Malaya Journal of Matematik, Vol. 8, No. 4, 2323-2328, 2020

https://doi.org/10.26637/MJM0804/0175

Radio even mean graceful labeling on some special graphs

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Abstract

Radio Even Mean Graceful Labeling of a connected graph G is a bijection ϕ from the vertex set V(G) to $\{2,4,6,...2|V|\}$ satisfying the condition $d(s,t)+\left\lceil \frac{\phi(s)+\phi(t)}{2} \right\rceil \geq 1+diam(G)$ for every s, t $\in$ V(G). A graph which admits radio even mean graceful labeling is called radio even mean graceful graph. In this paper we investigate the radio even mean graceful labeling on degree splitting of some special graphs.

Keywords

Radio mean graceful, degree splitting graphs, radio even mean graceful, labeling of graphs.

AMS Subject Classification

05C78.

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Article History: Received **11** October **2020**; Accepted **14** December **2020**

Contents

1. Introduction

graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Chartrand et al developed the concept of radio labeling in [1]. Somasundaram S and Ponraj introduce the notion of mean labeling of graphs in [13]. Radio mean labeling was introduced by Ponraj et al in [10]. Sampathkumar E and Walikar H B introduced the notion of the splitting graph of a graph in [12]. Ponraj R and S Somasundaram developed the concept of degree splitting of graphs in [9]. C David Raj, K Sunitha and A Subramanian found the radio odd mean and even mean labeling of some graphs in [5]. Y. Lavanya et al introduced the new concept of Radio mean graceful graphs in [8]. C David Raj, M. Deva Saroja and Brindha Mary V T investigated radio mean labeling on degree splitting of graphs in [3]. C. David Raj and Brindha Mary V T investigated the radio mean graceful labeling on degree splitting of cycle related graphs in [4]. Brindha Mary V T, C David Raj and C Jayasekaran investigated the Radio mean graceful labeling on splitting of wheel related graphs in [2]. we consider simple, finite, connected and undirected graphs throughout this paper. For any real x, $\lceil x \rceil$ is the smallest integer greater than or equal to x. For theoretic terminology, we refer to Harary [7] and for a detailed survey of graph labeling we refer to Gallian[6]. We denote the vertex set of G by V(G), distance between the vertices s and t by d(s, t), diameter of G by diam(G), degree splitting of graph G by DS(G) and order of a graph G by $|V|$.

2. Main Results

Definition 2.1. $[11]$. *The triangular book with n pages B(3, n) is a graph obtained from n – copies of cycle C_3 sharing a common edge. The common edge is called the spine or base of the book.*

Definition 2.2. *The quadrilateral book with n pages B(4, n) is a graph obtained from n copies of quadrilaterals sharing* *a common edge. The common edge is called the spine of the book. That is the quadrilateral book is the Cartesian product of a star graph $K_{1,n}$ and K_2.*

Definition 2.3. [11]. *The triangular book with book mark is a triangular book B(3, n) with a pendant edge attached at any one end vertices of the spine. It is denoted by $TB_n(u,v)(v,w)$.*

Definition 2.4. *The quadrilateral book with book mark is a quadrilateral book B(4, n) with a pendant edge attached at any one end vertices of the spine. It is denoted by $QB_n(u,v)(v,w)$.*

Definition 2.5. [9]*Let G = (V, E) be a graph with $V = S_1 \cup S_2 \cup ... S_i \cup T$ where each S_i is a set of vertices having atleast two vertices and having the same degree and $T = V - \cup S_i$. The degree splitting graph of G denoted by DS(G) is obtained from G by adding $w_1, w_2, ... w_t$ and joining w_i to each vertex of S_i, $1 \leq i \leq t$.*

Definition 2.6. *A Radio Even Mean Graceful Labeling of a connected graph G is a bijection $\phi : V(G) \rightarrow \{2,4,6,...,2|V|\}$ satisfying the condition $d(s,t) + \left\lceil \frac{\phi(s)+\phi(t)}{2} \right\rceil \geq 1 + diam(G)$, for every u, v ∈ V(G). A graph which admits radio even mean graceful labeling is called an radio even mean graceful graph.*

Theorem 2.7. *DS(B(3, n)) is a radio even mean graceful graph.*

Proof. Let $s, t, u_i, 1 \leq i \leq n$ be the vertices of B(3, n) in which s, t is joined with $u_i, 1 \leq i \leq n$. Introduce two new vertices v, w and join it with the vertices of B(3, n) of degree two and n + 1 respectively. The new graph thus obtained is DS(B(3, n)) whose vertex set is $V = \{s, t, u_i, 1 \leq i \leq n\} \cup \{v, w\}$.

