Malaya Journal of Matematik, Vol. 8, No. 4, 2302-2305, 2020

https://doi.org/10.26637/MJM0804/0170

Relatively prime geodetic number of graphs

C. Jayasekaran [1]* and A. Sheeba[2]

Abstract

In this paper we introduce relatively prime geodetic number of a graph G. Let G be a connected graph. A set $S \subseteq V$ is said to be a relatively prime geodetic set if it is a geodetic set with at least three elements and the shortest distance between any two pairs of vertices in S is relatively prime. The relatively prime geodetic set of G is denoted by $g_{rp}(G)$-set. The cardinality of a minimum relatively prime geodetic set is the relatively prime geodetic number and it is denoted $g_{rp}(G)$.

Keywords

Geodetic set, Geodetic Number, Relatively prime, Line graph.

AMS Subject Classification

05C12.

[1,2] *Department of Mathematics, Pioneer Kumaraswamy College, Nagercoil-629003, Kanyakumari District, Tamil Nadu, India. Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012.*

***Corresponding author**: [1] jayacpkc@gmail.com; [2]jelina.j@gmail.com

Article History: Received **10** July **2020**; Accepted **22** November **2020**

Contents

1. Introduction

By a graph $G = (V,E)$ we mean a finite, connected, undirected graph with neither loops nor multiple edges. The order $|V|$ and size $|E|$ of G are denoted by p and q respectively. For graph theoretic terminology we refer to West [6]. The open neighborhood of any vertex v in G is $N(v) = \{x/xv \in E(G)\}$ and closed neighborhood of a vertex v in G is $N[v] = N(v) \bigcup v$. The degree of a vertex in the graph G is denoted by $deg(v)$ and the maximum degree(minimum degree) in the graph G is denoted by $\triangle(G)$ $(\delta(G))$. For a set $S \subseteq V(G)$, the open (closed) neighborhood $N(S)(N[S])$ in G is defined as $N(S) = \bigcup_{v\in S} N(v)(N[S] = \bigcup_{v\in S} N[v])$.

In a connected graph G, the distance between two vertices x and y is denoted by $d(x,y)$ and is defined as the length of a shortest $x-y$ path in G. The diameter of a graph G is defined by $diam(G) = max_{x,y\in V(G)} d(x,y)$. Two vertices u and v are said to be antipodal vertices if $d(u,v) = diam(G)$. If $e = \{u,v\}$ is an edge of a graph G with $deg(u) = 1$ and $deg(v)1$, then we call e a pendant edge, u a pendent vertex and v a support vertex. A set of vertices is said to be independent if no two vertices in it are adjacent. A vertex v of G is said to be an extreme vertex if the subgraph induced by its neighborhood is complete. A vertex v is said to be full vertex if v is adjacent to all other vertices in G, that is, if $deg(v)= p–1$. An acyclic connected graph is called a tree. An x–y path of length $d(x,y)$ is called geodesic. A vertex v is said to lie on a geodesic P if v is an internal vertex of P. The closed interval $I[x,y]$, consists of x,y and all vertices lying on some x–y geodesic of G and for a non empty set $S \subseteq V(G)$, $I[S] = \cup_{x,y\in S} I[x,y]$.

A set $S \subseteq V(G)$ in a connected graph is a geodetic set of G if $I[S] = V(G)$. The geodetic number of G denoted by $g(G)$, is the minimum cardinality of a geodetic set of G. The geodetic number of a disconnected graph is the sum of the geodetic number of its component. A geodetic set of cardinality $g(G)$ is called $g(G)$ – set. Various concepts inspired by geodetic set are introduced in [1,2].

2. Definitions and Known results

Definition 2.1. *[4] The line graph $L(G)$ of a graph G is the graph whose vertices are the edges of G and two vertices of $L(G)$ are adjacent if the corresponding edges of S are adjacent.*

Theorem 2.2. *[3] Every geodetic set of a graph contains its extreme vertices.*

Theorem 2.3. *[3] Every end vertices of a graph G belongs to geodetic set of G.*

Theorem 2.4. *[3] For the complete graph K_n, $g(K_n) = n$.*

Theorem 2.5. *[3] For the wheel graph $W_{1,n}$, $g(W_{1,n})$=*
$$\begin{cases} 4 \ for \ n = 3 \\ \lceil \frac{n}{2} \rceil \ for \ n \geq 4. \end{cases}$$

