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Some results on strong 2 - vertex duplication self switching of some connected graphs

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Abstract

A vertex $v \in V(G)$ is said to be a self vertex switching of G if G is isomorphic to G^v, where G^v is the graph obtained from G by deleting all edges of G incident to v in G and adding all edges incident to v which are not in G. A vertex v' is the duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in $D(vG)$, which is the duplication graph of G. Duplication self vertex switching of various graphs are given in the literature. In this paper we discuss about the 2-vertex duplication self switching graphs.

Keywords

Switching, self vertex switching, duplication, 2- vertex self switching, $dSS_2(G), dss_2(G)$.

AMS Subject Classification

05C02.

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Contents

1. Introduction

Switching has been defined by Seidel [7] it is referred to as Seidel switching. For a finite undirected simple graph $G(V,E)$ with $|V(G)| = p$ and a set $\sigma \subseteq V$,the switching of G by σ is defined as the graph $G^{\sigma}(V,E)$, which is obtained from G by removing all edges between σ and its complementary $V-\sigma$ and adding all non-edges as edges between σ and $V-\sigma$. Switching is an equivalence relation. When $\sigma = v \in V$, the corresponding switching G^v is called a vertex switching and is denoted by G^v. Switching is an equivalence relation and the associated equivalence classes are called switching classes. A subset σ of $V(G)$ is said to be a self switching of G if $G \cong G^{\sigma}$.The set of all self switchings of G with cardinality k is denoted by $SS_k(G)$ and its cardinality by $ss_k(G)$. If $k = 1$, then we call the corresponding self switching as self vertex switching. We also call it as $|\sigma|$- vertex self switching[1]. When $|\sigma| = 2$, we call it as 2-vertex self switching.

C. Jayasekaran and G. Sumathy[1] has done a survey on self-vertex switching of graphs. The existence of self vertex switching like trees, path, complete graph unicycle, two cyclic, bicyclic but not a two cyclic graph with given number of vertices are analyzed.

The concept of duplication self vertex switching was introduced by C. Jayasekaran and V. Prabavathy [2,3]. The set of all duplication self vertex switching is denoted by $dSS_1(G)$. The number of duplication self vertex switching is denoted by $dss_1(G)$.

2. 2-vertex duplication self switching graphs:

Definition 2.1. *2-vertex duplication of a graph G is the duplication of any two vertices $u, v \in V(G)$ is u', v' such that x', y' are adjacent to all the vertices that are adjacent to u&v. It is denoted as $D((u,v)G)$.*

Example 2.2. *2-vertex duplication of the graph $G = P_6$ is given in the fig.1 to fig.4*

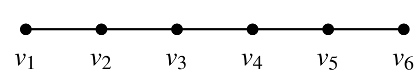

Figure 1. $G = P_6$

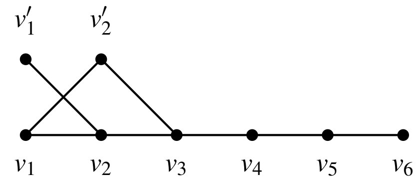

Figure 2. $D((v_1,v_2)G)$

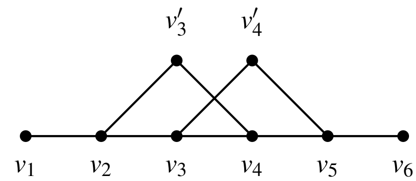

Figure 3. $D((v_3,v_4)G)$

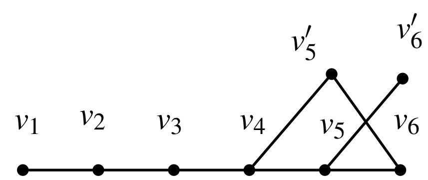

Figure. 4 $D((v_5,v_6)G)$

Definition 2.3. *The 2-vertex duplication switching of G by $\sigma = \{x,y\}$ is the graph obtained by duplicating any two vertices u,v then by removing all existing edges between and its complement $V-\sigma$ in $D((u,v)G)$ and also by adding edges between σ and $V-\sigma$ which are not in G ,without affecting the adjacency and non-adjacency of vertices in σ. It is denoted by $D((u,v)G)^{\sigma}$ or $D((u,v)G)^{\{x,y\}}$.*

Example 2.4. *2-vertex duplication switching of a graph G is given in the figures fig.5 to fig.6*

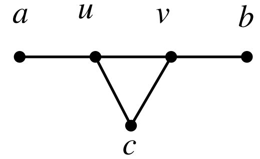

Figure. 5 G

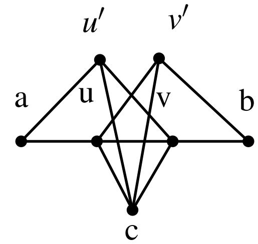

Figure.6 $D((u,v)G)$

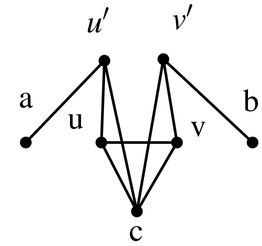

Figure.7 $D((u,v)G)^{\{u,v\}}$

Definition 2.5. *Let $\sigma = \{x,y\} \in V(G)$ is called a 2-vertex duplication self switching of a graph G if $D((u,v)G) \cong D((u,v)G)^{\sigma}$. If $\sigma = \{u,v\}$, then σ is called the strong 2- vertex duplication self switching of G.*