Clearly the diameter of $DS(B(3,n)) = \begin{cases} 1 & if \text{ n = 1} \\ 3 & if \text{ n} > 1 \end{cases}$

Define a bijection $\phi : V(DS(B(3,n))) \rightarrow \{2,4,6,...2|V|\}$ by $\phi(s) = 2n+4$, $\phi(t) = 2n+6$ $\phi(u_i) = 2i, 1 \leq i \leq n$, $\phi(v) = 2n+8$, $\phi(w) = 2n+2$.

Now we analyze the radio even mean graceful condition for ϕ,

Case 1: n = 2

Since the diam(DS(B(3, n))) = 1, it satisifes the radio even mean graceful condition for every pair of vertices.

Case 2: n > 1

Subcase(i): Examine the pair (s,t) :

$$d(s,t) + \left\lceil \frac{\phi(s)+\phi(t)}{2} \right\rceil \geq 1 + \left\lceil \frac{4n+10}{2} \right\rceil \geq 4 = 1 + diam(DS(B(3,n))).$$

Subcase(ii): Examine the pair (s,u_i), $1 \leq i \leq n$:

$$d(s,u_i) + \left\lceil \frac{\phi(s)+\phi(u_i)}{2} \right\rceil \geq 1 + \left\lceil \frac{2n+2i+4}{2} \right\rceil \geq 4.$$

Subcase(iii): Examine the pair (s,v) :

$$d(s,v) + \left\lceil \frac{\phi(s)+\phi(v)}{2} \right\rceil \geq 2 + \left\lceil \frac{4n+12}{2} \right\rceil \geq 4.$$

Subcase(iv): Examine the pair (s,w) :

$$d(s,w) + \left\lceil \frac{\phi(s)+\phi(w)}{2} \right\rceil \geq 1 + \left\lceil \frac{4n+6}{2} \right\rceil \geq 4.$$

Subcase(v): Examine the pair (t,u_i), $1 \leq i \leq n$:

$$d(t,u_i) + \left\lceil \frac{\phi(t)+\phi(u_i)}{2} \right\rceil \geq 1 + \left\lceil \frac{2n+2i+6}{2} \right\rceil \geq 4.$$

Subcase(vi): Examine the pair (t,v) :

$$d(t,v) + \left\lceil \frac{\phi(t)+\phi(v)}{2} \right\rceil \geq 2 + \left\lceil \frac{4n+14}{2} \right\rceil \geq 4.$$

Subcase(vii): Examine the pair (t,w) :

$$d(t,w) + \left\lceil \frac{\phi(t)+\phi(w)}{2} \right\rceil \geq 1 + \left\lceil \frac{4n+8}{2} \right\rceil \geq 4.$$

Subcase(viii): Examine the pair (u_i,u_j), $1 \leq i \leq n-1$, $i+1 \leq j \leq n$:

$$d(u_i,u_j) + \left\lceil \frac{\phi(u_i)+\phi(u_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{2i+2j}{2} \right\rceil \geq 4.$$

Subcase(ix): Examine the pair (u_i,v), $1 \leq i \leq n$:

$$d(u_i,v) + \left\lceil \frac{\phi(u_i)+\phi(v)}{2} \right\rceil \geq 1 + \left\lceil \frac{2n+2i+8}{2} \right\rceil \geq 4.$$

Subcase(x): Examine the pair (u_i,w), $1 \leq i \leq n$:

$$d(u_i,w) + \left\lceil \frac{\phi(u_i)+\phi(w)}{2} \right\rceil \geq 2 + \left\lceil \frac{2n+2i+2}{2} \right\rceil \geq 4.$$

Subcase(xi): Examine the pair (v,w) :

$$d(v,w) + \left\lceil \frac{\phi(v)+\phi(w)}{2} \right\rceil \geq 3 + \left\lceil \frac{4n+10}{2} \right\rceil \geq 4.$$

