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The highly D_2 – distance of irregular labeling fuzzy graphs on some special graphs

R. Suja Kumari [1]* and K. Chithamparathanu Pillai[2]

Abstract

This paper states a new concept of highly D_2 –distance of irregular labeling fuzzy graphs and highly totally D_2 –distance of irregular labeling fuzzy graphs with a path on four vertices, Barbell graph and a cycle of length $\geqslant 4$. Some properties related to neighbourly totally D_2 –distance of irregular labeling fuzzy graphs , highly totally D_2 –distance of irregular labeling fuzzy graphs and product fuzzy graphs are discussed with some special graphs. In addition, some more properties and examples of these graphs are studied. The highly D_2 –distance of irregular labeling fuzzy graphs and neighbourly D_2 –distance of irregular labeling fuzzy graphs are observed and viewed for highly totally D_2 –distance of irregular labeling fuzzy graphs and neighbourly totally D_2 –distance of irregular labeling fuzzy graphs.

Keywords

Highly D_2 –distance of irregular labeling fuzzy graphs, highly totally D_2 –distance of irregular labeling fuzzy graphs, neighbourly totally D_2 –distance of irregular labeling fuzzy graphs, neighbourly D_2 –distance of irregular labeling fuzzy graphs, Barbell graph and a cycle of length $\geqslant 4$.

AMS Subject Classification

05C12.

[1] *Department of Mathematics, Scott Christian College, Nagercoil-629003, Kanyakumari District, Tamil Nadu, India. Affiliated to Manonmaniam Sundaranar University, Abishekapatti-Tirunelveli-627012.*

[2] *Department of Mathematics, Lekshmipuram College, Neyyoor, Kanyakumari District, Tamil Nadu, India. Affiliated to Manonmaniam Sundaranar University, Abishekapatti-Tirunelveli-627012.*

***Corresponding author**: [1] sujakumari678@gmail.com; [2]kctlpc@gmail.com

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Contents

1. Introduction

Some various properties of fuzzy graphs [FGs] was introduced by Azriel Rosenfeld [2]. Ravi Narayanan, Santhi Maheswari [5] developed and discussed some properties of highly irregular and highly totally irregular fuzzy graphs. Some properties of fuzzy distance two labeling graphs was discussed by Anuj kumar, pradhan[1]. In this paper, we defines highly D_2 –distance of irregular labeling fuzzy graphs and highly totally D_2-distance of irregular labeling fuzzy graphs with a path on four vertices, Barbell graph and a cycle of length $\geqslant 4$. The condition that provides the connection between these two graphs and a characterization of a neighbourly D_2 –distance of irregular labeling fuzzy graphs, neighbourly totally D_2 –distance of irregular labeling fuzzy graphs on some special graphs such as a path on four vertices, Barbell graph and a cycle of length $\geqslant 4$. The following terminologies fuzzy graph and irregular labeling fuzzy graph are respectively abbreviated as FG and IRRLFG.

2. Preliminaries

Let $G = (A_N, B_A)$ be a FG on $G^* = (N, A)$ for which the membership function $A_N : N \longrightarrow [0,1]$ and $B_A : N \times N \longrightarrow [0,1]$. Then the graph G is said to be a D_2 –distance of irregular labeling fuzzy graph if the arcs and nodes of this graph G have different assignment such that the following conditions are satisfied. (1)$B_A(x,y) < A_N(x) \wedge A_N(y)$ for all $x, y \in N$; (2)$B_A(x,y) \leqslant |A_N(x) - A_N(y)|$, if $d(x,y) = 1$ and (3)$|B_A(x,y) - B_A(y,z)| \leqslant A_N(y)$, if $d(x,z) = 2$, where y is a node on the path connected by the nodes x and z. The graph $F(G) = (A_N, B_A)$ be a PFG then $B_A(x,y) \leqslant A_N(x) \times A_N(y)$ for all $x, y \in N$ .The degree of a node x in G[4] is defined as $d(x) = \sum B_A(xy)$ for all $xy \in A$ and total degree of a node x in G[4] is $td(x) = d(x) + A_N(x) \forall x \in N$. If every nodes of G is adjacent to the nodes with distinct degrees , then G is said to be highly irregular fuzzy graph and if every nodes of G is adjacent to the nodes with distinct total degrees, then G is said to be highly totally irregular fuzzy graph . A graph G is called neighbourly irregular fuzzy graph [5] if every two adjacent nodes of the graph G has distinct degree and if every two adjacent nodes has distinct total degrees then the graph G[5] is said to be neighbourly totally irregular fuzzy graph.

