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Some notes on isolated signed total domination number for digraphs

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Abstract

An isolated signed total dominating function (ISTDF) of a digraph is a function $f : V(D) \to \{-1,+1\}$ such that $\sum_u \in N-(v) \geq 1$ for every vertex $v \in V(D)$ and for at least one vertex of $w \in V(D), f(N^-(w)) = +1$. An isolated signed totaldomination number of D, denoted by $\gamma_{ist}(D)$, in the minimal weight of an isolated signed total dominating function of D. In this paper, we study some properties of ISTDF.

Keywords

Signed total dominating set, isolated vertex, digraph.

AMS Subject Classification 05C60.

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Contents

1. Introduction

In this paper, we consider $D = (V((D), A(D))$ be a digraph with p vertices and q arcs. For a vertex $v \in V(D)$, the set $I(v) = u : (u,v) \in V(D)$ is called the in-neighborhood of v. The in-degree of u is defined by $deg^-(v) = |I(v)|$. A general reference for graph theoretic notions is [3,6].

In 1995, J. E. Dunbar et al [4,5] defined signed dominating function of an undirected graph. A function $f : V(G) \to \{-1,+1\}$ is a signed dominating function of G, if for every vertex $v \in V(G), f(N[v]) \geq 1$. The signed domination number, denoted n by $\gamma_s(G)$, is the minimum weight of a signed dominating function on G [1,8,9].

An isolated signed dominating function [2,7] (ISDF) of a graph G is a SDF function such that $(N)f(N[w]) = +1$ for at leat one vertex of $w \in V(G)$. The weight f, denoted by $w(f)$ is the sum of the value for all $v \in V(G)$. An indated signed domination number of G , denoted by $\gamma_{is}(G)$ is the minimum weight of an ISDF of G.

An ISTDF of a graph G is a function $f : V(G) \to \{-1,+1\}$ such that $\sum_(v \in N(v)) f(v) \geq 1$ for every vertex $v \in V(G)$. and for at least one vertex $w \in V(G)$, $f(N(w)) = +1$. An isolated signed total domination number of C denoted (C), is the minimum weight of added signed total dominating function of G .

In this paper, we study an isolated signed total domination number (ISTDN) of a digraph. An ISTDN of a digraph D is a function $f : V(D) \to \{-1,+1\}$ such that $\sum_(u \in N^-(u)) f(u) \geq 1$ for every vertex $v \in V(D)$ and for atleast one vertex of $w \in V(D), f(N^-(w)) = +1$. In this paper, we concentrate on certain properties of ISTDF and we give ISTDN of few classes of graphs.

2. Main Results:

Theorem 2.1. *Let $n \geq 2$ be an integer. Then the digraph $D = \overrightarrow{P_n} \times \overleftrightarrow{P_2}$ admits ISTDF with ISTDN $\gamma_{ist}(D) = n$.*

Proof. Let $V(D) = \{a_i, b_i : 1 \leq i \leq n\}$ and $A(D) = \{(a_i, a_{i+1}), (b_i, b_{i+1}) : 1 \leq i \leq n-1\} \cup \{(a_i, b_i), (b_i, a_i) : 1 \leq i \leq n\}$. Let f be any ISTDF of D.

Note that $N^-(a_i) = \{a_{i-1}, b_i\}$ for $2 \leq i \leq n$. Suppose any one of the vertex a_{i-1} or b_i has -1, then $f(N^-(a_i)) \leq 0$ for some i. Therefore all the vertices of a_i and b_i must have $+1$.

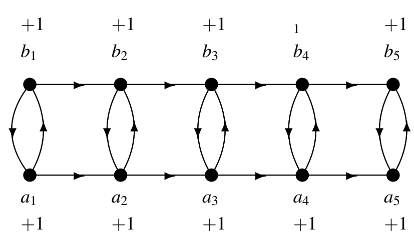
Since $N^-(b_i) = \{b_{i-1}, a_i\}$ for $2 \leq i \leq n$. Suppose any one of the vertex b_{i-1} or a_i has -1, then $f(N^-(b_i)) \leq 0$ for some i. Therefore all the vertices of b_i and a_i must have $+1$.

Next consider the vertex a_1. Since $N^-(a_1) = \{b_1\}$. Suppose the vertex b_1 has -1, then $f(N^-)(a_1) = -1$, a contradiction.

Now consider the vertex b_1. Since $N^-(b_1) = \{a_1\}$. Suppose the vertex a_1 has -1, then $f(N^-)(b_1) = -1$, a contradiction. Therefore all the vertices of $V(D)$ must have $+1$. In this case $f(N^-)(a_1) = f(N^-)(b_1) = 1$. Thus $w(f) \geq n$ and so $\gamma_{ist} \geq n$.
Define a function $g : V(D) \to \{-1,+1\}$ as follows $g(v) = +1$ for all $v \in V(D)$.
From the above labeling, $g(N^-(a_i)) = g(a_{i-1}) + g(b_i) = 1 + 1 = 2$ for $2 \leq i \leq n$ and $g(N^-(b_i)) = g(b_{i-1}) + g(a_i) = 1+1 = 2$ for $2 \leq i \leq n$.
In this case $g(N^-(a_1)) = g(N^-(b_1)) = 1$. Thus $w(g) \leq n$ and so $\gamma_{ist} \leq n$. □