Thus the radio even mean graceful condition is satisfied for every pair of vertices. Hence $DS(B(3,n))$ is a radio even mean graceful graph. □

Example:1

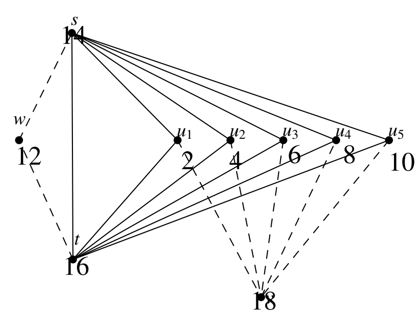

Figure: 1. Radio even mean graceful labeling of DS(B(3, 5))graph.

Theorem 2.8. *DS(B(4, n)) is a radio even mean graceful graph.*

Proof. Let $s, t, s_i, t_i, 1 \leq i \leq n$ be the vertices of B(4, n) in which s, t is joined with s_i, and $t_i, 1 \leq i \leq n$ respectively. Introduce two new vertices u, v and join it with the vertices of B(4, n) of degree n + 1 and two respectively. The new graph thus obtained is DS(B(4, n)) whose vertex set is $V = \{s,t,s_i,t_i, 1 \leq i \leq n\} \cup \{u,v\}$.

Clearly the diameter of $DS(B(4,n)) = \begin{cases} 2 & if \text{ n} = 1 \\ 3 & if \text{ n} > 1 \end{cases}$

Define a bijection $\phi : V(DS(B(4,n))) \to \{2,4,6,...2|V|\}$ by $\phi(s) = 4n+4$, $\phi(t) = 4n+6$ $\phi(s_i) = 2i, 1 \leq i \leq n$, $\phi(t_i) = 2n+2i, 1 \leq i \leq n$, $\phi(u) = 4n+2, \phi(v) = 4n+8$.

Now we analyze the radio even mean graceful condition for ϕ,

Case 1: n = 2

Since the diam(DS(B(4, n))) = 2, it obviously satisfies the radio even mean graceful condition for every pair of vertices.

Case 2: n > 1

Subcase(i): Examine the pair (s,t) :

$d(s,t) + \left\lceil \frac{\phi(s)+\phi(t)}{2} \right\rceil \geq 1 + \left\lceil \frac{8n+10}{2} \right\rceil \geq 4$

$= 1 + diam(DS(B(4,n)))$.

Subcase(ii): Examine the pair (s,s_i), $1 \leq i \leq n$:

$d(s,s_i) + \left\lceil \frac{\phi(s)+\phi(s_i)}{2} \right\rceil \geq 1 + \left\lceil \frac{4n+2i+4}{2} \right\rceil \geq 4.$

Subcase(iii): Examine the pair $(s,t_i), 1 \leq i \leq n$:

$d(s,t_i) + \left\lceil \frac{\phi(s)+\phi(t_i)}{2} \right\rceil \geq 2 + \left\lceil \frac{6n+2i+4}{2} \right\rceil \geq 4.$

Subcase(iv): Examine the pair (s,u) :

$d(s,u) + \left\lceil \frac{\phi(s)+\phi(u)}{2} \right\rceil \geq 2 + \left\lceil \frac{8n+6}{2} \right\rceil \geq 4.$

Subcase(v): Examine the pair (s,v) :

$d(s,v) + \left\lceil \frac{\phi(s)+\phi(v)}{2} \right\rceil \geq 1 + \left\lceil \frac{8n+12}{2} \right\rceil \geq 4.$

Subcase(vi): Examine the pair $(t,s_i) 1 \leq i \leq n$:

$d(t,s_i) + \left\lceil \frac{\phi(t)+\phi(s_i)}{2} \right\rceil \geq 2 + \left\lceil \frac{4n+2i+6}{2} \right\rceil \geq 4.$

Subcase(vii): Examine the pair $(t,t_i), 1 \leq i \leq n$:

$d(t,t_i) + \left\lceil \frac{\phi(t)+\phi(t_i)}{2} \right\rceil \geq 1 + \left\lceil \frac{6n+2i+6}{2} \right\rceil \geq 4.$

Subcase(viii): Examine the pair (t,u) :