Observation 2.6. *[5] For the complenent of path graph $\bar{P}_n$ of n vertices has $g(\bar{P}_n) = 3$, $n \geqslant 5$.*

3. Relatively Prime Geodetic Number of graphs

Definition 3.1. *Let G be a connected graph. A set $S \subseteq V$ is said to be a relatively prime geodetic set if it is a geodetic set with atleast three elements and the shortest distance between any two pairs of vertices in S is relatively prime. The relatively prime geodetic set of G is denoted by $g_{rp}(G)$-set. The cardinality of a minimum relatively prime geodetic set is the relatively prime geodetic number and it is denoted $g_{rp}(G)$.*

Example 3.2. *Consider the graph G given in figure 3.1. Here $\{1,5\}$ is a minimum geodetic set of G and hence $g(G)$ = 2. Now $\{1,5,6\}$ is a geodetic set of G with $d(1,5)$ = 4, $d(1,6)$ = 3 and $d(5,6)$ = 1. Also $(1,3)$ = 1, $(1,4)$ = 1 and $(3,4)$ = 1. Hence $\{1,5,6\}$ is a relatively prime geodetic set of G. Moreover it has the minimum cardinality with this property and hence $g_{rp}(G)$ = 3.*

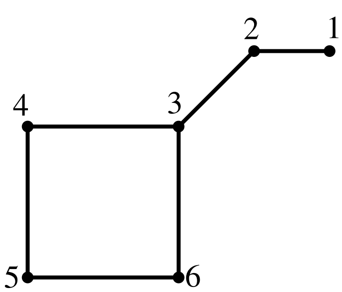

Figure 3.1

Theorem 3.3. *Let G be a connected graph of order n. Then*
(i) Each relatively prime geodetic set of G contains its extreme vertices.
(ii) Each end vertex of G belongs to relatively prime geodetic set of G.

Proof. Let G be a connected graph of order n. By definition, each relatively prime geodetic set is a geodetic set.
(i) Hence by Theorem 2.1, each relatively prime geodetic set of G contains its extreme vertices.
(ii) Further by Theorem 2.2, each end vertex of G belongs to relatively prime geodetic set of G. □

Theorem 3.4. *For the complete graph K_n $(n \geqslant 3)$, $g_{rp}(K_n) = n$.*

Proof. In a complete graph K_n every vertex is an extreme vertex. Therefore, the vertex set $V(K_n)$ is the unique relatively prime geodetic set of K_n and hence $g_{rp}(K_n) = n$. □

Theorem 3.5. *Let G be connected graph of order n. If $g_{rp}(G)$ exists, then $g(G) \leq g_{rp}(G) \leq n$.*

Proof. Let G be a connected graph of order n such that $g_{rp}(G)$ exists. Since every relatively prime geodetic set is a geodetic set, it follows that $g(G) \leq g_{rp}(G)$. Also any relatively prime geodetic set can have atmost n vertices and hence $g_{rp}(G) \leq n$. Thus, $g(G) \leq g_{rp}(G) \leq n$. □

Remark 3.6. For the complete graph K_n $(n \geq 3)$, $g(K_n) = g_{rp}(K_n) = n$. Hence all the inequalities in Theorem 3.5 become sharp. Now consider the graph G given in Figure 3.1. Here $S = \{1,5\}$ is a minimum geodetic set of G and so $g(G) = 2$. The set $S_1 = \{1,5,6\}$ is a minimum relatively prime geodetic set of G and so $g_{rp}(G) = 3$. Thus $g(G) < g_{rp}(G) < n$, and hence all the inequalities in Theorem 3.5 become strict.

Theorem 3.7. *For the path P_n of order n, $g_{rp}(P_n) = 3, n \geqslant 3$.*

Proof. Let $v_1v_2...v_n$ be the path P_n. By Theorem 3.3, the end vertices v_1 and v_n must be in any relatively prime geodetic set of P_n. By definition, any relatively prime geodetic set of P_n, contains at least three vertices. Consider $S = \{v_1, v_2, v_n\}$. Clearly S is a geodetic set of P_n. Now $d(v_1, v_n) = n-1$, $d(v_1, v_2) = 1$ and $d(v_2, v_n) = n-2$. Also, $(1, n-1) = (1, n-2) = (n-2, n-1) = 1$. Hence S is a minimum relatively prime geodetic set of P_n. Similarly, $S^* = \{v_1, v_{n-1}, v_n\}$ is a minimum relatively prime geodetic set of P_n. Hence $g_{rp}(P_n) = |S| = |S^*| = 3$. □

