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Some application of higher order sine function

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Abstract
In this paper, develop certain properties of sine function. We find the higher order difference operator value of sine function. Suitable examples are inserted to illustrate the main results.

Keywords
Difference operator, factorial polynomial, sine function, inverse difference operator, extorial function.

AMS Subject Classification
39A13, 33E12, 35K05.

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Contents

1. Introduction

The properties of difference equations is depends on the operator Δ defined as $\Delta u(k) = u(k+1) - u(k), k \in N(0)$. The difference equations in a different direction. The difference operator Δ denoted by Δ_l and its inverse operator Δ_l^{-1}. We obtain higher order difference operator and inverse operator values.

2. Higher Order Sine function

Definition 2.1. *The l- sine function denoted as*

$$\sin(k_l^{(m)}) = k_l^{(m)} - \frac{k_l^{(3m)}}{3!} + \frac{k_l^{(5m)}}{5!} - \frac{k_l^{(7m)}}{7!} + \cdots + \infty, \tag{2.1}$$

where $|l| \leq 1$ *and* $n, k \in \mathbb{R}$.

Lemma 2.2. *If* $|l| \leq 1$ *and k real variable, then the following holds.*
(i) $\sin(k_0^{(1)}) = \sin k$ *(ii)* $\sin((-k)_l^{(1)}) = -sin(k_{-l}^{(1)})$
(iii) $\sin((-k)_l^{(1)}) = -sin(k_l^{(1)})$ *(iv)* $\sin(k_2^{(m)}) = \sum_{t=0}^{\infty}(-1)^t \frac{k^{m(2t+1)}}{(2t+1)!}$

Proof. (i) We have $\sin(k_l^{(m)}) = k_l^{(m)} - \frac{k_l^{(3m)}}{3!} + \frac{k_l^{(5m)}}{5!} - \frac{k_l^{(7m)}}{7!} + \cdots + \infty \ldots$
$\sin(k_0^{(1)}) = k_0^{(1)} - \frac{k_0^{(3)}}{3!} + \frac{k_0^{(5)}}{5!} - \frac{k_0^{(7)}}{7!} + \cdots + \infty$
$\sin(k_0^{(1)}) = \sin k$

(ii) $\sin((-k)_l^{(1)})$
$\sin((-k)_l^{(1)}) = (-k)_l^{(1)} - \frac{(-k)_l^{(3)}}{3!} + \frac{(-k)_l^{(5)}}{5!} - \frac{(-k)_l^{(7)}}{7!} + \cdots + \infty$
$= -k_{-l}^{(1)} + \frac{k_{-l}^{(3)}}{3!} - \frac{k_{-l}^{(5)}}{5!} + \frac{k_{-l}^{(7)}}{7!} - \cdots + \infty$
$\sin((-k)_l^{(1)}) = -[k_{-l}^{(1)} + \frac{-k_{-l}^{(3)}}{3!} - \frac{-k_{-l}^{(5)}}{5!} + \frac{-k_{-l}^{(7)}}{7!} - \cdots + \infty]$
$\sin((-k)_l^{(1)}) = -sin(k_{-l}^{(1)})$

(iii) $\sin((-k)_l^{(1)}) = -sin(-k_l^{(1)})$
$\sin((-k)_l^{(1)}) = -k_{-l}^{(1)} + \frac{-k_{-l}^{(3)}}{3!} - \frac{-k_{-l}^{(5)}}{5!} + \frac{-k_{-l}^{(7)}}{7!} + \cdots + \infty$
$= -k_l^{(1)} + \frac{k_l^{(3)}}{3!} - \frac{k_l^{(5)}}{5!} + \frac{k_l^{(7)}}{7!} + \cdots + \infty$
$= -[k_l^{(1)} + \frac{-k_l^{(3)}}{3!} - \frac{-k_l^{(5)}}{5!} + \frac{-k_l^{(7)}}{7!} + \cdots - \infty]$
$\sin((-k)_l^{(1)}) = -sin(k_l^{(1)})$

(iv) $\sin(k_0^{(2)}) = k_0^{(2)} - \frac{k_0^{(6)}}{3!} + \frac{k_0^{(10)}}{5!} - \frac{k_0^{(14)}}{7!} + \cdots + \infty \ldots$
$= k^2 - \frac{k^{(6)}}{3!} + \frac{k^{(10)}}{5!} - \frac{k^{(14)}}{7!} + \cdots + \infty \ldots$
$= \sum_{t=0}^{\infty}(-1)^t \frac{k^{2(2t+1)}}{(2t+1)!}$

$\sin(k_0^{(3)}) = k_0^{(2)} - \frac{k_0^{(9)}}{3!} + \frac{k_0^{(15)}}{5!} - \frac{k_0^{(21)}}{7!} + \cdots + \infty \ldots$
$= k^3 - \frac{k^{(9)}}{3!} + \frac{k^{(15)}}{5!} - \frac{k^{(21)}}{7!} + \cdots + \infty \ldots$
$= \sum_{t=0}^{\infty}(-1)^t \frac{k^{3(2t+1)}}{(2t+1)!}$
Similarly,
$\sin(k_0^{(4)}) = \sum_{t=0}^{\infty}(-1)^t \frac{k^{4(2t+1)}}{(2t+1)!}$

