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Unlike degree Wiener index (UDWI) for graph structures

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Abstract
In this Paper we introduced a new concept Unlike degree Wiener index (UDWI) for some graph structures. We study the unlike degree Wiener Index for few graph structures such as Star graph, Path, Comb, Hurdle Graph.

Keywords
Wiener Index. **AMS Subject Classification**

05C12, 92E10.

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1. Introduction

In numerous number of topological indices the only that has been used in drug discovery Research is the Wiener Index .The Wiener Index of a graph (V,E) denoted by W(G), was introduced in 1947 by Chemist Harold Wiener .The Wiener index of a graph G is equal to the half -sum of the shortest distances between every pairs of vertices of G.
W(G)=$\frac{1}{2}\sum_{i=1}^{n} d(v_i v_j)$ where $d(v_i v_j)$ is the shortest distance between the vertices v_i and v_{ji} in Graph G.

Like wiener Index (LDWI) was introduced by J.Baskar Babujee and A.Subhashini recently in 2016 and it is defined as $W_{ud} = \frac{1}{2}\sum_k \sum_{i,j=1}^{n} d(v_i^k v_j^k)$ where $d(v_i^k, v_j^k)$ is the distance between the pair of vertices of same degree. In this paper we study Unlike Degree Wiener Index (UDWI) for few graphs structures derived from graph operators .

Definition 1.1. *Unlike Wiener index is defined as* $W_{ud} = \frac{1}{2}\sum_k \sum_{i,j=1}^{n} d(v_i^k v_j^k)$ *where* $d(v_i^k, v_j^k)$ *is the distance between the pair of vertices of unlike degree.*

2. Main Results

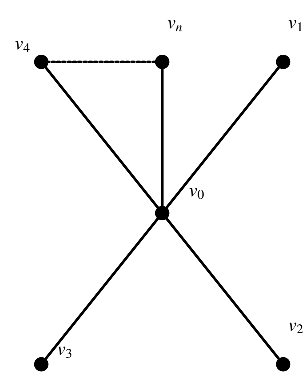
Theorem 2.1. *The Unlike degree Wiener index of star graph ,* S_n *is* $W_{ud}(S_n) = n$.

Proof. Let $S_n = (V,E)$

$V = \{v_0, v_1, v_2, ..., v_n\}$ have n+1 vertices and n edges.

$E = \{v_0 v_i; 1 \le i \le n\}$

All the vertices $v_1, v_2, ..., v_n$ has degree one except v_0



deg $(v_0) = n$ and $deg(v_i) = 1 \forall 1 \le i \le n$.

$W_{ud} = d(v_0, v_1) + dv_0, v_2) + ... + d(v_0, v_n)$

$= 1+1+1+...1(ntimes)$
$=n$

□

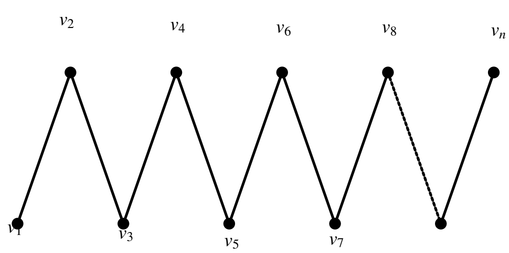
Theorem 2.2. *The unlike Degree Wiener index of Path graph P_n is $W_{ud}(P_n) = (n-1)(n-2)$.*

Proof. Let $P_n = (V,E)$ be the graph with vertex set and edge set as

$V = \{v_0, v_1, v_2, ..., v_n\}$ have n vertices and $n-1$ edges

$E = v_i v_{i+1}; 1 \leq i \leq n$

All the vertices have degree two except v_1 and v_n



$deg(v_1) = deg(v_n) = 1$
and $deg(v_i) = 2 \forall 2 \leq i \leq n-1$
$W_{ud}(P_n) = d(v_1,v_2) + d(v_1,v_3) + ... + d(v_1,v_{n-1})$
$+ d(v_2,v_n) + d(v_3,v_n) + ... + d(v_{n-1},v_n)$

$=1+2+3+...+(n-2times)+(n-2)+3+2+1$

$=2\times1+2\times2+2\times3+...+2(n-2)$

$=2[1+2+3+...+(n-2)]$

$=2\frac{(n-2)(n-2+1)}{2}$

$=(n-1)(n-2)$

□

Theorem 2.3. *The unlike degree Wiener index of comb graph, $(P_n \odot K_1) W_{ud}(P_n \odot K_1))$ = $n(2n-1)$.*

Proof. Let $(P_n \odot K_1) = (V,E)$ be the graph with vertex set and edge set as

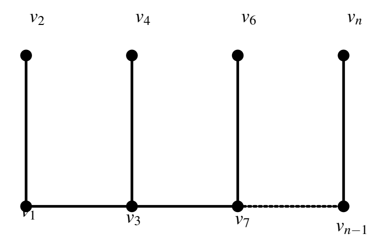
$V = \{v_1, v_2, v_3, ..., v_n\}$ have n vertices and n-1 edges