Theorem 3.8. *If $G = \bar{P}_n$, then $g_{rp}(\bar{P}_n)$ = $\begin{cases} 3 \ if \ n = 4 \\ 0 \ otherwise \end{cases}$.*

Proof. Let $v_1v_2...v_n$ be the path P_n. In $\bar{P}_n$, v_1 is adjacent to $v_3, v_4, ..., v_n$; v_n is adjacent to $v_1, v_2, ..., v_{n-2}$; v_2 is adjacent to $v_4, v_5, ..., v_n$; v_{n-1} is adjacent to $v_1, v_2, ..., v_{n-3}$ and v_i is adjacent to $v_1, v_2, ..., v_{i-2}, v_{i+2}, ..., v_n$, $3 \leq i \leq n-2$. By definition of $\bar{P}_n$, $|V(\bar{P}_n)| = n$ and $|E(\bar{P}_n)| = \binom{n}{2} - (n-1) = \dfrac{n(n-1)}{2} - (n-1) = \dfrac{(n-1)(n-2)}{2}$. We consider the following three cases.
Case 1. $n = 3$
Clearly $\bar{P}_3$ $=K_2 \bigcup K_1$ which contains an isolated vertex. Therefore, there doesnot exist geodetic set. Hence $g_{rp}(\bar{P}_3) = 0$.
Case 2. $n = 4$
Clearly $\bar{P}_4$=P_4. By Theorem 3.7, $g_{rp}(\bar{P}_4) = 3$.
Case 3. $n \geqslant 5$
By Observation 2.5,$g_{rp}(\bar{P}_n) = 3$. Then a minimum geodetic set for $\bar{P}_n$ is $S = \{v_i, v_j, v_k\}$ where $1 \leq i \neq j \neq k \leq n$. We consider the following three subcases.

Subcase 3.1. No two of v_i, v_j and v_k are consecutive

Then $I[v_i,v_j] = \{v_i,v_j\}, I[v_j,v_k] = \{v_j,v_k\}, I[v_i,v_k] = \{v_i,v_k\}$ in $\bar{P}_n$. Clearly $I[S] = \{v_i,v_j,v_k\} \neq V(\bar{P}_n)$. Hence, S is not a minimum geodetic set.

Subcase 3.2. Any two of v_i, v_j and v_k are consecutive

Then $S = \{v_i, v_{i+1}, v_k\}$ for $1 \leq i \leq n-1$.

If $i = 1$, then $S = \{v_1,v_2,v_k\}$, where $I[v_1,v_2] = V(\bar{P}_n)$ - $\{v_3\}$, $I[v_1,v_k] = \{v_1,v_k\}$, $I[v_2,v_k]$ =$\{v_2,v_k\}$ in $\bar{P}_n$. Clearly, $I[S] \neq V(\bar{P}_n)$.

If $i = n-1$, then $S = \{v_{n-1},v_n,v_k\}$, where $I[v_{n-1},v_n] = V(\bar{P}_n) - \{v_{n-2}\}, I[v_{n-1},v_k] = \{v_{n-1},v_k\}, I[v_n,v_k] = \{v_n,v_k\}$ in $\bar{P}_n$. Clearly, $I[S] \neq V(\bar{P}_n)$.

If $2 \leq i \leq n-2$, then $S = \{v_i,v_{i+1},v_k\}$, where $I[v_i,v_{i+1}] = V(\bar{P}_n)$ - $\{v_{i-1},v_{i+2}\}, I[v_i,v_k] = \{v_i,v_k\}, I[v_{i+1},v_k] = \{v_{i+1},v_k\}$ in $\bar{P}_n$. Clearly, $I[S] \neq V(\bar{P}_n)$. Clearly $I[S] = \{v_i,v_{i+1},v_k\}$ $\neq V(\bar{P}_n)$. Hence, S is not a minimum geodetic set.

Subcase 3.3. v_i, v_j and v_k are consecutive

Then $S = \{v_i,v_{i+1},v_{i+2}\}$ for $1 \leq i \leq n-2$.

