Malaya Journal of Matematik, Vol. 8, No. 4, 2282-2284, 2020

https://doi.org/10.26637/MJM0804/0164

Some standard results on even sum graphs

K. Chinju Krishna [1]*, C. David Raj[2] and K. Rubin Mary[3]

Abstract

A sum graph is a graph for which there is a labeling of its vertices with distinct positive integers so that two vertices are adjacent if and only if the sum of their labels is the label of another vertex. Integral sum graphs are defined similarly, except that the labels may be any integers. The concept of Even Sum Graphs was introduced by C. David Raj, et al. A graph G is called an even sum graph if there is a labeling θ of its vertices with distinct non - negative even integers, so that for any two distinct vertices u and v, uv is an edge of G if and only if $\theta(u)+\theta(v)=\theta(w)$ for some vertex w in G. The minimum number of isolated vertices required to make the graph G, an even sum graph is called the even sum number of G and is denoted by $\gamma(G)$.

Keywords

Even sum graph, path, umbrella graph, parachute graph.

AMS Subject Classification

05C05, 05C70, 05C75, 05C78.

[1] *Research Scholar, Registration Number: 18223232092004, Department of Mathematics, St. Jude's College, Thoothoor, Kanyakumari District, 629176, Tamilnadu, India.*

[2] *Department of Mathematics,Malankara Catholic College, Mariagiri, Kaliyakavilai, Tamilnadu, India*

[3] *Department of Mathematics, St. Jude's College, Thoothoor, Kanyakumari District, 629176, Tamilnadu, India.*

Affiliated to Manonmaniam Sundaranar University, Abishekapatti-Tirunelveli-627012.

***Corresponding author**: [1] chinjukrishna1991@gmail.com; [2] davidrajmccm@gmail.com, [3] rubyjudes@yahoo.com

Article History: Received **10** July **2020**; Accepted **22** November **2020**

Contents

1. Introduction

Graph Labeling was first introduced by Alexandra Rosa during 1960. In Graph Labeling, vast amount of literature is available. Labeling is an assignment of integers to the vertices or edges or both is subject to certain conditions motivated by practical problems. The concept of Sum Graphs and Integral Sum Graphs was introduced by F. Harary [6, 7, 8, 9]. The properties of Sum Graphs are investigated by many authors, including Chen. Z [1], Mary Florida. L [11], Nicholas. T [10], Soma Sundaram. S [10], Vilfred.V [10, 11,12], Surya Kala.V [12] and Rubin Mary. K [12]. In this paper we investigates different types of Even Sum Graphs [2, 3, 4, 5]. For all basic ideas, we refer [6,7].

Let P_m and P_n be two paths. Let the vertices of P_n be $u_1, u_2, \dots u_n$. Take n copies of P_m. Let the vertices of j^{th} copy be $v_{1j}, v_{2j}, \dots, v_{mj}, 1 \leq j \leq n$. Identify the end vertex v_{1j} of P_m with $u_j, 1 \leq j \leq n$, the resultant graph is $P_m * P_n$.

An Umbrella graph $U_{m,n}$ is obtained by joining each vertices of the path P_m with an end vertex of the path P_n.

Let g,b be the positive integers such that $g \geq 3$ and P_g denotes a path of order g with the vertex set $\{v_1, v_2, \dots, v_g\}$ and the edge set $E(P_g) = \{v_i v_{i+1} / 1 \leq i \leq g-1\}$. Then the graph $1 * P_g$ has the vertex set $V(1*P_g) = \{v\} \cup V(P_g)$ *and let* $E(1* P_g) = E(P_g) \cup \{vv_i / 1 \leq i \leq g\}$. Finally C_{g+b} denote the cycle of order $(g+b)$ with $V(C_{g+b}) = \{v_1, v_2, \dots, v_g, v'_1, v'_2, \dots, v'_b\}$ and $E(C_{g+b}) = E(P_g) \cup \{v_1 v'_1, v_g v'_b / 1 \leq i \leq b-1\} \cup \{v'_i v'_{i+1} / 1 \leq i \leq b-1\}$. Then the resultant graph is defined as a parachute $P_{g,b}$ and is given by $P_{g,b} = (V_{g,b}, E_{g,b})$ such that $g,b \in N, g \geq 3$ where $P_{g,b}$ is the amalgamation of $(1*P_g) \cup C_{g+b}$, obtained from the union of $1*P_g$ and C_{g+b} by passing them along P_g such that the intersection $(1*P_g) \cap C_{g+b}$ is equal to P_g.