Corollary 2.2. *The digraph $D = \overrightarrow{P_5} \times \overleftrightarrow{P_2}$ admits ISTDF with ISTDN $\gamma_{is}(D) = 10$.*

Proof. Let $V(D) = \{a_i, b_i : 1 \leq i \leq 5\}$ and $A(D) = \{(a_i, a_{i+1}), (b_i, b_{i+1}) : 1 \leq i \leq 4\} \cup \{(a_i, b_i), (b_i, a_i) : 1 \leq i \leq 5\}$. Let f be any ISTDF of D. Note that $N^-(a_i) = \{a_{i-1}, b_i\}$ for $2 \leq i \leq 5$. Suppose any one of the vertex a_{i-1} or b_i has -1, then $f(N^-(a_i)) \leq 0$ for some i. Therefore all the vertices of a_i and b_i must have $+1$ for $2 \leq i \leq 5$.
Since $N^-(b_i) = \{b_{i-1}, a_i\}$ for $2 \leq i \leq 5$. Suppose any one of the vertex b_{i-1} or a_i has -1, then $f(N^-(b_i)) \leq 0$ for some i. Therefore all the vertices of b_i and a_i must have $+1$ for $2 \leq i \leq 5$.
Next consider the vertex a_1. Since $N^-(a_1) = \{b_1\}$. Suppose the vertex b_1 has -1, then $f(N^-)(a_1) = -1$, a contradiction. Now consider the vertex b_1. Since $N^-(b_1) = \{a_1\}$. Suppose the vertex a_1 has -1, then $f(N^-)(b_1) = -1$, a contradiction. Therefore all the vertices of $V(D)$ must have $+1$. In this case $f(N^-)(a_1) = f(N^-)(b_1) = 1$. Thus $w(f) \geq n$ and so $\gamma_{ist} \geq 10$.
Define a function $g : V(D) \to \{-1,+1\}$ as follows $g(v) = +1$ for all $v \in V(D)$.
From the above labeling, $g(N^-(a_i)) = g(a_{i-1}) + g(b_i) = 1 + 1 = 2$ for $2 \leq i \leq 5$ and $g(N^-(b_i)) = g(b_{i-1}) + g(a_i) = 1+1 = 2$ for $2 \leq i \leq 5$.
In this case $g(N^-(a_1)) = g(N^-(b_1)) = 1$. Thus $w(g) \leq 10$ and so $\gamma_{ist} \leq 10$.



Therefore f is ISDF with ISDN $\gamma_{ist}(D) = 10$. □

Lemma 2.3. *Let $n = 4k, k \geq 1$ be an integer. Then the digraph $D = \overleftrightarrow{P_n^{(2)+}}$ admits ISTDF with $\gamma_{ist}(D) = 2k$.*

Proof. Let $V(D) = \{a_i : 1 \leq i \leq n\}$ and $A(D) = \{(a_i, a_{i+1}), (a_{i+1}, a_i : 1 \leq i \leq n-1\} \cup \{(a_1, a_i)\ i\ is\ odd\}$. Let f be a minimum ISTDF of D. Now we consider the vertex a_i for $i = 2, 4, \ldots, 4k$. Note that $N^-(a_i) = \{a_{i-1}, a_{i+1}\}$. Suppose any one of the vertex a_{i-1} or a_{i+1} has -1 sign, then $f(N^-(a_i)) \leq 0$, a contradiction.
Next consider the vertex a_1. Since $N^-(a_1) = \{a_2\}$. Suppose a_2 has -1, a contradiction. Now we consider the vertex a_n. Since $N^-(a_n) = \{a_{n-1}\}$. Suppose a_{n-1} has -1 sign, a contradiction.
Next consider the vertex a_3. Since $N^-(a_3) = \{a_1, a_2, a_4\}$. Already we know that a_1 and a_2 must be labeled with $+1$. Since f be a minimum ISTDF. Therefore the vertex v_4 must be labeled with -1sign. In this case $f(N^-(a_3)) = 1$.
Now we consider the vertex a_5. Note that $N^-(a_5) = \{a_1, a_5, a_6\}$. Already we know that a_1 has $+1$ sign and a_4 has -1. Suppose a_6 has -1, then $f(N^-(a_5)) \leq -1$, a contradiction. Therefore a_6 has $+1$ sign.
Next consider the vertex a_7. Since $N^-(a_7) = \{a_1, a_6, a_8\}$. Already we know that a_1 and a_6 must be labeled with $+1$. Since f be a minimum ISTDF. Therefore the vertex a_8 must be labeled with -1sign. In this case $f(N^-(a_7)) = 1$.
Continue the above process, we observe that, $f(a_{4i}) = -1$ for $1 \leq i \leq k$. Thus $w(f) = 3k(+1) + k(-1) = 2k$ and so $\gamma_{ist}(D) \geq 2k$.
Define a function $g : V(D) \to \{-1,+1\}$ as follows:

$$g(v) = \begin{cases} -1 & when\ v = a_{4i}\ for\ 1 \leq i \leq k \\ +1 & otherwise. \end{cases}$$

Form the above labeling, g is ISTDF of D. In this case $f(N^-(a_3)) = 1$. Thus $w(g) = 3k(+1) + k(-1) = 2k$ and so $\gamma_{ist}(D) \leq 2k$. □

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