If $i = 1$, then $S = \{v_1,v_2,v_3\}$, where $I[v_1,v_2] = V(\bar{P}_n)$ - $\{v_3\}, I[v_1,v_3]$ =$\{v_1,v_3\}, I[v_2,v_3] = V(\bar{P}_n)$ -$\{v_1,v_4\}$ in $\bar{P}_n$. Clearly, $I[S] = V(\bar{P}_n)$. Hence, S is a minimum geodetic set. Here, $d(v_1,v_2)$ = 2, $d(v_1,v_3)$ = 1, $d(v_2,v_3) = 2$. Also, $(d(v_1,v_2)$, $d(v_2,v_3)) = (2,2) = 2 \neq 1$. Hence, S cannot be a relatively prime geodetic set of $\bar{P}_n$.

If $i = n-2$, then $S = \{v_{n-2},v_{n-1},v_n\}$, where $I[v_{n-2},v_{n-1}]$ = $V(\bar{P}_n)$ - $\{v_{n-3},v_n\}$, $I[v_{n-2},v_n] = \{v_{n-2},v_n\}$, $I[v_{n-1},v_n]$ = $V(\bar{P}_n)$ - $\{v_{n-2}\}$ in $\bar{P}_n$. Clearly, $I[S] = V(\bar{P}_n)$. Hence, S is a minimum geodetic set. Here, $d(v_{n-2},v_{n-1}) = 2$, $d(v_{n-2},v_n)$ = 1, $d(v_{n-1},v_n)$ = 2. Also, $(d(v_{n-2},v_{n-1}),d(v_{n-1},v_n)) = (2,2)$ = $2 \neq 1$. Hence, S cannot be a relatively prime geodetic set of $\bar{P}_n$.

If $2 \leq i \leq n-3$, then $S = \{v_i,v_{i+1},v_{i+2}\}$, where $I[v_i,v_{i+1}]$ = $V(\bar{P}_n) - \{v_{i-1},v_{i+2}\}$, $I[v_i,v_{i+2}] = \{v_i,v_{i+2}\}$, $I[v_{i+1},v_{i+2}] = V(\bar{P}_n)$ -$\{v_i,v_{i+3}\}$ in $\bar{P}_n$. Clearly, $I[S] = V(\bar{P}_n)$. Hence, S is a minimum geodetic set. Here $d(v_i,v_{i+1}) = 2$, $d(v_i,v_{i+2}) =$ 1, and $d(v_{i+1},v_{i+2}) = 2$. Also, $(d(v_i,v_{i+1})$, $d(v_{i+1},v_{i+2}) =$ $(2,2) = 2 \neq 1$. Hence, S cannot be a relatively prime geodetic set of $\bar{P}_n$.

Thus, $g_{rp}(\bar{P}_n) = 0$ for $n \geq 5$. Hence the theorem. □

Theorem 3.9. *For cycle C_n of even order n, $g_{rp}(C_n) = 3$.*

Proof. Let $v_1v_2...v_nv_1$ be the cycle C_n of order n. Clearly $S = \{v_i, v_{i+\frac{n}{2}}\}$ where the suffices modulo n, is a minimum geodetic set of C_n and hence $g(C_n) = 2$. By the definition of relatively prime geodetic set of C_n, any relatively prime geodetic of C_n must contain at least 3 vertices of C_n. Let $S^{'} = \{v_i,v_{i+1},v_{i+\frac{n}{2}}\}$ where the suffices modulo n. Then $S^{'}$ is a geodetic set. Now $d(v_i,v_{i+1}) = 1$, $d(v_i,v_{i+\frac{n}{2}}) = \frac{n}{2}$ and $d(v_{i+1},v_{i+\frac{n}{2}}) = \frac{n}{2}-1$. Clearly $(1,\frac{n}{2}) = (1,\frac{n}{2}-1) = (\frac{n}{2},\frac{n}{2}-1) = 1$. Therefore $S^{'}$ is a minimum relatively prime geodetic set of C_n and hence, $g_{rp}(C_n) = 3$. □

Theorem 3.10. *For a star graph $K_{1,n}$,*

$$g_{rp}(K_{1,n}) = \begin{cases} 3 \; for \; n = 2 \\ 0 \; for \; n \geqslant 3 \end{cases}.$$

Proof. Let $v, v_1, v_2, ..., v_n$ be the vertices of a star graph $K_{1,n}$ in which v is the central vertex.