$d(t,u) + \left\lceil \frac{\phi(t)+\phi(u)}{2} \right\rceil \geq 2 + \left\lceil \frac{8n+8}{2} \right\rceil \geq 4.$

Subcase(ix): Examine the pair (t,v) :

$d(t,v) + \left\lceil \frac{\phi(t)+\phi(v)}{2} \right\rceil \geq 1 + \left\lceil \frac{8n+14}{2} \right\rceil \geq 4.$

Subcase(x): Examine the pair (s_i,s_j), $1 \leq i \leq n-1$, $i+1 \leq j \leq n$:

$d(s_i,s_j) + \left\lceil \frac{\phi(s_i)+\phi(s_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{2i+2j}{2} \right\rceil \geq 4.$

Subcase(xi): Examine the pair $(s_i,t_j), 1 \leq i,j \leq n$:

$d(s_i,t_j) + \left\lceil \frac{\phi(s_i)+\phi(t_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{2n+2i+2j}{2} \right\rceil \geq 4.$

Subcase(xii): Examine the pair $(s_i,u), 1 \leq i \leq n$:

$d(s_i,u) + \left\lceil \frac{\phi(s_i)+\phi(u)}{2} \right\rceil \geq 1 + \left\lceil \frac{4n+2i+2}{2} \right\rceil \geq 4.$

Subcase(xiii): Examine the pair $(s_i,v), 1 \leq i \leq n$:

$d(s_i,v) + \left\lceil \frac{\phi(s_i)+\phi(v)}{2} \right\rceil \geq 2 + \left\lceil \frac{4n+2i+8}{2} \right\rceil \geq 4.$

Subcase(xiv): Examine the pair $(t_i,t_j), 1 \leq i \leq n-1, i+1 \leq j \leq n$:

$d(t_i,t_j) + \left\lceil \frac{\phi(t_i)+\phi(t_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{4n+2i+2j}{2} \right\rceil \geq 4.$

Subcase(xv): Examine the pair $(t_i,u), 1 \leq i \leq n$:

$d(t_i,u) + \left\lceil \frac{\phi(t_i)+\phi(u)}{2} \right\rceil \geq 1 + \left\lceil \frac{6n+2i+2}{2} \right\rceil \geq 4.$

Subcase(xvi): Examine the pair $(t_i,v), 1 \leq i \leq n$:

$d(t_i,v) + \left\lceil \frac{\phi(t_i)+\phi(v)}{2} \right\rceil \geq 2 + \left\lceil \frac{6n+2i+8}{2} \right\rceil \geq 4.$

Subcase(xvii): Examine the pair (u,v) :

$d(u,v) + \left\lceil \frac{\phi(u)+\phi(v)}{2} \right\rceil \geq 3 + \left\lceil \frac{8n+10}{2} \right\rceil \geq 4.$

Thus the radio even mean graceful condition is satisfied for every pair of vertices. Hence $DS(B(4,n))$ is a radio even mean graceful graph. □

Example:2

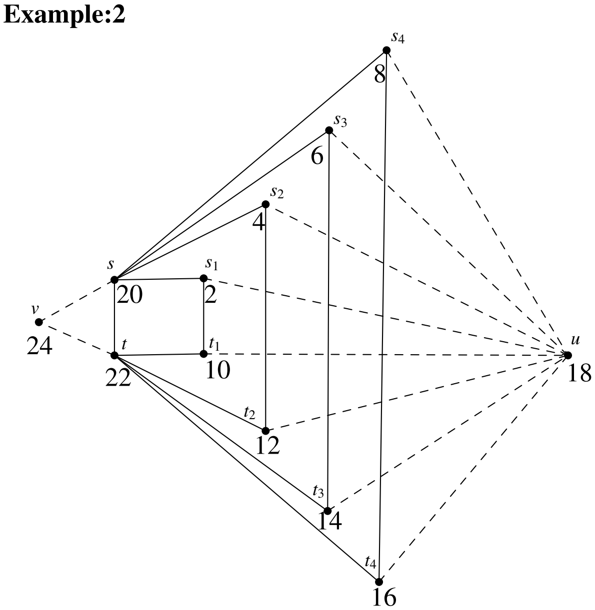

Figure: 2. Radio even mean graceful labeling of DS(B(4, 4))graph.

