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Inverse isolate domination number on a vertex switching of cycle related graphs

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Abstract

Let G be non-trivial graph. A subset $S \subset V(G)$ is called a isolate dominating set of G if is a dominating set and $\delta(< S >) = 0$. The set $S' \subset V(G) - S$ such that S' is a dominating set of G and $\delta(< S' >) = 0$, then S' is called an inverse isolate dominating set with respect to S. The minimum cardinality of an inverse isolate dominating set is called an inverse isolate dominating number and is denoted by $\gamma_0^{-1}(G)$. In this paper we find inverse isolate dominating number on vertex switching of some cycle related graphs.

Keywords

Domination, Isolate domination, Inverse domination, Switching.

AMS Subject Classification

05C15, 05C69.

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Contents

1. Introduction

We begin with finite connected and undirected graph $G = (V,E)$ without loops and multiple edges. A subset S of V is said to be a dominating set of G if every vertex in $V - S$ is adjacent to at least one vertex in S. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set in G[4,5].

A dominating set D is said to be a isolate dominating set if $\delta(\langle V - D\rangle) = 0$. The minimum cardinality of a isolate dominating set of a graph G is called the domination number of G and is denoted by $\gamma_0(G)$. Kulli V. R. et al introduced the concept of inverse domination in graphs [7]. Let D be a minimum dominating set of G. If $V - D$ contains a dominating set S, then S is called the inverse dominating set of G with respect to D. The inverse dominating number $\gamma^{-1}(G)$ is the minimum cardinality taken over all the minimal inverse dominating set of G. In this paper we discuss about inverse isolate domination number on vertex switching of cycle related graphs.

2. Definitions and Known results

Definition 2.1. *[6] Let S be a minimum isolate dominating set of a graph G. If $V - S$ contains a dominating set S' such that $\delta(< S' >) = 0$, then S' is called an inverse isolate dominating set with respect to S. If $\delta(< S' >) > 0$, then we call S' as weak inverse isolate a dominating set of G. The minimum cardinality of an inverse isolate dominating set is called an inverse isolate domination number and is denoted by $\gamma_0^{-1}(G)$ and the minimum cardinality of a weak inverse isolate dominating set is called a weak inverse isolate domination number and is denoted by $\gamma_{w0}^{-1}(G)$*

Definition 2.2. *[8] For a finite undirected graph $G(V,E)$ and a subset $\sigma \subseteq V$, the* **switching** *of G by σ is defined as the graph $G^\sigma(V,E')$ which is obtained from G by removing all edges between σ and its complement $V - \sigma$ and adding as edges all non-edges between σ and $V - \sigma$. For $\sigma = \{v\}$, we write G^v instead of $G^{\{v\}}$ and the corresponding switching is called as* **vertex switching**.

Theorem 2.3. *[6] Let G be any graph and S be an isolate dominating set of G. If G has an inverse isolate dominating* *set S' with respect to S, then the following conditions hold:*

1. *G contains no isolate vertices*
2. $\left|S'\right| \leq |V - S|$.

Theorem 2.4. *[5] For cycle C_n, $\gamma_0^{-1}(C_n) = \lceil \frac{n}{3} \rceil$.*

Definition 2.5. *[9] The Sunlet graph is the graph on 2n vertices obtained by attaching pendent edges to each vertex of cycle C_n.*

Definition 2.6. *[9] The Windmill graph $Wd(k,n)$ is an undirected graph constructed for $k \geq 2$ and $n \geq 2$ by joining n copies of the complete graph K_k at a shared universal vertex.*

Definition 2.7. *[2] Book graph B_n is a Cartesian product of S_n and P_2, where S_n is a star on n vertices and P_2 is the path graph on 2 vertices.*

Definition 2.8. *[1] The Helm graph H_n is the graph obtained from a wheel graph by adding a pendent edge at each node of the cycle.*