In general,
$\sin(k_0^{(m)}) = \sum_{t=0}^{\infty}(-1)^t \frac{k^{t(2t+1)}}{(2t+1)!}$

□

Lemma 2.3. *If* $|l| \le 1, m \ge 1$ *and k real variable, the* $\sin(k_l^{(m)}) = \sum_{t=0}^{\infty}(-1)^t \frac{k_1^{m(2t+1)}}{m(2t+1)!}$

Proof. We have $\sin(k_l^{(m)}) = k_l^{(m)} - \frac{k_l^{(3m)}}{3!} + \frac{k_l^{(5m)}}{5!} - \frac{k_l^{(7m)}}{7!} + \cdots + \infty$

$\sin(k_1^{(1)}) = k_1^{(1)} - \frac{k_1^{(3)}}{3!} + \frac{k_1^{(5)}}{5!} - \frac{k_1^{(7)}}{7!} + \cdots + \infty$
$= \sum_{t=0}^{\infty}(-1)^t \frac{k_1^{(2t+1)}}{(2t+1)!}$
$\sin(k_2^{(2)}) = k_2^{(2)} - \frac{k_2^{(6)}}{3!} + \frac{k_2^{(10)}}{5!} - \frac{k_2^{(14)}}{7!} + \cdots + \infty$
$= \sum_{t=0}^{\infty}(-1)^t \frac{k_1^{2(2t+1)}}{2(2t+1)!}$
$\sin(k_3^{(2)}) = k_3^{(3)} - \frac{k_3^{(9)}}{3!} + \frac{k_3^{(15)}}{5!} - \frac{k_3^{(21)}}{7!} + \cdots + \infty$
$= \sum_{t=0}^{\infty}(-1)^t \frac{k_1^{3(2t+1)}}{3(2t+1)!}$
In general,
$\sin(k_l^{(m)}) = \sum_{t=0}^{\infty}(-1)^t \frac{k_1^{m(2t+1)}}{m(2t+1)!}$

□

Lemma 2.4. *If* $|l| \le 1$ *and* $k > 0$*, we have* $\Delta_l(\sin(k_l^{(m)})) = mlk_l^{(m-1)}[1 - \frac{(k-(m-1)l)_l^{(2m)}}{2!} + \frac{(k-(m-1)l)_l^{(4m)}}{4!} - \frac{(k-(m-1)l)_l^{(6m)}}{6!} + \cdots + \infty]$

Proof. From the definition (2.1) we have
$\sin(k_l^{(m)}) = k_l^{(m)} - \frac{k_l^{(3m)}}{3!} + \frac{k_l^{(5m)}}{5!} - \frac{k_l^{(7m)}}{7!} + \cdots + \infty$
Taking Δ_l on both sides, we get
$\Delta_l(\sin(k_l^{(m)})) = \Delta_l[k_l^{(m)} - \frac{k_l^{(3m)}}{3!} + \frac{k_l^{(5m)}}{5!} - \frac{k_l^{(7m)}}{7!} + \cdots + \infty]$
$\Delta_l(\sin(k_l^{(m)})) = ml[k_l^{(m)} - \frac{k_l^{(3m)}}{3!} + \frac{k_l^{(5m)}}{5!} - \frac{k_l^{(7m)}}{7!} + \cdots + \infty]$
(2.2) We have, $k_l^{(m-1)} = k(k-l)(k-2l)\ldots(k-(m-2)l)$
$k_l^{(3m-1)} = k_l^{(m-1)}(k-(m-1)l)\ldots(k-(3m-2)l)$
$k_l^{(3m-1)} = k_l^{(m-1)}(k-(m-1)l)_l^{(2m)}$
Similarly, $k_l^{(5m-1)} = k_l^{(m-1)}(k-(m-1)l)_l^{(4m)}$
$k_l^{(7m-1)} = k_l^{(m-1)}(k-(m-1)l)_l^{(6m)}$
Substituting these values in equation (2.2), we get,
$\Delta_l(\sin(k_l^{(m)})) = mlk_l^{(m-1)}[1 - \frac{(k-(m-1)l)_l^{(2m)}}{2!} + \frac{(k-(m-1)l)_l^{(4m)}}{4!} - \frac{(k-(m-1)l)_l^{(6m)}}{6!} + \cdots + \infty]$
(2.3)

□

Theorem 2.5. *If* $|l| \le 1, k > 0$ *and* $n \ne m$*, we have* $\Delta_l^n(\sin(k_l^{(m)})) = ml^n k_l^{(m-n)} \sum_{t=0}^{\infty}((2t+1)m-1)((2t+1)m-2)\ldots((2t+1)m-(n-1))\frac{(k-(m-n)l)_l^{2tm}}{(2t)!}$