Theorem 2.9. *$DS(TB_n(t,s),(s,w)))$ is a radio even mean graceful graph.*

Proof. Let $s,t,w,u_i, 1 \le i \le n$ be the vertices of i^{th} triangle of $TB_n(t,s),(s,w))$ in which s, t is joined with $u_i, 1 \le i \le n$ and s with t, w respectively. Introduce a new vertex v and join it with the vertex of $TB_n(t,s),(s,w))$ of degree two. The new graph thus obtained is DS($TB_n(t,s),(s,w))$) whose vertex set is $V = \{s,t,w,u_i, 1 \le i \le n\} \cup \{v\}$.
Clearly the diameter of DS($TB_n(t,s),(s,w))$) = 3.
Define a bijection ϕ : V(DS($TB_n(t,s),(s,w)$)$\rightarrow \{2,4,6,...2|V|\}$ by $\phi(s) = 2n+8$, $\phi(t) = 2n+6$ $\phi(u_i) = 2i+4, 1 \le i \le n$, $\phi(v) = 4$, and$\phi(w) = 2$.
Now we analyze the radio even mean graceful condition for ϕ,

case(i): Examine the pair (s,t) :
$d(s,t) + \left\lceil \frac{\phi(s)+\phi(t)}{2} \right\rceil \ge 1 + \left\lceil \frac{4n+14}{2} \right\rceil \ge 4$
$= 1 + diam(DS(TB_n(t,s),(s,w)))$.

case(ii): Examine the pair (s,u_i), $1 \le i \le n$:
$d(s,u_i) + \left\lceil \frac{\phi(s)+\phi(u_i)}{2} \right\rceil \ge 1 + \left\lceil \frac{2n+2i+12}{2} \right\rceil \ge 4.$

case(iii): Examine the pair (s,v) :
$d(s,v) + \left\lceil \frac{\phi(s)+\phi(v)}{2} \right\rceil \ge 2 + \left\lceil \frac{2n+12}{2} \right\rceil \ge 4.$

case(iv): Examine the pair (s,w) :
$d(s,w) + \left\lceil \frac{\phi(s)+\phi(w)}{2} \right\rceil \ge 1 + \left\lceil \frac{2n+10}{2} \right\rceil \ge 4.$

case(v): Examine the pair $(t,u_i), 1 \le i \le n$:
$d(t,u_i) + \left\lceil \frac{\phi(t)+\phi(u_i)}{2} \right\rceil \ge 1 + \left\lceil \frac{2n+2i+10}{2} \right\rceil \ge 4.$

case(vi): Examine the pair (t,v) :
$d(t,v) + \left\lceil \frac{\phi(t)+\phi(v)}{2} \right\rceil \ge 2 + \left\lceil \frac{2n+10}{2} \right\rceil \ge 4.$

case(vii): Examine the pair (t,w) :
$d(t,w) + \left\lceil \frac{\phi(t)+\phi(w)}{2} \right\rceil \ge 2 + \left\lceil \frac{2n+8}{2} \right\rceil \ge 4.$

case(viii): Examine the pair $(u_i,u_j), 1 \le i \le n-1, i+1 \le j \le n$:
$d(u_i,u_j) + \left\lceil \frac{\phi(u_i)+\phi(u_j)}{2} \right\rceil \ge 2 + \left\lceil \frac{2i+2j+8}{2} \right\rceil \ge 4.$

case(ix): Examine the pair $(u_i,v), 1 \le i \le n$:
$d(u_i,v) + \left\lceil \frac{\phi(u_i)+\phi(v)}{2} \right\rceil \ge 1 + \left\lceil \frac{2i+8}{2} \right\rceil \ge 4.$

case(x): Examine the pair (u_i,w), $1 \le i \le n$:
$d(u_i,w) + \left\lceil \frac{\phi(u_i)+\phi(w)}{2} \right\rceil \ge 2 + \left\lceil \frac{2i+6}{2} \right\rceil \ge 4.$

case(xi): Examine the pair (v,w) :
$d(v,w) + \left\lceil \frac{\phi(v)+\phi(w)}{2} \right\rceil \ge 3 + \left\lceil \frac{6}{2} \right\rceil \ge 4.$