Case 1. $n = 2$

Clearly S= $\{v,v_1,v_2\}$ is the only relatively prime geodetic set, since $d(v,v_1)$ = 1, $d(v,v_2) = 1$, $d(v_1,v_2) = 2$ and $(1,1) =$ $(1,2) = 1$. Hence $g_{rp}(K_{1,2}) = 3$.

Case 2. $n \geqslant 3$

Suppose there exists a relatively prime geodetic set S of $K_{1,n}$. By Theorem 3.3(ii), the end vertices $v_1,v_2,...,v_n \in$ S. Since $d(v_1,v_2) = 2$ and $d(v_1,v_3) = 2$ we have $(d(v_1,v_2)$, $d(v_1,v_3)) = 2$. This implies that S cannot be a relatively prime geodetic set of $K_{1,n}$. Thus $g_{rp}(K_{1,n}) = 0$ for $n \geq 3$. Hence the theorem.

□

Theorem 3.11. *For a wheel $W_{1,n} = K_1 + C_{n-1} (n \geqslant 3)$, $g_{rp}(W_{1,n})$*

$$= \begin{cases} 4 \; if \; n = 3 \\ 0 \; otherwise \end{cases}.$$

Proof. Let $v_1v_2...v_{n-1}v_1$ be the outer cycle C_{n-1} and v be the central vertex of $W_{1,n}$. Then $d(v,v_i) = 1$ and $d(v_i,v_j) = 2$ for $i, j \in \{1,2,...,n-1\}$ and $i \neq j$.

Case 1. $n = 3$

Clearly, $W_{1,3} = K_4$. By Theorem 3.4, $g_{rp}(W_{1,3}) = 4$.

Case 2. $n = 4$

Clearly $S = \{v_1,v_3\}$ is a minimum geodetic set. By definition any relatively prime geodetic set must contains at least three vertices. Let $S^{'} = \{v_1,v_3,v\}$ is a geodetic set and $d(v_1,v) = 1, d(v_1,v_3) = 2, d(v,v_3) = 1$ and $(2,1) = (1,1) = 1$, hence $S^{'}$ is a minimum relatively prime geodetic set. Then $g_{rp}(W_{1,n}) = 3$.

Case 3. $n \geqslant 5$

In $W_{1,n}$, a minimum geodetic set is $S = \{v_i, v_{i+2}, v_{i+4}, ...,$ $v_{i+(\lceil\frac{n}{2}\rceil-1)2}\}$. Let $S^{'} = \{v_i,v_{i+2},v_{i+4},...,v_{i+(\lceil\frac{n}{2}\rceil-1)2},v\}$ is a geodetic set. Since $d(v_i,v) = 1$, $d(v_j,v_k) = 2$, where $v_j,v_k \in S$. Therefore the shortest distance between any two vertices in S is 2. Clearly any two of these shortest distances are not relatively prime. Hence it follows that $g_{rp}(W_{1,n}) = 0$ for $n \geqslant 5$. □

Theorem 3.12. *For a bistar graph $B_{m,n}$, $g_{rp}(B_{m,n})$ =*

$$\begin{cases} 3 \; if \; m = n = 1 \\ 0 \; otherwise \end{cases}.$$

Proof. Let u_0 and v_0 be the vertices of P_2. Let $u_1,u_2,...,u_m$ be the vertices attached with u_0 and let $v_1,v_2,...,v_n$ be the vertices attached with v_0. The resultant graph is a bistar graph $B_{m,n}$ with $V(B_{m,n}) = \{u_0,v_0,u_i,v_i, 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(B_{m,n})$ = $\{u_0v_0,u_0u_i,v_0v_j, 1 \leq i \leq m, 1 \leq j \leq n\}$. We consider the following three cases.

Case 1. $m = n = 1$

Then $B_{m,n} = B_{1,1} = P_4$. By Theorem 3.7, $g_{rp}(B_{m,n}) = 3$.

Case 2. $m = 1, n \geqslant 2$ or $n = 1, m \geqslant 2$

When $m = 1, n \geqslant 2$, let S be a relatively prime geodetic set of $B_{m,n}$. Then By Theorem 3.3(ii), $u_1,v_1,v_2,...,v_n \in S$. Since

$d(u_1,v_j)=3$, for $1\leq i\neq j\leq n$, we have $(d(u_1,v_1),d(u_2,v_2)) = (3,3)=3\neq 1$. This implies that S cannot be a relatively prime geodetic set of $B_{m,n}$. Similarly we can prove that there is no relatively prime geodetic set when $n=1, m\geqslant 2$. Hence $g_{rp}(B_{m,n})=0$.