Proof. By lemma (2.3)
$\Delta_l(\sin(k_l^{(m)})) = ml[k_l^{(m-1)} - 3\frac{(k_l^{(3m-1)})}{3!} + 5\frac{(k_l^{(5m-1)})}{5!} - 7\frac{(k_l^{(7m-1)})}{7!} + \cdots + \infty]$
Taking Δ_l on both sides, we get
$\Delta_l^2(\sin(k_l^{(m)})) = \Delta_l[ml(k_l^{(m-1)} - 3\frac{(k_l^{(3m-1)})}{3!} + 5\frac{(k_l^{(5m-1)})}{5!} - 7\frac{(k_l^{(7m-1)})}{7!} + \cdots + \infty)]$
$\Delta_l^2(\sin(k_l^{(m)})) = ml^2[(m-1)(k_l^{(m-2)} - (3m-1)\frac{(k_l^{(3m-2)})}{2!} + (5m-1)\frac{(k_l^{(5m-2)})}{4!} - (7m-1)\frac{(k_l^{(7m-2)})}{6!} + \cdots + \infty)]$
(2.4) Then,
$\Delta_l^2(\sin(k_l^{(n)})) = nl^2 k_l^{(n-2)}[(n-1) - ((3n-1)\frac{(3n-1)(k-(n-2)l)_l^2}{2!} + (5n-1)\frac{(5n-1)(k-(n-2)l)_l^{(4n)}}{4!} - (7n-1)\frac{(7n-1)(k-(n-2)l)_l^{(6n)}}{6!} + \cdots + \infty)]$
Again taking Δ_l on both sides in (2.4) we get, $\Delta_l^3(\sin(k_l^{(m)})) = \Delta_l[ml_2[(m-1)(k_l^{(m-2)} - (3m-1)\frac{(k_l^{(3m-2)})}{2!} + (5m-1)\frac{(k_l^{(5m-2)})}{4!} - (7m-1)\frac{(k_l^{(7m-2)})}{6!} + \cdots + \infty)]]$
$\Delta_l^2(\sin(k_l^{(m)})) = ml^3[(m-1)(m-2)lk_l(m-3) - (3m-1)(3m-2)l\frac{(k_l^{(3m-3)})}{2!} + (5m-1)(5m-2)l\frac{(k_l^{(5m-3)})}{4!} + (7m-1)(7m-2)l\frac{(k_l^{(7m-3)})}{6!} + \cdots + \infty)]$
(2.5)
We have $k_l^{(3m-3)} = k(k-l)\ldots(k-(m-4)l)(k-(m-3)l)\ldots(k-(3m-4)l)$
$k_l^{(3m-3)} = k_l^{(m-3)}(k-(m-3)l)_l^{(2m)}$
Similarly, $k_l^{(5m-3)} = k_l^{(m-3)}(k-(m-3)l)_l^{(4m)}$
$k_l^{(7m-3)} = k_l^{(m-3)}(k-(m-3)l)_l^{(6m)}$
Substituting these values in (2.5) we get,
$\Delta_l^3(\sin(k_l^{(m)})) = ml^3[(m-1)(m-2)lk_l^{(m-3)} - (3m-1)(3m-2)l\frac{(k_l^{(m-3)})(k-(m-3)l)_l^{(2m)}}{2!} + (5m-1)(5m-2)\frac{(k_l^{(m-3)})(k-(m-3)l)_l^{(4m)}}{4!} - (7m-1)(7m-2)\frac{(k_l^{(m-3)})(k-(m-3)l)_l^{(6m)}}{6!} + \cdots + \infty]$
$\Delta_l^3(\sin(k_l^{(m)})) = ml^3 k_l^{(m-3)} \sum_{t=0}^{\infty}(-1)^t((2t+1)m-1)((2t+1)m-2)\frac{(k-(m-3)l)_l^{2tm}}{(2t)!}$
Similarly,
$\Delta_l^4(\sin(k_l^{(m)})) = ml^4 k_l^{(m-4)} \sum_{t=0}^{\infty}(-1)^t((2t+1)m-1)((2t+1)m-2)\frac{(k-(m-4)l)_l^{2tm}}{(2t)!}$
In general,
$\Delta_l^m(\sin(k_l^{(m)})) = ml^n k_l^{(m-n)}[(m-1)(m-2)\ldots(m-(n-1)) - (3m-1)(3m-2)\ldots(3m-(n-1))\frac{(k-(m-n)l)_l^{(2m)}}{2!} + (5m-1)(5m-2)\ldots(5m-(n-1))\frac{(k-(m-n)l)_l^{(4m)}}{4!} - (7m-1)(7m-2)\ldots(7m-(n-1))\frac{(k-(m-n)l)_l^{(6m)}}{6!} + \cdots + \infty]$
ie) $\Delta_l^n(\sin(k_l^{(m)})) = ml^n k_l^{(m-n)} \sum_{t=0}^{\infty}((2t+1)m-1)((2t+1)m-2)\ldots((2t+1)m-(n-1))\frac{(k-(m-n)l)_l^{2tm}}{(2t)!}$

□

3. Conclusion

We have derived solutions of higher order difference equations by introducing trigonometric function.Several properties can be arriving apply these functions.

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