Thus the even radio mean graceful condition is satisfied for every pair of vertices. Hence DS($TB_n(t,s),(s,w))$) is a radio even mean graceful graph. □

Example:3

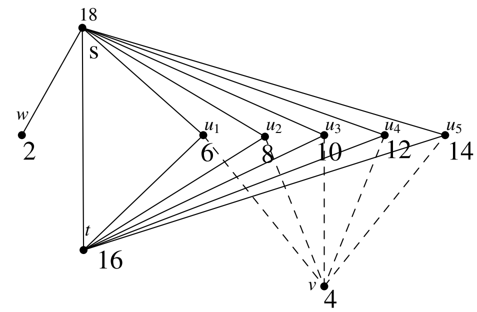

Figure: 3. Radio mean graceful labeling of DS($TB_5(t,s)(s,w)$))graph.

Theorem 2.10. *DS($QB_n(t,s)(s,w)$) is a radio even mean graceful graph.*

Proof. Let $s,t,v,w,s_i,t_i, 1 \le i \le n$ be the vertices of $QB_n(t,s)$ $(s,w))$ in which s is joined with w, s_i, and t with $t_i, 1 \le i \le n$. Also join s and t. Introduce a new vertex v and join it with the vertices of $QB_n(t,s)(s,w))$ of degree two. The new graph thus obtained is DS($QB_n(t,s)(s,w)$) whose vertex set is $V = \{s,t,v,w,s_i,t_i, 1 \le i \le n\} \cup \{v\}$. Clearly, the diameter of $DS(QB_n(t,s)$
(s,w))= 3. Define a bijection $\phi : V(DS(QB_n(t,s)(s,w))) \rightarrow \{2,4,6,...2|V|\}$ by $\phi(s) = 4n+4$, $\phi(t) = 4n+6$ $\phi(s_i) = 2i, 1 \le i \le n$, $\phi(t_i) = 2n+2i+2, 1 \le i \le n$, $\phi(v) = 2n+2$ and $\phi(w) = 4n+8$.
Now we analyze the radio even mean graceful condition for ϕ,

case(i): Examine the pair (s,t) :
$d(s,t) + \left\lceil \frac{\phi(s)+\phi(t)}{2} \right\rceil \ge 1 + \left\lceil \frac{8n+10}{2} \right\rceil \ge 4 =$
$1 + diam(DS(QB_n(t,s)(s,w)))$.

case(ii): Examine the pair (s,s_i), $1 \le i \le n$:
$d(s,s_i) + \left\lceil \frac{\phi(s)+\phi(s_i)}{2} \right\rceil \ge 1 + \left\lceil \frac{4n+2i+4}{2} \right\rceil \ge 4.$

case(iii): Examine the pair $(s,t_i), 1 \le i \le n$:
$d(s,t_i) + \left\lceil \frac{\phi(s)+\phi(t_i)}{2} \right\rceil \ge 2 + \left\lceil \frac{6n+2i+6}{2} \right\rceil \ge 4.$

case(iv): Examine the pair (s,w) :
$d(s,w) + \left\lceil \frac{\phi(s)+\phi(w)}{2} \right\rceil \ge 2 + \left\lceil \frac{8n+12}{2} \right\rceil \ge 4.$

case(v): Examine the pair (s,v) :
$d(s,v) + \left\lceil \frac{\phi(s)+\phi(v)}{2} \right\rceil \ge 1 + \left\lceil \frac{6n+6}{2} \right\rceil \ge 4.$

case(vi): Examine the pair $(t,s_i) 1 \le i \le n$:

$d(t,s_i)+\left\lceil\frac{\phi(t)+\phi(s_i)}{2}\right\rceil \geq 2+\left\lceil\frac{4n+2i+6}{2}\right\rceil \geq 4.$

case(vii): Examine the pair $(t,t_i), 1 \leq i \leq n$:

$d(t,t_i)+\left\lceil\frac{\phi(t)+\phi(t_i)}{2}\right\rceil \geq 1+\left\lceil\frac{6n+2i+8}{2}\right\rceil \geq 4.$

case(viii): Examine the pair (t,w) :

$d(t,w)+\left\lceil\frac{\phi(t)+\phi(w)}{2}\right\rceil \geq 2+\left\lceil\frac{8n+14}{2}\right\rceil \geq 4.$

case(ix): Examine the pair (t,v) :

