Malaya Journal of Matematik, Vol. 8, No. 4, 2292-2295, 2020
https://doi.org/10.26637/MJM0804/0167

Relatively prime inverse domination of a graph

C. Jayasekaran [1]* and L. Roshini[2]

Abstract
Let G be non-trivial graph. A subset D of the vertex set V(G) of a graph G is called a dominating set of G if every vertex in V - D is adjacent to a vertex in D. The minimum cardinality of a dominating set is called the domination number and is denoted by $\gamma(G)$. If V - D contains a dominating set S of G, then S is called an inverse dominating set with respect to D. In an inverse dominating set S, every pair of vertices u and v in S such that (deg u, deg v) = 1, then S is called relatively prime inverse dominating set. The minimum cardinality of a relatively prime inverse dominating set is called relatively prime inverse dominating number and is denoted by γ_{rp}^{-1} (G). In this paper we find relatively prime inverse dominating number of some graphs.

Keywords
Domination, Inverse domination, Relatively prime domination.

AMS Subject Classification
05C15, 05C69.

[1,2] *Department of Mathematics, Pioneer Kumaraswamy College Nagercoil-629003, Kanyakumari District, Tamil Nadu, India. Affiliated to Manonmaniam Sundaranar University, Abishekapatti-Tirunelveli-627012.*
***Corresponding author**: [1] jayacpkc@gmail.com; [2]jerryroshini92@gmail.com
Article History: Received **10** July **2020**; Accepted **22** November **2020**

Contents

1. Introduction

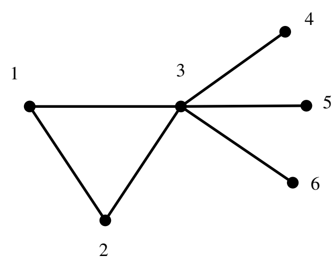
By a graph, we mean a finite undirected graph with neither loops nor multiple edges. For graph theoretic terminology, we refer to the book by Chartrand and Lesniak [1]. All graphs in this paper are assumed to be non-trivial. In a graph $G = (V,E)$, the degree of a vertex v is defined to be the number of edges incident with v and is denoted by deg v. The subgraph induced by a set S of vertices of a graph G is denoted by $\langle S\rangle$ with $V(\langle S\rangle) = S$ and $E(\langle S\rangle) = \{uv \in (G)/u,v \in S\}$. The study of domination and related subset problems is one of the fastest growing areas in graph theory. For a detailed survey of domination one can see [3, 4]. A set D of vertices of graph G is said to be a dominating set if every vertex in $V-D$ is adjacent to a vertex in D. A dominating set D is said to be a minimal dominating set if no proper subset of D is a dominating set. The minimum cardinality of a dominating set of a graph G is called the domination number of G and is denoted by γ(G). Kulli V. R. et al introduced the concept of inverse domination in graphs [6]. Let D be a minimum dominating set of G. If $V-D$ contains a dominating set S, then S is called the inverse dominating set of G with respect to D. The inverse dominating number γ^{-1}(G) is the minimum cardinality taken over all the minimal inverse dominating set of G. Bistar $B_{m,n}$ is the graph obtained by joining the center(apex) vertex of $K_{1,m}$ and $K_{1,n}$ by an edge [2]. The $n-$barbell graph $BB_{n,n}$ is the simple graph obtained by connecting two copies of a complete graph K_n by a path P_2 [7]. C. Jayasekaran et.a introduced the concept of relatively prime domination. A set $S \subseteq V$ is said to be relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices u and v in S such that $(degu, degv) = 1$. The minimum cardinality of a relatively prime dominating set of a graph G is called the relatively prime domination number of G and is denoted by $\gamma_{rpd}(G)$ [5]. The purpose of this paper is to introduce the concept of relatively prime inverse domination of graphs.

2. Relatively prime inverse domination of some graphs

Definition 2.1. *Let D be a minimum dominating set of a graph G. If $V-D$ contains a dominating set S of G, then S is called an inverse dominating set with respect to D. In an inverse dominating set S, every pair of vertices u and v in S*

such that $(deg u, deg v) = 1$, then S is called relatively prime inverse dominating set. The minimum cardinality of relatively prime inverse dominating set is called relatively prime inverse dominating number and is denoted by $\gamma_{rp}^{-1}(G)$.