Definition 2.9. *The Jelly fish graph J(m,n) is obtained from a 4-cycle $v_1v_2v_3\ v_4v_1$ by joining v_1 and v_3 with an edge and appending m pendent edges to v_2 and n pendent edges to v_4.*

Definition 2.10. *[3] The Barbell graph $BB_{n,n}$ is the simple graph obtained by connecting two copies of a complete graph K_n by a bridge.*

3. Inverse isolate domination number on a vertex switching of graphs

Theorem 3.1. *Let G be the sunlet S_n graph and v be any vertex of G. Then* $\gamma_0^{-1}(G^v) = \begin{cases} n-1 \text{ if } d_G(v) = 1 \\ 0 \text{ if } d_G(v) = 3 \end{cases}$.

Proof. Let G be the sunlet graph S_n obtained from a cycle C_n by attaching a pendent edge at each vertex of the cycle. Clearly $V(G) = \{u_i, v_i / 1 \leq i \leq n\}$ and $E(G) = \{u_iv_i, u_iu_{i\pm1} / 1 \leq i \leq n\}$, where $d_G(u_i) = 3$ and $d_G(v_i) = 1$. Let G^v be the vertex switching of G with respect to v. By the definition of vertex switching, v is non adjacent to vertices in $N_G(v)$ and adjacent to all the vertices of $\overline{N_G(v)}$. Let S be a minimum isolate dominating set of G^v and S' be a corresponding minimum inverse dominating set. Now we consider the following two cases.

Case 1. $d_G(v) = 1$

Here $v = v_i$, $1 \leq i \leq n$. In G, v_i is adjacent to u_i only, which implies that v_i is adjacent to all vertices other than u_i in G^v and hence a minimum isolate dominating set S contains only the two vertices v_i and u_i. Then the corresponding minimum inverse dominating set S' is either $\{u_{i+1}, v_{i+2}, u_{i+3}, v_{i+4}, ..., u_{i+(n-1)}\}$ or $\{v_{i+1}, u_{i+2}, v_{i+3}, u_{i+4}, ..., v_{i+(n-1)}\}$ where the suffixes modulo n. Clearly $\delta(\langle S' \rangle) = 0$. This implies that S' is a minimum inverse isolate dominating set of G^v. Hence $\gamma_0^{-1}(G^v) = n-1$.

Case 2. $d_G(v) = 3$

Here $v = u_i$, $1 \leq i \leq n$. Clearly, G^v is disconnected graph with an isolate vertex v_i. By Theorem 2. 3, there does not exist any inverse dominating set. Hence $\gamma_0^{-1}(G^v) = 0$.

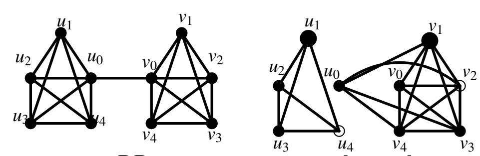
Thus the theorem following from cases 1 and 2. □

Theorem 3.2. *Let G be the barbell graph $BB_{n,n}$ and v be any vertex of G. Then $\gamma_0^{-1}(G^v) = 2$.*

Proof. Consider two copies of K_n with vertex set $\{u_0, u_1, ..., u_{n-1}\}$ and $\{v_0, v_1, ..., v_{n-1}\}$. Join u_0 and v_0, we get the barbell graph $G = BB_{n,n}$ with vertex set $V(G) = \{u_i, v_i / 0 \leq i \leq n-1\}$ and edge set $E(G) = \{u_0v_0, u_iu_j, v_iv_j\ /1 \leq i < j \leq n-1\}$. Also $d_G(u_0) = d_G(v_0) = n$, $d_G(u_i) = d_G(v_i) = n-1$. Let G^v be the vertex switching graph of G with respect to v. Let S be a minimum isolate dominating set of G^v and S' be a corresponding minimum inverse dominating set. We now consider the following two cases.