$d(t,v)+\left\lceil\frac{\phi(t)+\phi(v)}{2}\right\rceil \geq 1+\left\lceil\frac{6n+8}{2}\right\rceil \geq 4.$

case(x): Examine the pair (s_i,s_j), $1 \leq i \leq n-1, i+1 \leq j \leq n$:

$d(s_i,s_j)+\left\lceil\frac{\phi(s_i)+\phi(s_j)}{2}\right\rceil \geq 2+\left\lceil\frac{2i+2j}{2}\right\rceil \geq 4.$

case(xi): Examine the pair $(s_i,t_j), 1 \leq i,j \leq n$:

$d(s_i,t_j)+\left\lceil\frac{\phi(s_i)+\phi(t_j)}{2}\right\rceil \geq 1+\left\lceil\frac{2n+2i+2j+2}{2}\right\rceil \geq 4.$

case(xii): Examine the pair $(s_i,w), 1 \leq i \leq n$:

$d(s_i,w)+\left\lceil\frac{\phi(s_i)+\phi(w)}{2}\right\rceil \geq 2+\left\lceil\frac{4n+2i+8}{2}\right\rceil \geq 4.$

case(xiii): Examine the pair $(s_i,v), 1 \leq i \leq n$:

$d(s_i,v)+\left\lceil\frac{\phi(s_i)+\phi(v)}{2}\right\rceil \geq 1+\left\lceil\frac{2n+2i+2}{2}\right\rceil \geq 4.$

case(xiv): Examine the pair $(t_i,t_j), 1 \leq i \leq n-1, i+1 \leq j \leq n$:

$d(t_i,t_j)+\left\lceil\frac{\phi(t_i)+\phi(t_j)}{2}\right\rceil \geq 2+\left\lceil\frac{4n+2i+2j+4}{2}\right\rceil \geq 4.$

case(xv): Examine the pair $(t_i,w), 1 \leq i \leq n$:

$d(t_i,w)+\left\lceil\frac{\phi(t_i)+\phi(w)}{2}\right\rceil \geq 3+\left\lceil\frac{6n+2i+10}{2}\right\rceil \geq 4.$

case(xvi): Examine the pair $(t_i,v), 1 \leq i \leq n$:

$d(t_i,v)+\left\lceil\frac{\phi(t_i)+\phi(v)}{2}\right\rceil \geq 1+\left\lceil\frac{4n+2i+4}{2}\right\rceil \geq 4.$

case(xvii): Examine the pair (v,w) :

$d(v,w)+\left\lceil\frac{\phi(v)+\phi(w)}{2}\right\rceil \geq 3+\left\lceil\frac{6n+10}{2}\right\rceil \geq 4.$

Thus the even radio mean graceful condition is satisfied for every pair of vertices.Hence $DS(QB_n(t,s)(s,w)))$ is a radio even mean graceful graph.

□

Example:4

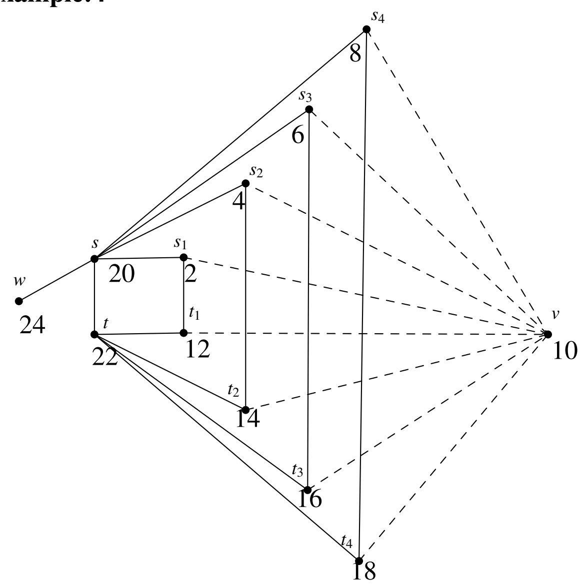

Figure: 4. Radio even mean graceful labeling of DS($QB_4(t,s)(s,w)$))graph.

3. Acknowledgement

The author is thankful to the referee for his valuable suggestions which improved the presentation of the paper.

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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