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# Mean labeling pattern of some graphs

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**Abstract**
The concept of mean labeling was introduced by S. Somasundaram and Ponraj in 2003.Many research papers have published in this topic. In this paper we have established a general format for labeling of $T_{m,1}; T(n); B_{n,n}; D_nt; K_2 \odot C_n$.

**Keywords**
Labeling, Mean labeling, Mean graph.

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## Contents



## 1. Introduction

Throughout this paper by a graph we mean a finite simple graph. For basic definitions and notations in graph theory follow Bondy Murthi [2], S.Somasundaram and Ponraj [7] introduced the concept of mean labeling of graphs. In this paper we investigate the existence of mean labeling of some graph.

**Definition 1.1.** *The pan graph is the graph obtained by joining a cycle graph $C_m$ with $K_1$ as a bridge the m-pan graph is isomorphic with the (m,1) tadpole graph.*

**Definition 1.2.** *The graph G of (V, E) obtained from path by attaching exactly two pendent edges to each internal vertices of the path is called twig graph it is denoted by T(n).*

**Definition 1.3.** *The Bistar $B_{n,n}$ is the graph obtained by making adjacent the two central vertices of $K_{1,n}$ and $K_{1,n}$.*

**Definition 1.4.** *The graph $D_nt(n \geq 3, t \geq 1)$ is obtained from the cycle $C_n$ by attaching a path of length t to any on vertices of $C_n$.*

**Definition 1.5.** *The graph $K_2 \odot C_n$ is obtained by attaching the cycle $C_n$ at each end point of $K_2$.*

## 2. Main Result

**Theorem 2.1.** *The graph $T_{m,1}$ is a mean graph.*

*Proof.* Let $G = T_{m,1}$ be a graph with $|V(G)| = m+1$ ; $|E(G)| = m+1$;
The vertex set of the graph is defined by $V(G) = \{v_i/i = 1,2,3,...,k-1,k,k+1,...,m+1\}$
Here $k = \lfloor \frac{n}{2} \rfloor$
$N(v_i) = \{v_{i-1}; v_{i+1}/1 \geq i \geq m+1\}; N(v_1) = \{v_2, v_m\}$;
$N(v_{m+1}) = \{v_k\}$;

The edge set is defined by ,$E(G)= \begin{cases} e_i \ /i = 1,2,3,...,k \\ e_j \ /j = 1,2,3,...,k \end{cases}$.

Labeling pattern of the graph is given below,

$$L(v_{n-i}) = \begin{cases} n-(i-j)/i = m-1 \ to \lceil \frac{m+1}{2} \rceil, j = 2 \ to \ \lceil \frac{m+1}{2} \rceil \\ n-(j-i)/i = \lceil \frac{m-1}{2} \rceil \ to \ 0, \ j = \lceil \frac{m+1}{2} \rceil \ to \ m \end{cases}.$$

$L(v_{m+1}) = v_k + 1$
$L(e_i) = \{2i/i = 1,2,3,...,k\}$
$L(e_i^{'}) = \{2j-1/i = 1,2,3,...,k+1\}$
The graph satisfies the mean labeling condition.
Therefore, the graph $G$ is mean graph. □

**Theorem 2.2.** *The graph $D_nt(n \geq 3)$ is a mean graph.*

*Proof.* Let $G = D_nt$ be a graph with $n+t$ vertices and edges.
The vertex set is defined by

$$V(G)= \begin{cases} v_i/i = 1,2,3,...,n \\ v_i^{'}/i = 1,2,3,...,t \end{cases}.$$

The edge set is defined by

$E(G) = \begin{cases} e_i/i = 1,2,3,...,k \\ f_i/i = 1,2,3,...,k \\ x_i/i = 1,2,3,...,k \end{cases}$.
$N(v_i) = \{v_(i-1, v_{i+1}; 1 < i < n\}$ ;
$N(v_i^{'}) = \{v_{i-1}^{'}, v_{i+1}^{'}; 1 < i < n\}$
$N(v_n) = \{v_{n-1}, v_1\}; N(v_1^{'}) = \{v_2^{'}, v_k\}$
$N(v_t^{'}) = \{v_{t-1}^{'}\}$
$f(e_i) = \{v_i v_(i+1); 1 \leq i \leq x\}$ ; $f(e_i^{'}) = \{v_i v_{i-1}/i \leq i \leq k\}$