Case 1. $v = u_0$ or v_0

Any minimum isolate dominating set S is $\{u_i, v_j\}$ where $1 \leq i, j \leq n-1$ and a corresponding minimum inverse dominating set S' is $\{u_l, v_m\}$ where $1 \leq l, m \leq n-1$ and $i \neq l$, $j \neq m$. Clearly $\delta(\langle S' \rangle) = 0$. This implies that S' is a minimum inverse isolate dominating set of G^v. Hence $\gamma_0^{-1}(G^v) = |S'| = 2$.



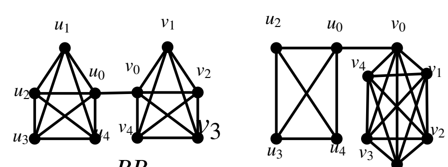
$BB_{5,5}$ $[BB_{5,5}]^{u_0}$

*Fig.*3.1

Case 2. $v = u_i$ or v_i, $1 \leq i \leq n-1$

Without loss of generality, let $v = u_i$. Any minimum isolate dominating set S is $\{v, u_j\}$ where $1 \leq j \leq n-1$ and $v \neq u_j$ and a corresponding minimum inverse dominating set S' is $\{u_l, v_m\}$ where $l \neq j$, $v \neq u_l$, $1 \leq l, m \leq n-1$. Clearly $\delta(\langle S' \rangle) = 0$. This implies that S' is a minimum inverse isolate dominating set of G^v. Therefore, $\gamma_0^{-1}(G^v) = 2$.

Thus the theorem follows from cases 1 and 2. □



$BB_{5,5}$ $BB_{5,5}^{u_1}$

*Fig.*3.2

Theorem 3.3. *Let G be the helm graph and v be any vertex of G. Then* $\gamma_0^{-1}(G^v) = \begin{cases} 0 \text{ if } d_G(v) = 3 \\ n \text{ otherwise} \end{cases}$.

Proof. Let $v_1v_2...v_nv_1$ be the cycle C_n. Add a vertex u which is adjacent to v_i, $1 \leq i \leq n$. The resultant graph is the wheel W_n. For $1 \leq i \leq n$, add u_i which is adjacent to v_i. The resultant graph G is the helm graph H_n. Clearly $V(G) = \{u, u_i, v_i \;/ 1 \leq i \leq n\}$ and $E(G) = \{uv_i,\, v_iv_{i+1},\, u_iv_i, uu_n, u_nv_n,\, v_1v_n / 1 \leq i \leq n-1\}$, $d_G(u_i) = 1$, $d_G(v_i) = 3$ and $d_G(u) = n$. Let G^v be a vertex switching of G with respect to v. Let S be a minimum isolate dominating set of G^v and $S^{'}$ be a corresponding minimum inverse dominating set. We now consider two following cases.

Case 1. $d_G(v) = 1$

In this case $v = u_i$, $1 \leq i \leq n$. Clearly a minimum isolate dominating set S of G^v is $\{u_i, v_i\}$, $1 \leq i \leq n$ and a corresponding minimum inverse dominating set $S^{'}$ is $\{u, u_j / 1 \leq j \leq n$ and $i \neq j\}$. Clearly $\delta(\langle S^{'} \rangle) = 0$. This implies that $S^{'}$ is a minimum inverse isolate dominating set of G^v. Hence $\gamma_0^{-1}(G^v) = n$.