Example 2.6. *The graph $G = C_4$ has strong 2-vertex duplication self switching are given in the figures 8 to 9.*

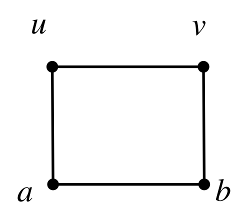

Figure. 8 G

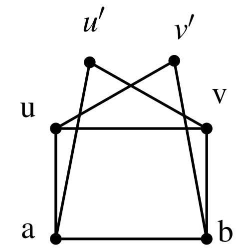

Figure.9 $D((u,v)G))$

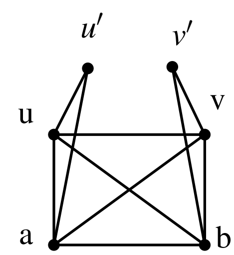

Figure.10 $D((u,v)G)^{\{u,v\}}$

Result 2.7. *Let G be a (p,q) graph. Then $D((u,v)G)$ is a $(p+2, q+d_G(u)+d_G(v))$ graph.*

Theorem 2.8. *If $\sigma = \{u,v\} \subseteq V$ is a strong 2-vertex duplication self switching of a graph G,then $d_G(u)+d_G(v) = p$ if $uv \in E(G)$.*

Proof. Let $\sigma = \{u,v\} \subseteq V$ be a strong 2-vertex duplication self switching of a graph G. By the definition, $D((u,v)G) =$

$D((u,v)G)^{\sigma}$ and hence $|E(D(u,v)G)| = |E(D(u,v)G)^{\sigma}|$.
That implies $q + d_G(u) + d_G(v) = q + d_G(u) + d_G(v) + [p + 2 - 1 - d_{D((u,v)G)(u)}] - d_{D((u,v)G)}(u) + [p + 2 - 1 - d_{D((u,v)G)(v)}] - d_{D((u,v)G)}(v) + 2$
$0 = p + 1 - [d_G(u) + 1] - (d_G(u) + 1) + p + 1 - [d_G(v) + 1] - (d_G(v) + 1) + 2$
$0 = 2p + 2 - 2d_G(u) - 2 - 2d_G(v) - 2 + 2$
$0 = 2p - 2d_G(u) - 2d_G(v)$
$d_G(u) + d_G(v) = p$

□

Note 2.9. *The converse of the above theorem need not be true. For example, let us consider the graph G with 5 vertices given in fig.12. In this graph the vertices a and b are adjacent with $d_G(a) + d_G(b) = 5 = p$. Therefore the graph $D((a,b)G)$ and $D((a,b)G)^{\{a,,b\}}$ is given in the fig. 12 and 13. Thus $D((a,b)G) \ncong D(a,b)G)^{\{a,b\}}$.*

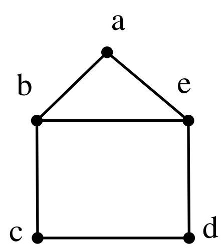

Figure. 11 G

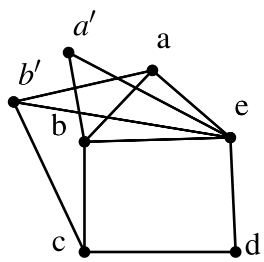

Figure.12 $D((a,b)G)$

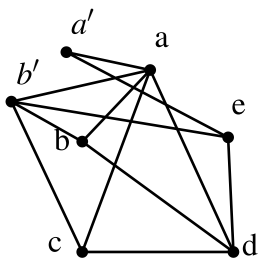

Figure.13 $D((a,b)G)^{\{a,b\}}$

Theorem 2.10. *For $p \geq 4, dss_2(W_p) = 0$*

Proof. Let K_1 be the central vertex w and let $v_1v_2 \dots v_{(p-1)}v_1$ be the cycle of $C_{(p-1)}$.
W_p is the join of K_1 and C_{p-1}. Then $V(W_p) = w, v_1, v_2, v_3, \dots v_{p-1}$ and $E(W_p) = \{v_iv_{i+1}, v_1v_{p-1}, wv_i, wv_{p-1}/1 \leq i \leq p-2\}$. Let $\sigma = \{u,v\}$ be the subset of $V(W_p)$.
Let us consider $uv \in E(W_p)$.
Then either $\sigma = \{w, v_i\}$ for some $i, 1 \leq i \leq p-1$. In this graph $d_G(u) + d_G(v) \neq p$. Hence by the theorem 2.8 σ is not a 2-vertex duplication self switching of G.
If $\sigma \subseteq V(C_{p-1})$, then $d_G(u) + d_G(v) \neq p$ then by the theorem 2.8 σ is not a 2-vertex duplication self switching of G.
Thus $dss_2(W_p) = 0$. □

3. Conclusion

In this paper we have proved some results of 2-vertex duplication self switching graphs and also we discussed about the 2-vertex duplication of connected graphs.

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