2. Main Results

Theorem 2.1. *A graph $P_m * P_n$ is an even sum graph.*

Proof. Let $u_1, u_2, \dots, u_n$ be the vertices of P_n. Take n copies of P_m. Let the vertices of j^{th} copy of P_m be $v_{1j}, v_{2j}, \dots v_{mj}, 1 \leq j \leq n$. Identify the end vertex v_{1j} of P_m with $u_j, 1 \leq j \leq n$, the resultant graph is $P_m * P_n$. Define a function

$\theta : V(G) \to 2Z^{+} \cup \{0\}$ by $\theta(v_{11}) = 2; \theta(v_{12}) = 4;$
$\theta(v_{1j}) = \theta(v_{1(j-2)}) + \theta(v_{1(j-1)}),\ 3 \leq j \leq n;$
$\theta(v_{21}) = \theta(v_{1(n-1)}) + \theta(v_{1n});$
$\theta(v_{2j}) = \theta(v_{(n-1)(j-1)}) + \theta(v_{n(j-1)}),\ 2 \leq j \leq n;$
$\theta(v_{ij}) = \theta(v_{(i-1)j}) + \theta(v_{(i-2)j}), 1 \leq i \leq m, 3 \leq j \leq n;$
$\theta(v_{(m-1)n}) = 0; \theta(v_{mn}) = \theta(v_{(m-2)n}) + \theta(v_{(m-3)n}).$
Then the labels are distinct and $\theta(u) + \theta(v) = \theta(w)$ for some vertices u, v, w in G. Hence $P_m * P_n$ is an even sum graph. □

Example 2.2. *Even sum graph of $P_6 * P_5$ is shown in Figure 1.*

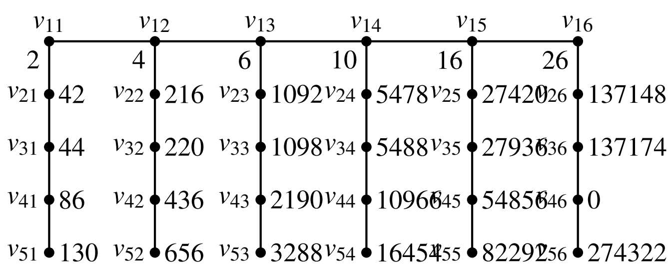

Figure 1. $P_6 * P_5$

Theorem 2.3. *A graph $G = (U_{m,n}) \cup 1K_1$ is an even sum graph.*

Proof. Let $x_i, 1 \leq i \leq m$ and $y_i, 1 \leq i \leq n$ be the vertices of $U_{m,n}$. Let w_0 be an isolated vertex of G. Let $E(G) = \{x_i x_{i+1}, / 1 \leq i \leq m-1\} \cup \{x_i y_i / 1 \leq i \leq m\} \cup \{x_i y_{i+1}, / 1 \leq i \leq n-1\}$. Define a function $\theta : V(G) \to 2Z^{+} \cup \{0\}$by $\theta(x_1) = 2; \theta(x_2) = 4; \theta(x_i) = \theta(x_{i-2}) + \theta(x_{i-1}), 3 \leq i \leq m; \theta(y_1) = 0; \theta(y_2) = \theta(x_{n-1}) + \theta(x_n); \theta(y_3) = \theta(y_2) + 2;\ \theta(y_i) = \theta(y_{i-2}) + \theta(y_{i-1}), 4 \leq i \leq n; \theta(w_0) = \theta(y_{n-1}) + \theta(y_n)$. Then the labels are distinct and $\theta(u) + \theta(v) = \theta(w)$ for some vertices u, v, w in G. Hence the graph G is an even sum graph. □

Example 2.4. *An Even sum graph of$G = U_{6,5} \cup 1K_1$ is shown in the Figure 2.*