Example 2.2. *Consider the graph G given figure 2. 1. Clearly $D = \{3\}$ is a minimum dominating set and $S = \{2,4,5,6\}$ is minimum relatively prime inverse dominating set of G and hence $\gamma_{rp}^{-1}(G)$=4.*



G

Figure 2.1

Theorem 2.3. *For a path P_n,* $\gamma_{rp}^{-1}(P_n) = \begin{cases} 2 \; if \; 3 \leq n \leq 5 \\ 3 \; if \; n = 6,7 \\ 0 \; otherwise \end{cases}$.

Proof. Let P_n be the path $v_1v_2v_3...v_n$. Let D be a minimum domination set of P_n. If $n \equiv 0(mod 3)$, then $D = \{v_2, v_5, v_8, ..., v_{n-1}\}$ and if $n \not\cong 0(mod 3)$, then $D = \{v_2, v_5, v_8, ..., v_n\}$ and hence $\gamma = \lceil \frac{n}{3} \rceil$. Now, we consider the following three cases.

Case 1. $n = 3$

Clearly, $D = \{v_2\}$. Now $S = \{v_1, v_3\}$ is the inverse dominating set which is minimal and $(deg\ v_1, deg\ v_3)$ = (1, 1) =1. Therefore, S is the minimum relatively prime inverse dominating set of P_3 and hence $\gamma_{rp}^{-1}(P_3)$= 2.

Case 2. n = 4 or 5

Clearly, $D = \{v_2, v_n\}$. Now $S = \{v_1, v_{n-1}\}$ is an inverse dominating set which is minimal and $(deg\ v_1, deg\ v_{n-1})$ = (1, 2) = 1. Therefore S is a minimum relatively prime inverse dominating set of P_n and hence $\gamma_{rp}^{-1}(P_n)$ = 2.

Case 3. $n = 6$

In this case $D = \{v_2, v_5\}$ is a minimum dominating set. Clearly, $S = \{v_1, v_3, v_6\}$ is a minimum inverse dominating set. Since $(deg v_1, deg v_3)$ = (1, 2) = 1, $(deg\ v_1, deg\ v_6)$= (1, 1) = 1 and $(d(v_3), d(v_6))$ = (2, 1) = 1, S is a minimum relatively prime inverse dominating set of P_6. Therefore, $\gamma_{rp}^{-1}(P_6)$ =3.

Case 4. $n = 7$

In this case $D = \{v_2, v_5, v_6\}$ is a minimum dominating set. Clearly, $S = \{v_1, v_3, v_7\}$ is a minimum inverse dominating set. Since $(deg\ v_1, deg\ v_3)$ = (1, 2) = 1, $(deg\ v_1, deg\ v_7)$ = (1, 1) = 1 and $(d(v_3), d(v_7))$ = (2, 1) = 1, S is a minimum relatively prime inverse dominating set of P_7. Therefore $\gamma_{rp}^{-1}(P_7)$ =3.

Case 5. $n \geq 8$

Let S be an inverse dominating set of P_n with respect to the dominating set D. Then S contains at least two vertices of degree 2. This implies that S is not a relatively prime dominating set of P_n. Hence, $\gamma_{rp}^{-1}(P_n)$ =0.

The theorem follows from cases 1, 2, 3, 4 and 5. □

Theorem 2.4. *For a path P_n,*

$$\gamma_{rp}^{-1}(\bar{P}_n) = \begin{cases} 2 \; if n > 3 \\ 0 \; otherwise \end{cases} .$$