3. The highly D_2 –distance of irregular and highly totally D_2 –distance of irregular labeling fuzzy graphs

Let G be a FG on G^*. A graph G is said to be highly totally D_2–distance of irregular labeling fuzzy graph , if every node of G is adjacent to nodes with distinct total degrees and if the nodes and arcs of this graph satisfies the following conditions:

(1)$B_A(x,y) < A_N(x) \wedge A_N(y)$ for all $x, y \in N$; (2)$B_A(x,y) \leqslant |A_N(x) - A_N(y)|$, if $d(x,y) = 1$ and (3) $|B_A(x,y) - B_A(y,z)| \leqslant A_N(y)$, if $d(x,z) = 2$, where y is a node on the path connected by the nodes x and z.

We are giving suitable examples for above definition.

Example 3.1. *Let us consider a PFG, G on G^*. Define $A_N(s) = 0.3, A_N(t) = 0.4, A_N(u) = 0.5, A_N(v) = 0.6, A_N(w) = 0.7, B_A(s,t) = 0.06, B_A(t,u) = 0.02, B_A(u,v) = 0.01, B_A(v,w) = 0.03, B_A(w,s) = 0.04$. Here $td(s) = 0.4, td(t) = 0.48, td(u) = 0.53, td(v) = 0.64, td(w) = 0.77$ and (1)$B_A(s,t) < A_N(s) \wedge A_N(t)$ for all $s,t \in N$; (2) $B_A(s,t) = 0.06 \leqslant 0.3 = |A_N(s) - A_N(t)$, if $d(s,t) = 1$ and $|B_A(s,t) - B_A(t,u)| = 0.04 \leqslant A_N(t)$ if $d(s,u) = 2$.*

Similarly, the above conditions are satisfied for the remaining nodes. So the graph G is a D_2–distance of irregular labeling fuzzy graph. Now, every node of G is adjacent to nodes with distinct total degrees . Hence the graph G is highly totally D_2 – distance of irregular labeling fuzzy graph .

4. The highly D_2 – distance of irregular labeling fuzzy graphs on a path with four vertices

Theorem 4.1. *Let $G : (A_N, B_A)$ be a product fuzzy graph such that $G^* : (N, A)$ is a path with four vertices and B_A is a constant function , then the graph G is a highly D_2 – distance of irregular labeling fuzzy graph.*

Proof. Assume that G is a product fuzzy graph then $B_A(x,y) \leqslant A_N(x) \times A_N(y)$ for all $x, y \in N$, and $A_N(x), A_N(y) \in [0,1]$. Now, $B_A(x,y) < A_N(x) \wedge A_N(y); B_A(x,y) \leqslant |A_N(x) - A_N(y)|$, if $d(x,y) = 1$ and $|B_A(x,y) - B_A(y,z)| \leqslant A_N(y)$ if $d(x,z) = 2$ for all x, y and z in A, where y is a node on the path connected by the two nodes x and z . Clearly all the conditions of D_2–distance of labeling fuzzy graphs are satisfied by such a PFG. So G is a D_2–distance of labeling fuzzy graph. Also every node of G is adjacent to nodes with distinct degrees . Hence the graph G is highly D_2– distance of irregular labeling fuzzy graph . But B_A is a constant function. □

Theorem 4.2. *Let G be a product fuzzy graph such that G^* is a path with four vertices. If each nodes and arcs of the graph G assigns a distinct membership values, then the graph G is a highly totally D_2–distance of irregular labeling fuzzy graph.*