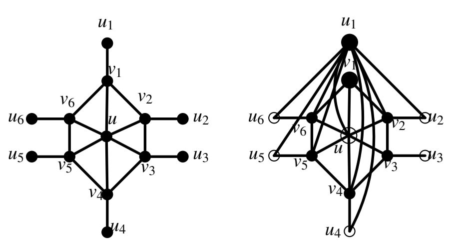
Case 2. $d_G(v) = n$

In this case $v = u$. Clearly a minimum isolate dominating set S of G^v is either $\{u, v_i, v_{i+3}, v_{i+6}, ..., v_{i+(n-3)}\}$ for $n \equiv 0(mod 3)$ or $\{u, v_i, v_{i+3}, v_{i+6},\, ..., v_{i+(n-2)}\}$ for $n \not\equiv 0(mod 3)$, where the suffixes modulo n and $1 \leq i \leq n$. The corresponding minimum inverse dominating set $S^{'}$ is $\{u_i / 1 \leq i \leq n\}$. Clearly $\delta(\langle S^{'} \rangle) = 0$. This implies that $S^{'}$ is a minimum inverse isolate dominating set of G^v. Hence $\gamma_0^{-1}(G^v) = n$.

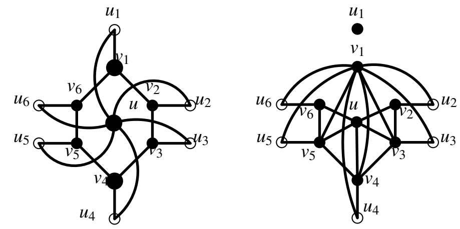
Case 3. $d_G(v) = 3$

Here $v = v_i$, $1 \leq i \leq n$. In this case G^v is disconnected graph with an isolate vertex u_i. By Theorem 2.3, there does not exist any inverse dominating set. Hence $\gamma_0^{-1}(G^v) = 0$.
The theorem follows from cases 1, 2 and 3. □

Example 3.4. Consider the graph $G = H_6$ and G^v be the vertex switching of G with respect to the vertex v. If $v = u_1$, then $S = \{u_1, v_1\}$ is the minimum isolate dominating set and $S^{'} = \{u, u_2, u_3, u_4, u_5, u_6\}$ is the minimum inverse dominating set with respect to S. If $v = u$, then $S = \{u, v_1, v_4\}$ is the minimum isolate dominating set and $S^{'} = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is the minimum inverse dominating set with respect to S. If $v = v_1$, then G^{v_1} has an isolate vertex u_1. Hence $\gamma_0^{-1}(G^{u_1}) = \gamma_0^{-1}(G^u) = 6$, $\gamma_0^{-1}(G^{v_1}) = 0$.



H_6 $\quad$ $H_6^{u_1}$



H_6^u $\quad$ *Fig.*3.3 $\quad$ $H_6^{v_1}$

Theorem 3.5. *Let G be a jelly fish graph $J(m,n)$ and v be any vertex of G. Then*

1. *for m = n,*

$$\gamma_0^{-1}(G^v) = \begin{cases} n \; for \; v = v_{2i} \; or \; v_{4i}, 1 \leq i \leq m \\ 2 \; for \; v = v_1 \; or \; v_3 \; and \; m = n = 1 \\ 3 \; for \; v = v_1 \; or \; v_3 \; and \; m = n > 1 \\ 0 \; for \; v = v_2 \; or \; v_4 \end{cases}$$

2. *for $m \neq n$,*

$$\gamma_0^{-1}(G^v) = \begin{cases} n \; for \; v = v_{4j}, 1 \leq j \leq n \\ m \; for \; v = v_{2i}, 1 \leq i \leq m \\ 2 \; for \; v = v_1 \; or \; v_3 \; and \; m \; or \; n = 1 \\ 3 \; for \; v = v_1 \; or \; v_3 \; and \; m \; or \; n > 1 \\ 0 \; for \; v = v_2 \; or v_4 \end{cases}$$

Proof. Consider the 4-cycle $v_1v_2v_3v_4v_1$ and join v_1 and v_3 with an edge. Join m end vertices to v_2, denote v_{2i}, $1 \leq i \leq m$ and n end vertices to v_4, denote v_{4j}, $1 \leq j \leq n$, then the graph G is the jelly fish graph $J(m,n)$ with vertex set $V(G) = \{v_{2i}, v_{4j}, v_k / 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq 4\}$ and edge set $E(G) = \{v_2v_{2i}, v_4v_{4j}, v_kv_{k+1}, v_1v_4,\, v_1v_3 \;/ 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq 4\}$. Let G^v is a vertex switching of G with respect to the vertex v. By the definition of vertex switching v is adjacent to the vertices in $\overline{N_G(v)}$ in G^v. Let S be a minimum isolate dominating set of G^v and $S^{'}$ be a corresponding minimum inverse dominating set with respect to S. We now consider the following seven cases.