Proof. Let $\bar{P}_n$ be the complement of path P_n. If n = 2, then $\bar{P}_2 = \bar{K}_2$ which is a regular graph of degree 0. This implies that $\gamma(P_n) = 2$ and hence $\gamma_{rp}^{-1}(\bar{P}_2) = 0$. If $n = 3$, then $\bar{P}_3 = K_2 \cup K_1$. Clearly D contains the isolated vertex K_1 and a vertex of K_2. This implies that $\bar{P}_3$ has no inverse dominating set and hence $\gamma_{rp}^{-1}(\bar{P}_3) = 0$. Let $n > 3$. Now v_1 is adjacent to each vertex except v_2 in $\bar{P}_n$. Hence $D = \{v_1, v_2\}$ is a minimum dominating set of $\bar{P}_n$. Now v_3 is adjacent to all vertices except v_2 and v_4 and v_n is adjacent to all vertices except v_{n-1} in $\bar{P}_n$. Hence $\{v_3, v_n\}$ is a minimum dominating set in $V - D$. Now $(deg\ v_3, deg\ v_4) = (n-3, n-2) = 1$. Hence $\{v_3, v_4\}$ is a minimum relatively prime dominating set in $V - D$. This implies that $\gamma_{rp}^{-1}(\bar{P}_n)$= 2. Thus the theorem is proved. □

Theorem 2.5. *For a complete bipartite graph $K_{n,m}$,* $\gamma_{rp}^{-1}(K_{n,m})$

$$= \begin{cases} 2 \; if \; (n,m) = 1 \\ 0 \; if \; (n,m) \neq 1 \end{cases} .$$

Proof. Let $K_{n,m}$ be the complete bipartite graph and U, V be the bipartition of the vertex set of $K_{n,m}$ with $|U| = n, |V| = m$. Clearly $D = \{u, v / u \in U, v \in V\}$ is a minimum dominating set of $K_{n,m}$. Now $S = \{u', v'\}$ where $u' (\neq u) \in U, v' (\neq v) \in W$, is a minimum inverse dominating of $K_{n,m}$. Also $(d(u'), d(v')) = (m, n)$. If $(m, n) = 1$, then S is a relatively prime dominating set, otherwise S is not a relatively prime dominating set. Therefore

$$\gamma_{rp}^{-1}(K_{n,m}) = \begin{cases} 2 \; if \; (n,m) = 1 \\ 0 \; otherwise \end{cases} .$$ □

Theorem 2.6. *For a fan graph F_n,* $\gamma_{rp}^{-1}(F_n) = \begin{cases} 2 \; if \; n = 3,4,5 \\ 0 \; otherwise \end{cases}$.

Proof. Let F_n be a fan graph which is obtained by joining all the vertices of P_n to the vertex in K_1 i.e., $F_n = P_n + K_1$. Let, $V(F_n) = \{v_i / 0 \leq i \leq n\}$ and $E(F_n) = \{v_0v_i, v_iv_{i+1}, v_0v_n / 0 \leq i \leq n-1\}$. In F_n, $D = v_0$ is the minimum dominating set. Now we consider the following cases.

Case 1. $n = 2$

Then, $F_2 = C_3$. In C_3, all vertices have degree 2 and hence there is no relatively prime dominating set and hence $\gamma_{rp}^{-1}(F_2)$=0. .

Case 2. $n = 3$

Cleary either $\{v_1, v_2\}$ or $\{v_2, v_3\}$ is a minimum inverse dominating set with respect to D. Now, $(d(v_1), d(v_2))$ = (2,3) =1 and $(d(v_2), d(v_3))$ = (3,2)=1. Hence either $\{v_1, v_2\}$ or $\{v_2, v_3\}$ is a minimum relatively prime inverse dominating set. Therefore, $\gamma_{rp}^{-1}(F_3)$ =2.

Case 3. $n = 4$

In this case a minimum inverse dominating set $S = \{v_2, v_4\}$ in F_4 with respect to D. Also, $(d(v_2), d(v_4))$ = (3,2) = 1. Hence S is a minimum relatively prime inverse dominating set and $\gamma_{rp}^{-1}(F_4) = 2$

Case 4. $n = 5$

Here $S = \{v_2, v_5\}$ is a minimum inverse dominating set in F_5 with respect to D. Also, $(d(v_2), d(v_5))$ = (3,2) = 1.Therfore, S is a minimum relatively prime inverse dominating set. Hence $\gamma_{rp}^{-1}(F_5) = 2$