Case 3. $m,n\geqslant 2$

Suppose there exists a relatively prime geodetic set S. Then By Theorem 3.3(ii), $u_1,u_2,...,u_m,v_1,v_2,...,v_n\in S$. Since $d(u_i,v_j)=3$, for $1\leq i\leq m, 1\leq j\leq n$, we have $(d(u_1,v_1), d(u_2,v_2)) = (3,3)=3\neq 1$. This implies that S cannot be a relatively prime geodetic set of $B_{m,n}$. Hence $g_{rp}(B_{m,n}) = 0$. □

Theorem 3.13. *Let $L(P_n)$ be the line graph of P_n. Then $g_{rp}(L(P_n))=3$ for $n\geq 4$.*

Proof. Let the path P_n have vertex set $\{v_i/1\leq i\leq n\}$ and the edge set $\{v_iv_{i+1}/1\leq i\leq n-1\}$. By the definition of line graph, the edges $\{v_iv_{i+1}/1\leq i\leq n-1\}$ in P_n are considered as the vertices $\{u_i/1\leq i\leq n-1\}$ in $L(P_n)$ and two vertices of $L(G)$ are joined by an edge if and only if the corresponding edges of G are adjacent in G. Hence $L(P_n)$ is a path with $n-1$ vertices. By Theorem 3.7, $g_{rp}(L(P_n)) = g_{rp}(P_{n-1}) = 3$ for $n-1\geqslant 3$ and hence $n\geqslant 4$. □

Theorem 3.14. *Let $L(K_{1,n})$ be the line graph of $K_{1,n}$. Then $g_{rp}(L(K_{1,n}))=n$.*

Proof. Let $v,v_1,...,v_n$, be the vertices of $K_{1,n}$ with v as the central vertex. By definition of line graph, clearly $L(K_{1,n})) = K_n$. By Theorem 3.4, $g_{rp}(L(K_{1,n})) = g_{rp}(K_n) = n$ for $n\geqslant 3$. □

Theorem 3.15. *Let $L(C_n)$ be the line graph of C_n of even order n, $g_{rp}(L(C_n))$ = 3 for $n\geqslant 4$.*

Proof. Let the cycle C_n have a vertex set $\{v_i/1\leq i\leq n\}$ and the edge set $\{v_iv_{i+1}\leq i\leq n-1\}\bigcup\{v_1v_n\}$. By the definition of line graph the edges $\{v_iv_{i+1}\leq i\leq n-1\}\bigcup\{v_1v_n\}$ in C_n are considered as the vertices $\{u_i/1\leq i\leq n\}$ in $L(C_n)$ and two vertices of $L(G)$ are joined by an edge if and only if the corresponding edges of G are adjacent in G. Hence $L(C_n)$ is an even cycle with n vertices and n edges. By Theorem 3.9, $g_{rp}(L(C_n)) = g_{rp}(C_n) = 3$ for $n\geqslant 4$. □

4. Conclusion

In this paper, we have found the relatively prime geodetic number of some standard graphs like cycle graph, path graph, wheel graph, bistar fish graph, star graph and complete graph.

References

[1] F. Buckley and F. Harary: Distance in Graphs, Addition-Wesley, Redwood City, 1990.

[2] G. Chartrand, F. Harary and P. Zhang: On The Geodetic Number of a Graph, *Networks* 39, (2002), 1-6.

[3] G. Chartrand, E.M. Palmer and P. Zhang: The Geodetic Number of a Graph, *A Survey, Congr. Number,* 156,(2002), 37 - 58.

[4] F. Harary, C. J. A. Nash-Williams: On Eulerian and Hamiltonian Graphs and Line Graphs, *Canadian Mathematical Bulletin*, 8, (1965), 701-709.

[5] S. Robinson Chellathurai and S. Padma Vijaya, : The Geodetic Domination number for the product of Graphs, *Transaction on Combinatorics* 3(4), (2014), 19-30.

[6] D.B. West: Introduction to Graph Theory, Second Ed., Prentice-Hall. Upper Saddle River, NJ, 2001.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