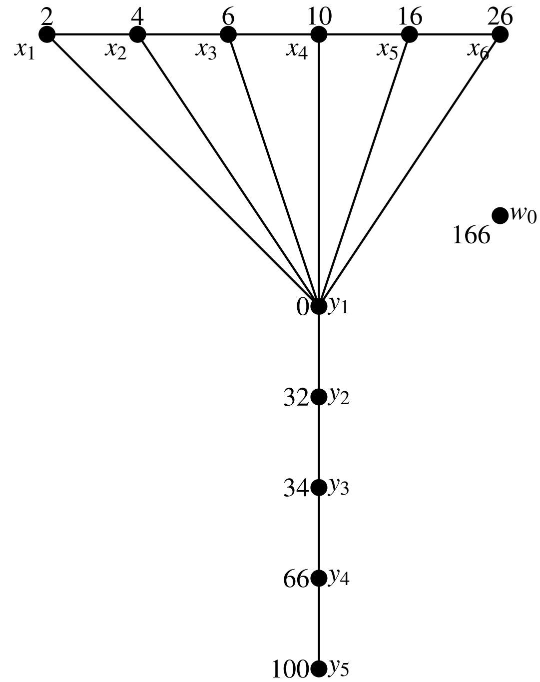

Figure 2. $G = U_{6,5} \cup 1K_1$

Theorem 2.5. *A graph $G = P_{g,b} \cup 2K_1$ is an even sum graph.*

Proof. Let $v_1, v_2, \ldots, v_g, v'_1, v'_2, \ldots, v'_b$ be the vertices of C_{g+b} where C_{g+b} denote the cycle of order $(g+b)$. Join a vertex v to each vertices of P_g. The resultant graph is $1 * P_g$. Add two isolated vertices w_1, w_2 to $P_{g,b}$. The graph so obtained is G whose edge set is $E(G) = \{vv_i, v_i v_{i+1} / 1 \leq i \leq g-1\} \cup \{v'_j v'_{j+1}, v_1 v'_1, v'_b v_g / 1 \leq i \leq g-1,\ 1 \leq j \leq b-1\}$. Define a function $\theta : V(G) \to 2Z^{+} \cup \{0\}$ by $\theta(v) = 0; \theta(v_1) = 2; \theta(v_2) = 4; \theta(v_i) = \theta(v_{i-2}) + \theta(v_{i-1}), 3 \leq i \leq g; \theta(v'_1) = \theta(v_{n-1}) + \theta(v_n);\ \theta(v'_2) = \theta(v_1) + \theta(v'_1), \theta(v'_j) = \theta(v'_{j-2}) + \theta(v'_{j-1}), 3 \leq j \leq b; \theta(w_1) = \theta(v'_{b-2}) + \theta(v'_{b-1});$
$\theta(w_2) = \theta(v'_{b-1}) + \theta(v'_b)$. Then the labels are distinct and $\theta(u) + \theta(v) = \theta(w)$ for some vertices u, v, w in G. Hence the graph G is an even sum graph. □

Example 2.6. *An even sum graph of $G = P_{6,5} \cup 2K_1$ is shown in the Figure 3.*

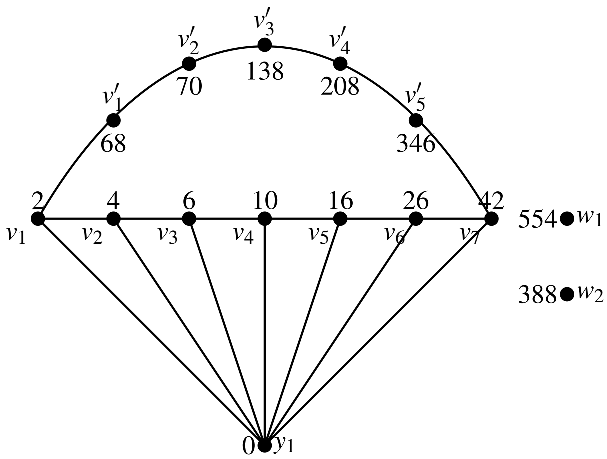

Figure 3. $G = P_{6,5} \cup 2K_1$

References

[1] Chen. Z, Harary's: Conjecture on Integral Sum Graphs, *Discrete Mathematics*, 160, pp. 241 - 244,(1990).

[2] David Raj. C, Rubin Mary K and Chinju Krishna K,: Sum Graphs and its Related Concepts, *International Journal of Aquatic Science,* Communicated.

[3] David Raj. C, Chinju Krishna. K and Rubin Mary. K,: A Study on Even Sum Graphs, *South East Asian Journal of Mathematics and Mathematical Sciences*, Communicated.

[4] Chinju Krishna. K, David Raj. C, Rubin Mary. K,: Path Related Even Sum Graphs, *Turkish Online Journal of Qualitative Inquiry,* 12(7),(2020) 6341 - 6344.

[5] Chinju Krishna. K, David Raj. C, Rubin Mary. K,: Some Results on Even Sum Graphs,*Advances and Applications in Mathematical Science*, Communicated.

[6] Gallian. J. A,: A Dynamic Survey on Graph Labeling,, Electronic J. Comb., 19 (2013),DS6.

[7] Harary. F: Graph Theory , Addison Weseley, Reading Mass, 1969 .

[8] Harary. F : Sum Graphs and Difference Graphs, Cong. Num., 72 (1990), 101 - 108.

[9] Harary. F: Sum Graphs Over all Integers, *Discrete Mathematics*, 124 (1994), 99 - 105.

[10] Nicholas T, Soma Sundaram. S and Vilfred V: Some Results on Sum Graphs, *J. Comb.Inf. and System Sci.*, 26(2001), pp. 135 - 142 .

[11] Vilfred. V, Surya Kala. V and Rubin Mary K: More on Integral Sum Graphs Gm,n,*Proc. International Conference. on Applied Mathematics and Theoretical Computer Science*, St. Xaviers College of Engineerig., Nagercoil., TN, India, pp. 134 - 141, (2013).

Malaya Journal of Matematik
ISSN(O):2321 – 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 – 3786