Case 5. $n \geq 6$

Let S be an inverse dominating set of F_n with respect to the dominating set D. Then S contains at least two vertices of degree three. This implies that S is not a relatively prime dominating set of F_n. Hence, $\gamma_{rp}^{-1}(F_2) = 0$

The theorem follows from cases 1 to 5. □

Theorem 2.7. *For a For a bistar tree $B_{m,n}$, $\gamma_{rp}^{-1}(B_{m,n}) = m + n$.*

Proof. Let $B_{m,n}$ be a bistar tree of order $m+n+2$ with vertex set $V(B_{m,n}) = u_i, v_j / 0 \leq i \leq m, 0 \leq j \leq n$ and edge set $E(B_{m,n}) = \{u_0v_0,\ u_0u_i,\ v_0v_j / 1 \leq i \leq m, 0 \leq j \leq n\}$. The minimum dominating set of $B_{m,n}$ is $D = \{u_0, v_0\}$. Since $V - D$ contains all end vertices, $V - D$ itself is a minimum inverse dominating set of $B_{m,n}$. Clearly $(d(u_i), d(u_j)) = (d(u_i), d(v_j)) = (d(v_i), d(u_j))$ = 1. Hence $V - D$ is the minimum relatively prime inverse dominating set and so $\gamma_{rp}^{-1}(B_{m,n}) = m + n$. □

Theorem 2.8. *If $G_1 \cong G_2$, then $\gamma_{rp}^{-1}(G_1) = \gamma_{rp}^{-1}(G_2)$.*

Proof. Let $G_1 \cong G_2$. Let f be an isomorphism between graphs G_1 and G_2. Let $\{v_1, v_2, ..., v_m\}$ be a minimum relatively prime inverse dominating set of G_1. Since f is an isomorphism $\{f(v_1), f(v_2), ..., f(v_m)\}$ is a minimum inverse dominating set of G_2. Since isomorphism, preserves degree of the vertices $(d(v_i), d(v_j)) = (d(f(v_i), d(f(v_j)) = 1$ for $i \neq j, 1 \leq i \leq j \leq m$, Therefore, $\{f(v_1), f(v_2), ..., f(v_n)\}$ is

theorem. □

Theorem 2.9. *For $K_m \cup K_n$,*

$$\gamma_{rp}^{-1}(K_m \cup K_n) = \begin{cases} 2 \ iff \ (m-1, n-1) = 1 \\ 0 \ otherwise \end{cases}.$$

Proof. Let $V(K_m) = \{v_1, v_2, ..., v_m\}$ and $V(K_n) = \{u_1, u_2, ..., u_n\}$. Clearly $D = \{v_i, u_j\}$ is a minimum dominating set of $K_m \cup K_n$ where $1 \leq i \leq m$ and $1 \leq j \leq n$. Now $S = \{v_x, u_y\}$ where $1 \leq x \leq m, x \neq i$ and $1 \leq y \leq n, y \neq j$ is a minimum inverse dominating set of $K_m \cup K_n$ with respect to D and $(d(v_x), d(u_y))$ = $(m-1, n-1)$. If $(m-1, n-1) = 1$, then S is a minimum relatively prime inverse dominating set. Otherwise S is not a relatively prime inverse dominating set. Therefore, $\gamma_{rp}^{-1}(K_m \cup K_n)$

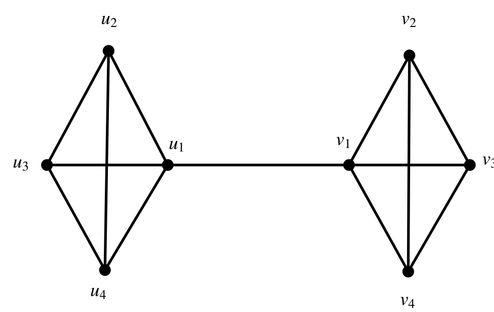
$$= \begin{cases} 2 \ iff \ (m-1, n-1) = 1 \\ 0 \ otherwise \end{cases}.$$ □

