac{s_{lpk}}{u_{lpk}} \text{ for fixed } p\right\}$. If there is a tie then select one to which maximum allocation.
8. Do the procedure of Step 4 to Step 6 until supply and demand requirement satisfied.

Table 4.1.

Origin/ Destination	W_1	W_2	....	W_y	Supply	Max.Value
U_1	$\frac{s_{111}}{u_{111}}\ \frac{s_{112}}{u_{112}} \dots \frac{s_{11r}}{u_{11r}}$	$\frac{s_{121}}{u_{121}}\ \frac{s_{122}}{u_{122}} \dots \frac{s_{12r}}{u_{12r}}$	....	$\frac{s_{1y1}}{u_{1y1}}\ \frac{s_{1y2}}{u_{1y2}} \dots \frac{s_{1yr}}{u_{1yr}}$	a_1	$\gamma_{11}\ \gamma_{12} \dots \gamma_{1r}$
U_2	$\frac{s_{211}}{u_{211}}\ \frac{s_{212}}{u_{212}} \dots \frac{s_{21r}}{u_{21r}}$	$\frac{s_{221}}{u_{221}}\ \frac{s_{222}}{u_{222}} \dots \frac{s_{22r}}{u_{22r}}$	....	$\frac{s_{2y1}}{u_{2y1}}\ \frac{s_{2y2}}{u_{2y2}} \dots \frac{s_{2yr}}{u_{2yr}}$	a_2	$\gamma_{21}\ \gamma_{22} \dots \gamma_{2r}$
....	....	....	....	...	...	...
U_x	$\frac{s_{x11}}{u_{x11}}\ \frac{s_{x12}}{u_{x12}} \dots \frac{s_{x1r}}{u_{x1r}}$	$\frac{s_{x21}}{u_{x21}}\ \frac{s_{x22}}{u_{x22}} \dots \frac{s_{x2r}}{u_{x2r}}$	....	$\frac{s_{xy1}}{u_{xy1}}\ \frac{s_{xy2}}{u_{xy2}} \dots \frac{s_{xyr}}{u_{xyr}}$	a_x	$\gamma_{2r}\ \gamma_{2r} \dots \gamma_{xr}$
Demand	b_1	b_2	.....	b_y		
Max.Value	$d_{11}\ d_{12} \dots d_{1r}$	$d_{21}\ d_{22} \dots d_{2r}$	....	$d_{y1}\ d_{y2} \dots d_{yr}$		

5. Numerical Example

To show the effectiveness of the proposed algorithm numerical example is solved by using proposed algorithm. Consider the following multi objective fuzzy fractional transportation problem. Here three objectives are considered, first objective is concerned with transportation cost which is the ratio of actual cost and preferred cost. Second objective is about time of transportation which is the ratio of actual transportation time and preferred transportation time. Third objective is related to damage cost which is the ratio of actual cost and preferred cost. All the parameters of the problem are described as dual hesitant fuzzy number.

Table 5.1. Transportation Cost

Origin/ Destination	A	B	C	Supply
D	$\frac{(10;0.7,0.6,0.3;0.2,0.1,0.6)}{(6;0.8,0.7,0.6;0.1,0.2,0.6)}$	$\frac{(13;0.9,0.7,0.6;0.1,0.2,0.3}{(7;0.7,0.5;0.1,0.2)}$	$\frac{(30;0.6,0.5;0.2,0.4)}{(26;0.7,0.4;0.2,0.5)}$	(12;0.7;0.2)
E	$\frac{(14;0.6,0.4,0.3;0.3,0.5,0.6)}{(19;0.9,0.7;0.1,0.2)}$	$\frac{(10;0.7,0.6;0.2,0.3)}{(5;0.8,0.75;0.1,0.15)}$	$\frac{(20;0.7,0.6,0.5;0.1,0.25,0.37)}{(5;0.8,0.6;0.1,0.2)}$	(15;0.6;0.3)
F	$\frac{(25;0.7,0.6,0.5;0.2,0.1,0.3)}{(17;0.8,0.7;0.2,0.25)}$	$\frac{(30;0.4,0.3;0.5,0.3)}{(16;0.65,0.7;0.2,0.25)}$	$\frac{(16;0.75,0.3;0.2,0.6)}{(22;0.5,0.5;0.5,0.4)}$	(16;0.9,0.8;0.1,0.2)
Demand	(9;0.9;0.1)	(13;0.8;0.1)	(21;0.5,0.4;0.2,0.5)	