Proof. Let G be a product fuzzy graph such that G^* is a path with four vertices , then $B_A(x,y) \leqslant A_N(x) \times A_N(y)$ for all $x, y \in N$. If each nodes and arcs of the graph G assigns a distinct membership values then, $B_A(x,y) < A_N(x) \wedge A_N(y), B_A(x,y) \leqslant |A_N(x) - A_N(y)|$, if $d(x,y) = 1$ and $|B_A(x,y) - B_A(y,z)| \leqslant A_N(y)$ if $d(x,z) = 2$. Also, $td(x) = d(x) + A_N(x) \forall x \in N$ and every nodes of G is adjacent to nodes with distinct degrees . Hence the graph G is a highly totally D_2 – distance of irregular labeling fuzzy graph. □

Theorem 4.3. *Let G be a product fuzzy graph such that G^* is a path with four vertices. If the membership value A_N is not constant. Then the graph G is a highly totally D_2–distance of irregular labeling fuzzy graph.*

Proof. Let G be a product fuzzy graph such that G^* is a path with four vertices, then $B_A(x,y) \leqslant A_N(x) \times A_N(y)$ for all $x, y \in N$. Hence, if the membership values A_N is not constant then all the condition of the D_2 – distance of labeling fuzzy graph will be satisfied by such a product fuzzy graph and every node of G is adjacent to nodes with distinct total degrees. Hence the graph G is a highly totally D_2– distance of irregular labeling fuzzy graph. □

Remark 4.4. *Let G be a PFG such that G^* is a path with four vertices. If each arcs of the graph assign a distinct membership values then the graph G is highly totally irregular FG. But it is not D_2 – distance of labeling fuzzy graph.*
For example:

Let us consider a PFG, G on G^* with a path on four vertices. Define $A_N(s) = 0.3, A_N(t) = 0.3, A_N(u) = 0.3, A_N(v) = 0.3, B_A(s,t) = 0.06, B_A(t,u) = 0.12, B_A(u,v) = 0.02$. The graph

G is highly totally irregular FG, since every nodes of G is adjacent to nodes with distinct total degrees. But it is not D_2 – distance of labeling fuzzy graph, since every nodes of G is not satisfied the condition, $B_A(s,t) \leqslant |A_N(s) - A_N(t)|$, if $d(s,t) = 1$.

Theorem 4.5. *Let G be a product fuzzy graph such that G^* is a path with four vertices. If the middle arc of the membership value is less than the membership value of remaining arcs, then the graph G is a highly D_2–distance of irregular labeling fuzzy graph.*
For example:

Let us consider a PFG, G on G^* with a path on four vertices. Define $A_N(s) = 0.3, A_N(t) = 0.4, A_N(u) = 0.5, A_N(v) = 0.6, B_A(s,t) = 0.03, B_A(t,u) = 0.02, B_A(u,v) = 0.03$. The graph G is highly irregular D_2 – distance of labeling FG, since every nodes of G is adjacent to nodes with distinct degrees.

Remark 4.6. *Let G be a product fuzzy graph such that G^* is a path with four vertices. If the middle arc of the membership value is less than the membership value of remaining arcs, then the graph G is a highly totally D_2–distance of irregular labeling fuzzy graph.*

Theorem 4.7. *Let G be a product fuzzy graph such that G^* is a path with four vertices. If the alternate arcs have same membership values, then the graph G is a highly D_2–distance of irregular labeling fuzzy graph.*
For example:

Let us consider a PFG, G on G^* with a path on four vertices. Define $A_N(s) = 0.3, A_N(t) = 0.4, A_N(u) = 0.5, A_N(v) = 0.6, B_A(s,t) = 0.03, B_A(t,u) = 0.02, B_A(u,v) = 0.03$. The graph G is highly irregular D_2 – distance of labeling FG, since every nodes of G is adjacent to nodes with distinct degrees.