Theorem 2.10. *For For a barbell graph $BB_{n,n}$,*

$$\gamma_{rp}^{-1}(BB_{n,n}) = \begin{cases} 2 \ if \ n \geq 2 \\ 0 \ otherwise \end{cases}.$$

Proof. Let $\{v_1, v_2, ..., v_n\}$ and $\{u_1, u_2, ..., u_n\}$ be the vertex sets of two copies of K_n. Join v_1u_1. The resultant graph is a barbell graph. If $n = 1$, then $BB_{1,1} = K_2 = P_2$. By Theorem 2.3, $\gamma_{rp}^{-1}(B_1) = 0$. Let $n \geq 2$. A minimum dominating set $D = \{v_1, u_i\}, 2 \leq i \leq n$. A minimum inverse dominating with respect to D is $S = \{v_i, u_1\}, 2 \leq i \leq n$. Clearly $(d(v_i), d(u_1)) = (n-1, n)$ =1. Hence S is a minimum relatively prime inverse dominating set. Therefore, $\gamma_{rp}^{-1}(BB_{n,n}) = 2$. □

Example 2.11. *Consider the graph $G = BB_{4,4}$, given in figure 2.2. Clearly $D = \{v_1, u_2\}$ is a minimum dominating set and $S = \{v_2, u_0\}$ is a minimum relatively prime inverse dominating set of G. Therefore, $\gamma_{rp}^{-1}(BB_{4,4}) = 2$.*



$G = B_n$

Figure 2.2

Theorem 2.12. *Let G be a connected graph of order n. If $\gamma_{rp}^{-1}(G)$ exists, then $\gamma^{-1}(G) \leq \gamma_{rp}^{-1}(G)$.*

Proof. Let G be a connected graph of order n such that $\gamma_{rp}^{-1}(G)$. exists. Since every relatively prime inverse dominating set is an inverse dominating set, it follows that $\gamma^{-1}(G) \leq \gamma_{rp}^{-1}(G)$. □

Theorem 2.13. *For the path graph $P_n (n \geq 3), \gamma^{-1}(P_n) = \gamma_{rp}^{-1}(P_n)$. Hence the inequality in Theorem 2.11. becomes sharp. Now consider the graph G given in figure.2.3. Here $S = \{v_3\}$ is a minimum inverse dominating set of G and so $\gamma^{-1}(G) = 1$. The set $S_1 = \{v_2, v_3\}$ is a minimum relatively prime inverse dominating set of G and so $\gamma_{rp}^{-1}(G) = 2$. Thus $\gamma^{-1}(G) < \gamma_{rp}^{-1}(G)$ and hence the inequality in Theorem 2.11, becomes strict.*

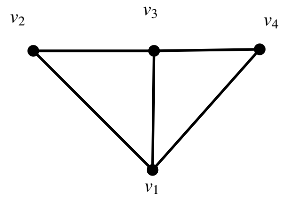


Figure 2.3

3. Conclusion

In this paper, we introduced the concept of relatively prime inverse domination of a graphs and found relatively prime inverse domination of a graph of some standed graphs like path graph, complement of path graph, complete bipatite graph, fan graph, bistar graph and barbell graph.

References

[1] G. Chartrand, Lesniak: Graphs and Digraphs, fourth ed.,CRC press, BoCa Raton, 2005.

[2] J. A. Gullian: A dynamic survey of graph labeling, *The electronics Journal of Combinatories*, 17(2014).

[3] W. Haynes, S. T. Hedetniemi, P. J. Slater: Domination in Graphs, Advanced Topices, Marcel Dekker, New York, 1998.

[4] S. T. Hedetniemi, R. Laskar (Eds.): Topics in domination in graphs, *Discrete Math.* 86(1990).

[5] C. Jayasekaran and A. Jancy vini: Results on relatively prime dominating sets in graphs, *Annals of pure and Applied Mathematics*, 14, (3), (2017), 359 – 369.

[6] V. R. Kulli and S. C. Sigarkant: Inverse domination in graphs, *Nat. Acad Sci. Letters*, 14, (1991), 473-475.

[7] Weisstein, Eric W: Barbell Graph, from Mathworld.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 – 3786
Malaya Journal of Matematik
ISSN(O):2321 – 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