$E = \{v_i v_{i+1}, v_i v_{i+2}; 1 \leq i \leq n\}$

$deg(v_2) = deg(v_4) = ... = deg(v_n) = 1$

$deg(v_1) = deg(v_{n-1}) = 2$

$deg(v_3) = deg(v_5) = ... = deg(v_{n-3}) = 3$



$W_{ud}(P_n \odot K_1)) = d(v_1,v_2) + d(v_3,v_4) + d(v_5,v_6) + ...+$ $d(v_{n-1},v_n) + d(v_1,v_3) + d(v_1,v_5) +...+ d(v_1,v_{n-3}) +$ $d(v_{n-1},v_{n-3}) +...+ d(v_{n-1},v_5)+ d(v_{n-1},v_3) + d(v_1,v_4) + d(v_1,v_6)$ $+ ...+ d(v_1,v_n) + d(v_{n-1},v_{n-2})+ ... +d(v_{n-1},v_6) + d(v_{n-1},v_4)$ $+ d(v_{n-1},v_2)$

$= [1+1+...+1] + [1+2+3+...+(n-2)] + [2+3+...+ n] + [2+3+4+...+n]$

$=n + 2[\frac{(n-2)(n-2+1)}{2}] + [\frac{n(n+1)}{2} - 1]$

$=n + (n-2)(n-1) + (n(n+1)-2)$

$=n + (n-2)(n-1) + n^2 + n - 2$

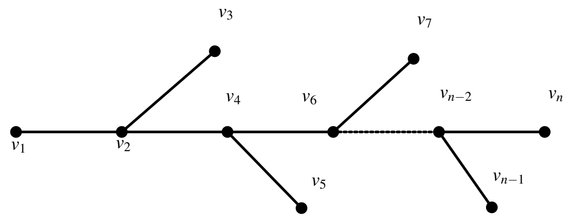
$= n + n^2 - 2n - n + 2 + n^2 + n - 2$

$=2n^2 - n = n(2n-1)$

□

Theorem 2.4. *The unlike degree Wiener index of hurdle graph Hdn is , $W_{ud}(Hdn) = n^2 - 2$*

Proof. Let $(Hdn) = (V,E)$ be the graph with vertex set and edge set as
$V = \{v_1, v_2, v_3, ..., v_n\}$ have n vertices and n-1 edges
$E = \{v_i v_{i+1}, v_i v_{i+2}; 1 \leq i \leq n\}$
$deg(v_1) = deg(v_3) = deg(v_5) = deg(v_7) = ...$
$= deg(v_n) = 1$
$deg(v_2) = deg(v_4) = ... = deg(v_{n-2}) = 3$



$W_{ud}(Hdn) = d(v_1,v_2) + d(v_1,v_4) + ... + d(v_1,v_{n-2}) + d(v_n,v_{n-2}) +...+ d(v_n,v_2) + d(v_n,v_1) + d(v_2,v_3) + d(v_4,v_5)+ d(v_{n-1},v_{n-2})$
$=[1+2+3+...+1(n-1)]+[1+2+3+...+(n-1)]+[1+1+...+n](n-2)times$

$= 2[\frac{(n-1)(n-1+1)}{2}]+(n-2)\times 1$

$=n(n-1)+(n-2)$

$= n^2-n+n-2$

$= n^2-2.$

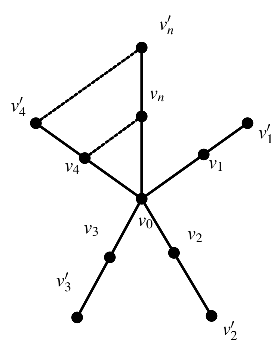
□

Theorem 2.5. *The Unlike degree Wiener Index of Double Graph $K_{1,n,n}$ is $W_{ud}(K_{1,n,n}) = 4n$*

Proof. $V = (v_0,v_1,v_2,...,v_n,v_1^{'},v_2^{'},...,v_n^{'})$

$E = \{v_0v_i, vv_i^{'}; 1 \leq i \leq n\}$

The vertices $V_1,V_2,...,V_n$ has degree two and the vertices $v_1^{'},v_2^{'},...,v_n^{'}$ has degree one except $v_0^{'}$
$degdeg(V_0) = n$
$degdeg(V_i) = 2; \forall 1 \leq i \leq n$
and $deg(v_i^{'}) = 1; \forall 1 \leq i \leq n$



$W_{ud}(K_{1,n,n}) = d(v_0,v_1)+d(v_0,v_2)+...+d(v_0,v_n)+d(v_0,v_n^{'})$
$+ d(v_1,v_1^{'})+d(v_2,v_2^{'}) + ... + d(v_n,v_n^{'})$
$= (1+1+1+...+1)(ntimes)+(2+2+2+...+2)(ntimes)+(1+1+1+...+1)(ntimes)$

$= n+2n+n$

=4n

□

3. Conclusion

We have studied Unlike degree Wiener Index for various graph structures derived using graph operators. In future a comparison study will be carried out with Wiener index and Unlike degree Wiener Index for various compounds.

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