Remark 4.8. *Let G be a product fuzzy graph such that G^* is a path with four vertices. If the alternate arcs have same membership values, then the graph G is a highly D_2–distance of irregular labeling fuzzy graph.*

5. The highly D_2 – distance of irregular labeling fuzzy graphs on Barbell graph

Theorem 5.1. *Let $G : (A_N, B_A)$ be a PFG such that $G^* : (N, A)$ is a Barbell graph $B_{n,n}$ of order $2n$. If B_A is a constant function, then the graph G is a highly D_2 – distance of irregular labeling FG.*
For example:

Let us consider a PFG, G on G^* is a Barbell graph. Define $A_N(s) = 0.3, A_N(t) = 0.4, A_N(u) = 0.5, A_N(v) = 0.6, A_N(w) = 0.7, B_A(s,t) = 0.02, B_A(t,u) = 0.02, B_A(t,v) = 0.02, B_A(v,w) = 0.02, B_A(v,x) = 0.02$. The graph G is highly D_2 – distance of irregular labelling FG, but B_A is a constant function.

Theorem 5.2. *Let $G : (A_N, B_A)$ be a PFG such that $G^* : (N, A)$ is a Barbell graph. If B_A is a constant function, then the graph G is a highly totally D_2 – distance of irregular labeling FG.*
For example:

From the above example, G is a highly totally D_2 – distance of irregular labeling FG.

Remark 5.3. *Let $G : (A_N, B_A)$ be a PFG such that $G^* : (N, A)$ is a Barbell graph. Then the graph G is a neighbourly totally D_2 – distance of irregular labeling FG.*

Theorem 5.4. *Let $G : (A_N, B_A)$ be a PFG such that $G^* : (N, A)$ is a Barbell graph. If the pendant edges have the same membership values less than (or) equal to membership values of the middle edge, then the graph G is not neighbourly D_2 – distance of irregular labeling fuzzy graph.*
For example:

Let us consider a PFG, G on G^* is a Barbell graph. Define $A_N(s) = 0.3, A_N(t) = 0.5, A_N(u) = 0.6, A_N(v) = 0.8, A_N(w) = 0.9, B_A(s,t) = 0.02, B_A(t,u) = 0.02, B_A(t,v) = 0.03, B_A(v,w) = 0.02, B_A(v,x) = 0.02$. The graph G is not neighbourly D_2 – distance of irregular labelling FG.

Remark 5.5. *Let $G : (A_N, B_A)$ be a PFG such that $G^* : (N, A)$ is a Barbell graph. Then the graph G is neighbourly totally D_2 – distance of irregular labeling fuzzy graph.*

Remark 5.6. *Let $G : (A_N, B_A)$ be a PFG such that $G^* : (N, A)$ is a Barbell graph. Then the graph G is a highly totally D_2 – distance of irregular labeling fuzzy graph.*

Remark 5.7. *Let $G : (A_N, B_A)$ be a PFG such that $G^* : (N, A)$ is a Barbell graph. Then the graph G is not a highly D_2 – distance of irregular labeling fuzzy graph.*

6. The highly D_2 – distance of irregular labeling fuzzy graphs on a cycle with some specific membership function

Theorem 6.1. *Let $G : (A_N, B_A)$ be a PFG such that $G^* : (N, A)$ is a cycle of length $\geqslant 4$. If B_A is a constant function, then the graph G is not neighbourly D_2 – distance of irregular labeling FG.*
For example :

Let us consider a PFG, G on G^* is a cycle of length $\geqslant 4$. Define $A_N(s) = 0.3, A_N(t) = 0.5, A_N(u) = 0.6, A_N(v) = 0.8, B_A(s,t) = 0.03, B_A(t,u) = 0.03, B_A(u,v) = 0.03, B_A(v,s) = 0.03$. The graph G is not neighbourly D_2 – distance of irregular labeling FG.