Case 1. $m = n$ and $v = v_{2i}$ or v_{4j}

Clearly $G^{v_{2i}} \cong G^{v_{4i}}$, $1 \leq i \leq m$. With out loss of generality we may take $v = v_{2i}, 1 \leq i \leq m$. If $m = n = 1$, then a minimum isolate dominating set $S = \{v, v_2\}$ and a minimum inverse dominating set $S^{'}$ with respect to S is $\{v_1, v_{41}\}$. If $m = n \geq 2$, then a minimum isolate dominating set S of $G^{v_{2i}}$ is $\{v, v_2\}$ and a corresponding minimum inverse dominating set $S^{'}$ is $\{v_4, v_{2i} / 1 \leq i \leq m$ and $v_{2i} \neq v\}$. In all possible cases $\delta(\langle S^{'} \rangle) = 0$. This implies that $S^{'}$ is a minimum inverse isolate dominating set of $G^{v_{2i}}$. Hence $\gamma_0^{-1}(G^{v_{2i}}) = n$.

Case 2. $m = n = 1$ and $v = v_1$ or v_3

In this case $G^{v_1} \cong G^{v_3}$. With out loss of generality we may take $v = v_1$ and $m = n = 1$, then $G^{v_1} \cong C_6$ and hence by Theorem 2.4, $\gamma_0^{-1}(G^{v_1}) = 2$.

Case 3. $m = n > 1$ and $v = v_1$ or v_3

In this case $G^{v_1} \cong G^{v_3}$. With out loss of generality we may take $v = v_1$ and $m = n \geq 2$, then a minimum isolate dominating set S of G^{v_1} is $\{v, v_3\}$ and a corresponding minimum inverse dominating set $S^{'}$ is either $\{v_2, v_4, v_{2i}\}$ or $\{v_2, v_4, v_{4i}\}$, $1 \leq i \leq m$. Also $\delta(\langle S^{'} \rangle) = 0$. This implies that $S^{'}$ is a minimum inverse isolate dominating set of G^{v_1}. Hence $\gamma_0^{-1}(G^{v_1}) = 3$.

Case 4. $m = n$ and $v = v_2$ or v_4

Here $G^{v_2} \cong G^{v_4}$. Let $v = v_2$. Then the graph $G^v \cong G^{v_2}$ contains isolate vertices $v_{2i}, 1 \leq i \leq m$. By Theorem 2.3, there does not exist any inverse isolate dominating set. Hence $\gamma_0^{-1}(G^v) = 0$.

Case 5. $m \neq n$ and $v = v_{4j}$

Here a minimum isolate dominating set S of $G^{v_{4j}}$ is $\{v, v_4\}$ and a corresponding minimum inverse dominating set $S^{'}$ is $\{v_2, v_{4j}/1 \leq j \leq n\}, v_{4j} \neq v$. Clearly $\delta(\langle S^{'} \rangle) = 0$. This implies that $S^{'}$ is a minimum inverse isolate dominating set of $G^{v_{4j}}$. Hence $\gamma_0^{-1}(G^{v_{4j}}) = n$.