Table 5.2. Transportation Time

Origin/ Destination	A	B	C	Supply
D	$\frac{(16;0.73,\ 0.65;0.21,\ 0.3)}{(25;0.7,\ 0.6,\ 0.5;\ 0.2,\ 0.3,\ 0.4)}$	$\frac{(33;0.5,\ 0.4;\ 0.4,\ 0.5)}{(28;0.7,\ 0.6,\ 0.5;\ 0.2,\ 0.1,\ 0.3)}$	$\frac{(24;0.8,\ 0.7,\ 0.75;0.1,\ 0.2,\ 0.15)}{(35;0.6,\ 0.55,\ 0.59;0.4,\ 0.3,\ 0.35)}$	(12; 0.7; 0.2)
E	$\frac{(3;0.6,\ 0.5;\ 0.4,\ 0.3)}{(5;0.7,\ 0.5;0.2,\ 0.4)}$	$\frac{(21;0.8,\ 0.6;0.1,\ 0.2)}{(8;0.7,\ 0.4,\ 0.3;0.1,0.3,\ 0.5)}$	$\frac{(12;0.9,\ 0.8,\ 0.7;\ 0.1,\ 0.2,\ 0.25)}{(10;\ 0.8,\ 0.7;\ 0.2,\ 0.25)}$	(15; 0.6; 0.3)
F	$\frac{(30;0.5,\ 0.3;0.2,\ 0.4)}{(22;0.7,\ 0.68;0.2,\ 0.28)}$	$\frac{(19;0.5,\ 0.6;\ 0.2,\ 0.4)}{(12;0.8,\ 0.7;0.2,0.3)}$	$\frac{(37;0.7,\ 0.6,\ 0.5;0.3,\ 0.4,\ 0.2)}{(14;0.4,\ 0.3,\ 0.2,0.1;0.5,0.4,0.6,0.7)}$	(16; 0.9, 0.8; 0.1, 0.2)
Demand	(9; 0.9 ; 0.1)	(13; 0.8; 0.1)	(21; 0.5, 0.4; 0.2, 0.5)	

Table 5.3. Damage Cost

Origin/ Destination	A	B	C	Supply
D	$\frac{(28;0.6,0.5,0.4;0.2,0.4,0.5)}{(29;0.8,0.7;0.2,0.15)}$	$\frac{(19;0.75,0.61;0.2,0.3)}{(12;0.8,0.6;0.2,0.4)}$	$\frac{(26;0.85,0.8;0.1,0.18)}{(16;0.7,0.6;0.2,0.3)}$	(12;0.7;0.2)
E	$\frac{(25;0.5,0.4;0.1,0.4)}{(16;0.8,0.7,0.6;0.2,0.25,0.3)}$	$\frac{(34;0.6,0.7;0.2,0.3)}{(24;0.7,0.8;0.2,0.1)}$	$\frac{(26;0.6,0.5,0.4;0.2,0.3,0.4)}{(16;0.8,0.7,0.65;0.2,0.25,0.3)}$	(15;0.6;0.3)
F	$\frac{(13;0.6,0.5,0.4;0.4,0.25,0.35)}{(17;0.7,0.8;0.1,0.15)}$	$\frac{(30;0.8,0.75,0.6;0.1,0.18,0.28)}{(12;0.5,0.4,0.3;0.2,0.1,0.5)}$	$\frac{(34;0.8,0.9;0.1,0.18)}{(15,0.7,0.6;0.2,0.3)}$	(16;0.9,0.8;0.1,0.2)
Demand	(9;0.9;0.1)	(13;0.8;0.1)	(21;0.5,0.4;0.2,0.5)	

Step 1: By using the ranking function the given fuzzy parameters of the multi objective transportation problem can be written as follows:

Table 5.4. Transportation Cost, Transportation Time and Damage Cost

Origin/ Destination	A	B	C	Supply
D	$\frac{10.233}{6.467}$ $\frac{13.967}{19.65}$ $\frac{25.4}{17.525}$	$\frac{13.533}{7.3}$ $\frac{10.4}{5.65}$ $\frac{29.95}{16.3}$	$\frac{30.25}{26.2}$ $\frac{20.36}{5.55}$ $\frac{16.125}{22.1}$	(12;0.7;0.2)
E	$\frac{16.435}{25.3}$ $\frac{3.2}{5.3}$ $\frac{30.1}{22.45}$	$\frac{33}{28.4}$ $\frac{21.55}{8.667}$ $\frac{19.25}{12.5}$	$\frac{24.6}{35.23}$ $\frac{12.617}{10.525}$ $\frac{37.3}{13.6}$	(15;0.6;0.3)
F	$\frac{28.133}{29.575}$ $\frac{25.2}{16.45}$ $\frac{13.167}{17.625}$	$\frac{19.43}{12.4}$ $\frac{34.5}{24.6}$ $\frac{30.717}{12.133}$	$\frac{26.685}{16.4}$ $\frac{26.2}{16.467}$ $\frac{34.71}{15.2}$	(16;0.9,0.8;0.1,0.2)
Demand	(9;0.9;0.1)	(13;0.8;0.1)	(21;0.5,0.4;0.2,0.5)	