Remark 6.2. *Let $G : (A_N, B_A)$ be a PFG such that $G^* : (N, A)$ is a cycle of length $\geqslant 4$. If B_A is a constant function, then the graph G is not highly D_2 – distance of irregular labeling FG.*

Remark 6.3. *Let $G : (A_N, B_A)$ be a PFG such that $G^* : (N, A)$ is a cycle of length $\geqslant 4$. If B_A is a constant function, then the graph G is highly totally D_2 – distance of irregular labeling FG.*

Remark 6.4. *Let $G : (A_N, B_A)$ be a PFG such that $G^* : (N, A)$ is a cycle of length $\geqslant 4$. If B_A is a constant function, then the graph G is neighbourly totally D_2– distance of irregular labeling FG.*

Theorem 6.5. *Let G be a product fuzzy graph such that G^* is a cycle of length $\geqslant 4$. If the alternate arcs have same membership values, then the graph G is not a highly D_2–distance of irregular labeling fuzzy graph.*
For example:

Let us consider a PFG, G on G^* is a cycle of length $\geqslant 4$. Define $A_N(s) = 0.4, A_N(t) = 0.5, A_N(u) = 0.6, A_N(v) = 0.8, B_A(s,t) = 0.03, B_A(t,u) = 0.02, B_A(u,v) = 0.03, B_A(v,s) = 0.02$. The graph G is not highly D_2 – distance of irregular labeling FG.

Remark 6.6. *Let G be a product fuzzy graph such that G^* is a cycle of length $\geqslant 4$. If the alternate arcs have same membership values and if A_N is a constant function, then the graph G is not a highly totally D_2–distance of irregular labeling FG.*

Remark 6.7. *Let G be a product fuzzy graph such that G^* is a cycle of length $\geqslant 4$. If the alternate arcs have same membership values, then the graph G is a highly totally D_2–distance of irregular labeling FG.*

Theorem 6.8. *Let G be a product fuzzy graph such that G^* is a cycle of length $\geqslant 4$. If the alternate arcs have same membership values, then the graph G is not neighbourly D_2–distance of irregular labeling fuzzy graph.*

Remark 6.9. *Let G be a product fuzzy graph such that G^* is a cycle of length $\geqslant 4$. If the alternate arcs have same membership values, then the graph G is neighbourly totally D_2–distance of irregular labeling fuzzy graph.*

7. Conclusion

In this paper, a new idea of the highly D_2 – distance of irregular labeling fuzzy graph and neighbourly D_2–distance of irregular labeling fuzzy graph are compared through some special graphs such as Barbell graph, a cycle of length $\geqslant 4$ and a path on four vertices. Some various properties of highly D_2 – distance of irregular labeling fuzzy graphs and neighbourly D_2–distance of irregular labeling fuzzy graph are studied through various examples and the results are examined for highly totally D_2 – distance of irregular labeling fuzzy graphs and neighbourly totally D_2–distance of irregular labeling fuzzy graph.

References

[1] ANUJ KUMAR , PRADHAN . P , : *Some properties of fuzzy distance two labeling graph*, International Journal of computer sciences and Engineering, Vol -7, Issue – 5, May -2019.

[2] AZRIEL ROSENFELD . A, FUZZY GRAPHS , IN : ZADEH . L .A, FU . K . S, SHIMURA .M (EDS) ,: *Fuzzy sets and their Applications* , Academic press, New York,(1975), pp .77 – 95.

[3] NAGOOR GANI . A , LATHA . S . R ,: *On Irregular Fuzzy Graphs*, Applied Mathematics Sciences, Vol .6, 2012, no.11, 517 – 523.

[4] NAGOOR GANI . A , RADHA . K ,: *On regular Fuzzy Graphs*, Journal of Physical Sciences, Vol . 12, 2008, 33 – 40.

[5] RAVI NARAYANAN .S , SANTHI MAHESWARI . N . R, : *Highly Totally Irregular Fuzzy Graphs*, J .Comp and Math. Sci. Vol – 4 (5), 445 – 449 (2013).

[6] SEKAR . C , SANTHI MAHESWARI . N . R, : *On (2, K) – Regular and totally (2, K) – Regular Fuzzy Graphs*, International Journal of Mathematics and Soft Computing, Vol.4, No.2 (2014), 59 – 69.

[7] SUJA KUMARI .R , CHITHAMPARATHANU PILLAI . K, : *The highly D_2 – distance of irregular labelling fuzzy graphs*, communicated.

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