Case 6. $m \neq n$ and $v = v_{2i}$

Here a minimum isolate dominating set S of $G^{v_{2i}}$ is $\{v, v_2\}$ and a corresponding minimum inverse dominating set $S^{'}$ is $\{v_4, v_{2i}/1 \leq i \leq m\}, v_{2i} \neq v$. Clearly $\delta(\langle S^{'} \rangle) = 0$. This implies that $S^{'}$ is a minimum inverse isolate dominating set of $G^{v_{2i}}$. Hence $\gamma_0^{-1}(G^{v_{4j}}) = m$.

Case 7. $m \neq n$, $v = v_1$ or v_3 and m or $n = 1$

In this case $G^{v_1} \cong G^{v_3}$. With out loss of generality we may take $v = v_1$ and $m = 1$. Then the minimum isolate dominating set S of G^{v_1} is $\{v_1, v_3\}$ and the corresponding minimum inverse dominating set $S^{'} = \{v_2, v_4\}$. Clearly $\delta(\langle S^{'} \rangle) = 0$. This implies that $S^{'}$ is a minimum inverse isolate dominating set of G^{v_1}. Hence $\gamma_0^{-1}(G^{v_1}) = 2$.

Case 8. $m \neq n$ and $v = v_1$ or v_3, $m, n > 1$

In this case $G^{v_1} \cong G^{v_3}$. With out loss of generality we may take $v = v_1$. Then a minimum isolate dominating set S of G^{v_1} is $\{v, v_3\}$ and a corresponding minimum inverse dominating set $S^{'}$ is either $\{v_2, v_4, v_{2i}\}$ or $\{v_2, v_4, v_{4i}\}$, $1 \leq i \leq m$. Also $\delta(\langle S^{'} \rangle) = 0$. This implies that $S^{'}$ is a minimum inverse isolate dominating set of G^{v_1}. Hence $\gamma_0^{-1}(G^{v_1}) = 3$.

Case 9. $m \neq n$ and $v = v_2$ or v_4

In this case, $G^{v_2} \cong G^{v_4}$. Let $v = v_2$. Then the graph contains isolate vertices $v_{2i}, 1 \leq i \leq m$. By Theorem 2.3, there does not exist any inverse isolate dominating set. Hence $\gamma_0^{-1}(G^v) = 0$.

Thus the theorem follows from the above nine cases. □

Theorem 3.6. *Let G be the book graph B_n and G^v is a vertex switching of G with respect to v, then*

$$\gamma_0^{-1}(G^v) = \begin{cases} 3 \; if \; n = 2 \; and \; d_G(v) = 2 \\ n \; otherwise \end{cases}.$$

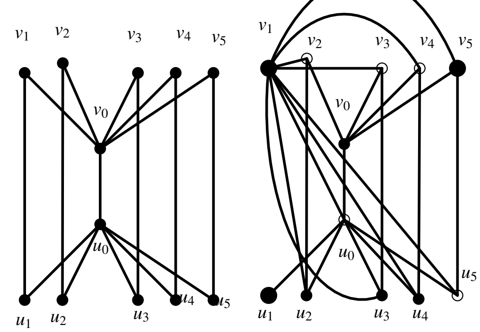
Proof. Let $v_0, v_1, ..., v_n$ and $u_0, u_1, ..., u_n$ be the two copies of star $K_{1,n}$ with central vertices v_0 and u_0 respectively. Join u_i with v_i for $i, 0 \leq i \leq n$. The resultant graph G is the book graph B_n with vertex set $V(G) = \{u_0, v_0, u_i, v_i/1 \leq i \leq n\}$ and edge set $E(G) = \{u_0v_0, u_iv_i, u_0u_i, v_0v_i/1 \leq i \leq n\}$. Let G^v is a vertex switching of G with respect to v. By the definition of vertex switching v is adjacent to vertices in $\overline{N_G(v)}$ in G^v. Let S be a minimum isolate dominating set of G^v and $S^{'}$ be a corresponding minimum inverse dominating set with respect to S. We now consider the following three cases.