$f(e_i^{''}) = \{v_i v_{i-1}\}$
The labeling pattern is given below,
$ML(v_i) = \begin{cases} n-(i-j)/i = n-1 \ to \ \lceil \frac{n+1}{2} \rceil \ and \ j = 2 \ to \ \lceil \frac{n+1}{2} \rceil \\ n-(j-i)/i = \lceil \frac{n-1}{2} \rceil \ to \ 0 \ and \ j = \lceil \frac{n+1}{2} \rceil \ to \ n \end{cases}$
$ML(v_i^{'}) = \{v_{\lfloor \frac{n}{2} \rfloor + i}/i - 1,2,3,...,t\}$
$f(v_i^{'}) = \{v_{\frac{n}{2}} v_i^{'}\}$
$ML(e_i) = \{2i/i = 1,2,3,...,k\}$
$ML(e_i^{'}) = \{2i-1/i = 1,2,3,...,t\}$
$ML(e_i^{''}) = \{n+i/i = 1,2,3,...,t\}$
The graph satisfies the mean labeling condition.
Therefore, the graph G is mean graph. □

**Theorem 2.3.** *The Bistar $B_{n,m}$ is a mean graph where $n \geq 2$.*

*Proof.* Let $G = B_{n,m}$ be a graph $|V(G)| = 2n+2$ and $|E(G)| = 2n+1$.
$v^{'}$ is adjacent to $v_i$ and $u^{'}$.
$u^{'}$ is adjacent to $u_i$ and $v^{'}$.
The vertex set is $V(G) = \begin{cases} v^{'}, u^{'} \\ v_i/i = 0,1,2,3,...,n-1 \\ u_i/i = 0,1,2,3,...,n \end{cases}$.
$N(v_i) = v^{'}; N(u_i) = u^{'}; N(v^{'}) = u^{'}$.
$ML(v^{'}) = 1; ML(v_i) = 2i/i = 1,2,3,...,n-1$.
$ML(u_n) = 2i+1/i = 1,2,3,...,n-1; ML(u_n) = 2n$.
$ML(u^{'}) = 2n+1$.
$ML(e_i) = i/i = 1,2,3,...,2n+1$.
Clearly, the vertex and edge satisfies the mean labeling condition.
Therefore, the graph $G = B_{n,m}$ is mean graph. □

**Theorem 2.4.** *The graph $K_2 \odot C_n (n \geq 3)$ is mean graph for all n.*

*Proof.* Let $G = K_2 \odot C_n (n \geq 3)$ be a graph $|V(G)| = 2n; |E(G)| = 2n+1$.
The vertex set of the graph is
$V(G) = \begin{cases} v_i/i = 1,2,3,...,n \\ v_i^{'}/i = 1,2,3,...,n \end{cases}$
The edge set of the graph is
$E(G) = \begin{cases} e_i/i = 1,2,3,...,n \\ e_i^{'}/i = 1,2,3,...,n \end{cases}$.
$N(v_i) = \{v_{i-1}, v_{i+1}; 1 < i < n\}$.
$N(v_1) = \{v_n, v_2\}$.
$N(v_n) = \{v_1, v_{n-1}\}$.
$N(v_k) = \{v_{k-1}, v_{k+1}, v_{n+1}\}$ here $k = \frac{n}{2}$.
Vertex labeling of the graph is given by
$L(v_i) = \begin{cases} n-(i-j)/i = n-1 \ to \ \lceil \frac{n+1}{2} \rceil, j = 2 \ to \ \lceil \frac{n+1}{2} \rceil \\ n-(j-i)/i = \lceil \frac{n-1}{2} \rceil \ to \ 0, j = \lceil \frac{n+1}{2} \rceil \ to \ n \end{cases}$.
$L(v_i^{'}) = \{n+i/i = 1,2,3,...,n\}$.
The edge labeling is given below,
$L(e_{n-i}) = \begin{cases} 2(n-i)/i = n-1 \ to \ \lfloor \frac{n+1}{2} \rfloor \\ 2i+1/i = \lfloor \frac{n-1}{2} \rfloor \ to \ 0 \end{cases}$.
$L(e_i^{'}) = \{n+i/i = 1,2,3,...,n\}$.
The graph satisfies the mean labeling condition.
Hence, $K_2 \odot C_n$ is mean graph. □

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**Theorem 2.5.** *The graph $T(n)(n \geq 3)$ is mean graph.*

*Proof.* Let $G = T(n)$ be a graph with $k = 3n-5$ edges $m = 3n-4$ vertices.
The vertex set of G is defined by
$V(G) = \{v_i/i = 0,1,2,3,...,n\}$.
The edge set of G is defined by
$E(G) = \{e_i/i = 1,2,3,...,n\}$.
Labeling pattern of the graph is given below,
$ML(v_i) = \begin{cases} v_0 = 0 \\ v_i = i/i = 1,2,3,...,m-1 \\ v_m = v_{m-1} + 2 \end{cases}$.
$ML(e_i) = \{e_i = i/i = 1,2,3,...,k\}$. The vertex and edge labeling are distinct.
Therefore, the graph $G = T(n)$ is mean graph. □

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