Step 2: Finding the fractional value of every value in the above table, reduced table as follows :

Table 5.5

Origin/ Destination	A	B	C	Supply
D	1.58 0.71 1.45	1.85 1.84 1.84	1.15 3.66 0.73	(12;0.7;0.2)
E	0.65 0.60 0.60	1.14 2.49 0.79	0.7 0.2 2.4	(15;0.6;0.3)
F	0.95 1.53 0.75	1.57 1.40 2.53	1.63 1.59 2.21	(16;0.9,0.8;0.1,0.2)
Demand	(9;0.9;0.1)	(13;0.8;0.1)	(21;0.5,0,4;0.2,0.5)	

Step 3: Find the maximum value for each row and each column and the maximum value are written in the above table:

Table 5.6

Origin/ Destination	A	B	C	Supply	Max. Value
D	1.58 0.71 1.45	1.85 1.84 1.84	1.15 3.66 0.73	(12;0,7;0.2)	1.85 3.66 1.84
E	0.65 0.60 0.60	1.14 2.49 0.79	0.7 0.2 2.4	(15;0.6;0.3)	1.14 2.49 2.4
F	0.95 1.53 0.75	1.57 1.40 2.53	1.63 1.59 2.21	(16;0.9,0.8; 0.1,0.2)	1.63 1.59 2.53
Demand	(9;0.9;0.1)	(13;0.8;0.1)	(21;0.5,0.4;0.2,0.5)		
Max.Value	1.58 1.53	1.85 2.49	1.63 3.66		

Step 4: Choose the maximum value (L) among the largest value of each row and column. That is

$$L = \max\{1.85, 3.66, 1.84, 1.14, 2.49, 2.4, 1.63, 1.59, 2.53, 1.58, 1.53, 1.45, 1.85, 2.49, 2.53, 1.63, 3.66, 2.4\}$$
$$L = 3.66$$

Step 5: The cell allocation of 3.66 is (1,3) of second objective. Find the summation of all objective values of 3^{rd} column. That is,

$$0.7 + 0.2 + 2.4 = 3.3 \text{ and } 1.63 + 1.59 + 2.21 = 5.43$$

Among them choose the minimum value . Here the minimum value is 3.3 which lies in the cell (2,3). Allocate min{(15; 0.6; 0.3), (21; 0.5, 0.4; 0.2, 0.5)} in the cell (2,3). Delete the second row.

Remaining is $(21;0.5,0.4;0.2,0.5) - (15;0.6;0.3) = (6;0.3,0.24;0.44,0.65)$

Repeating the procedure till all the optimum solution is obtained. The optimum solution is given as follows:

$$X_{11} = (9;0.15,0.12,0.13,0.17;0.64,0.78,0.68,0.8)$$
$$X_{12} = (3;0.22,0.18,0.19,0.13;0.58,0.12,0.6,0.75)$$
$$X_{23} = (15;0.6;0.3)$$
$$X_{32} = (10;0.27,0.22,0.24,0.19;0.5,0.69,0.55,0.72)$$
$$X_{33} = (6;0.3,0.24;0.44,0.65)$$

The objective values are

$$Z_1 = \frac{(825\ ;0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5\ 0.6;0.1.0.2,\ 0.3,\ 0.5,\ 0.6,\ 0.7)}{(444\ ;0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,;0.2,\ 0.3,\ 0.4,\ 0.5\ ,\ 0.6)}$$
$$Z_2 = \frac{(835\ ;0.1,\ 0.2,\ 0.3,0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8\ ;0.1,\ 0.2,\ 0.3\ ,\ 0.4,\ 0.5,\ 0.6)}{663;0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6\ ;0.2,\ 0.1,\ 0.3,\ 0.4,\ 0.5,\ 0.7,\ 0.8)}$$
$$Z_3 = \frac{(1203;0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,0.9;0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,)}{(747;0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,;0.2,\ 0.3,\ 0.4,\ 0.5\ ,\ 0.6)}$$

Using the developed algorithm the numerical problem solved. The obtained solution is again the hesitant fuzzy element which is the advantage of proposed method.

6. Conclusion

In this paper a new algorithm is proposed to find the optimal solution of the multi objective dual hesitant fuzzy transportation problem. The proposed algorithm is very simple and easy to understand. This algorithm gives the better solution in dual hesitant fuzzy environment. The numerical example is solved to explain the algorithm. The solution of the problem is again the hesitant fuzzy element which is the main advantage of the proposed method. The proposed algorithm gives the better solution than the existing one.

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