Case 1. $n = 2$ and $d_G(v) = 2$

In this case v is either v_i or u_i, $i = 1, 2$. With out loss of generality we may take $v = v_1$. The unique minimum isolate dominating set S of G^{v_1} is $\{u_0, v_2\}$ and a corresponding minimum inverse dominating set $S^{'}$ is $\{v_0, u_1, u_2\}$. Clearly $\delta(\langle S^{'} \rangle) = 0$. This implies that $S^{'}$ is a minimum inverse isolate dominating set of G^{v_1}. Hence $\gamma_0^{-1}(G^{v_1}) = 3$.

Case 2. $n \geq 3$ and $d_G(v) = 2$

Here v is either u_i or $v_i, 1 \leq i \leq n$. Let $v = v_1$. Then the unique minimum isolate dominating set S of G^{v_1} is $\{v_1, u_1, v_n\}$ and a corresponding minimum inverse dominating set $S^{'}$ is $\{u_0, u_n, v_i/2 \leq i \leq n-1\}$. Also $\delta(\langle S^{'} \rangle) = 0$. Therefore, $S^{'}$ is a minimum inverse isolate dominating set of G^{v_1}. Hence $\gamma_0^{-1}(G^{v_1}) = n$.

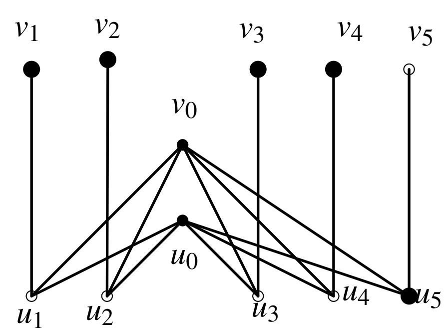


B_5 $[B_5]^{v_1}$

$Fig.3.4$

Case 3. $n \geq 3$ and $d_G(v) = n$

In this case v is either u_0 or v_0. Let $v = v_0$. Now a minimum isolate dominating set S of G^{v_0} is $\{v_i, u_n/1 \leq i \leq n-1\}$ and a corresponding minimum inverse dominating set $S^{'}$ is $\{u_i, v_n/1 \leq i \leq n-1\}$. Also $\delta(\langle S^{'} \rangle) = 0$. This implies that $S^{'}$ is a minimum inverse isolate dominating set of G^{v_0}. Hence $\gamma_0^{-1}(G^{v_0}) = n$.



$Fig.3.5$:$[B_5]^{v_0}$

Thus the theorem follows from cases 1, 2 and 3. □

Theorem 3.7. *If G is a windmill graph Wd(k,n) with shared vertex v_0 and G^v is a vertex switching of G with respect to v. Then*

1. *for* $v = v_0$, $\gamma_0^{-1}(G^v) = 0$.
2. *for* $v \neq v_0$, $\gamma_0^{-1}(G^v) = \begin{cases} n-1 \; for \; k=2 \\ n \; for \; k \geq 3 \end{cases}$

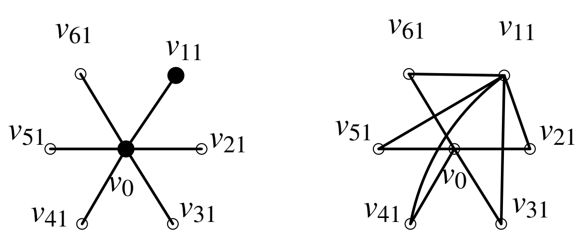
Proof. Consider the n-copies of K_k and joining them at a shared universal vertex v_0. The resultant graph G is the windmill graph $Wd(k,n)$ of order $(k-1)n+1$ with vertex set $V(G) = \{v_0, v_{ij}/1 \leq i \leq n, 1 \leq j \leq k-1\}$ and edge set $E(G) = \{v_0v_{ij}, v_{ij}v_{lm}/1 \leq i \leq n, 1 \leq j \leq k-1\}$, $i \neq l$ and $j \neq m$. Let G^v is a vertex switching of G with respect to v. By the definition of vertex switching v is adjacent to vertices $\overline{N_G(v)}$ in G^v. Let S be a minimum isolate dominating set of G^v and S' be a corresponding minimum inverse dominating set with respect to S. Clearly $v \neq v_0$ and $w \neq v_0$, $G^u \cong G^w$. We now consider the following three cases.

Case 1. $v = v_0$

In this case the resulting graph G^{v_0} contains an isolate vertex v_0. By Theorem 2.3, there does not exist any inverse isolate dominating set. Hence $\gamma_0^{-1}(G^{v_0}) = 0$.

Case 2. $v \neq v_0$ and $k = 2$

In this case $v = v_{i1}, 1 \leq i \leq n$. Let $v = v_{11}$. Then the unique minimum isolate dominating set S of G^v_{11} is $\{v_0, v_{11}\}$ and a corresponding minimum inverse dominating set S' obtained by taking all the vertices not in S. Clearly $\delta(\langle S' \rangle) = 0$. Therefore, S' is a minimum inverse isolate dominating set of $G^{v_{11}}$. Hence $\gamma_0^{-1}(G^{v_{11}}) = n-1$.



$Wd(2,6)$ $[Wd(2,6)]^{v_{11}}$

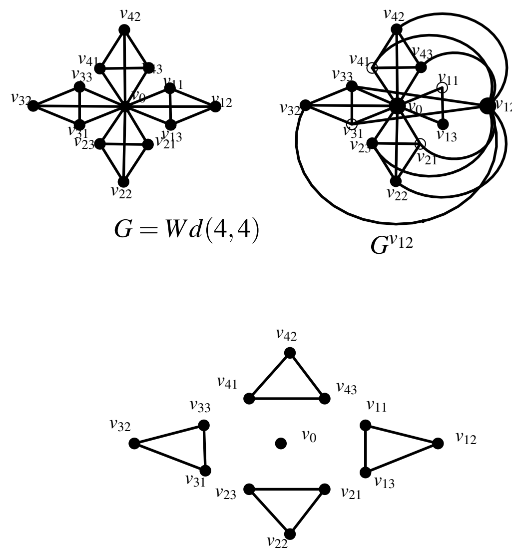
Fig.3.6

Case 3. $v \neq v_0$ and $k \geq 3$

In this case the unique minimum isolate dominating set S of G^v is $\{v_0, v\}$ and a corresponding minimum inverse dominating set S' is obtained by taking a vertex from each copy of K_k distinct from v and v_0. Clearly $\delta(\langle S' \rangle) = 0$. This implies that S' is a minimum inverse isolate dominating set of G^v. Hence $\gamma_0^{-1}(G^v) = n$.

Thus the theorem follows from cases 1, 2 and 3. □

Example 3.8. Consider the graph $G = Wd(4,4)$ and $G^{v_{12}}$ be the vertex switching of G with respect to the vertex v_{12}, G^{v_0} be the vertex switching of G with respect to the vertex v_0 given in Fig.2.18. Clearly $S = \{v_0, v_{12}\}$ is the minimum isolate dominating set and $S' = \{v_{ij}/1 \leq i \leq 4\}$, $1 \leq j \leq 3$ is a minimum inverse dominating set with respect to S. Since $\delta(\langle S' \rangle) = 0$. Hence $\gamma_0^{-1}(G^{v_{12}}) = 4$ and G^{v_0} has an isolate vertex u_0 such that $\gamma_0^{-1}(G^{v_0}) = 0$.



$G = Wd(4,4)$ $G^{v_{12}}$

G^{v_0}

Fig.3.7

4. Conclusion

In this paper, we have found the inverse isolate domination number on vertex switching of some standard graphs like Sunlet graph, barbell graph, Helm graph, Jelly fish graph, Book graph and Windmill graph.

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