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Isolate restrained domination in graphs

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Abstract

A dominating set D of a graph G is said to be a restrained dominating set(RDS) of G if every vertex of $V-D$ has a neighbor in $V-D$. A RDS is said to be an isolate restrained dominating set(IRDS) if $<D>$ has at least one isolated vertex.

The minimum cardinality of a minimal IRDS of G is called the isolate restrained domination number(IRDN), denoted by $\gamma_{r,0}(G)$. This paper contains basic properties of IRDS and gives the IRDN for the families of graphs such as paths, cycles, complete k-partite graphs and some other graphs.

Keywords

Restrained domination, isolate domination, isolate domination number.

AMS Subject Classification

05C69.

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Contents

1. Introduction

The concept of isolate domination in graphs has been introduced by Hameed and Balamurugan [7]. A dominating set D of a graph G is said to be an isolate dominating set(IDS) if $<D>$ has at least one isolated vertex [7]. The minimum cardinality of a minimal isolate dominating set of G is called the isolate domination number $\gamma_0(G)$. An isolate dominating set of cardinality $\gamma_0(G)$ is called a γ_0-set.

A dominating set D of a graph G is said to be RDS if every vertex of $V-D$ has a neighbor in $V-D$. By using the concepts of isolate domination and restrained domination, we introduce a new domination parameter, namely "Isolate Restrained Domination(IRD)". A RDS is said to be an isolate restrained dominating set(IRDS) if $<D>$ has at least one isolated vertex.

The minimum cardinality of a IRDS of G is called the IRDN, denoted by $\gamma_{r,0}(G)$.

2. Basics of isolate restrained domination

In this section, the IRD number of paths, cycles and complete $k-$partite graphs have been obtained. Also some properties of IRDS are given.

Remark 2.1. *If a graph G of order greater than 2 such that $\delta(G)\geq 2$ and has a full vertex, say x. Then $\{x\}$ is an IRD set and $\gamma_{r,0}(G)=1$. Thus the complete graphs(with order not equal to 2) and wheels admit IRDS with IRDN 1.*

Theorem 2.2. *For any graph G, we have $\gamma_r(G)\leq\gamma_{r,0}(G)$ and $\gamma_0(G)\leq\gamma_{r,0}(G)$.*

Proof. Since every IRDS of G is also an RDS of G, we have $\gamma_r(G)\leq\gamma_{r,0}(G)$.
Since every IRDS of G is also an IDS of G, we have $\gamma_0(G)\leq\gamma_{r,0}(G)$. ☐

The following results are proved in [4].

Proposition 2.3. *[4] If $n\neq 2$ is a positive integer, then $\gamma_r(K_n)=1$.*

Proposition 2.4. *[4] If $n>2$ is an integer, then $\gamma_r(K_{1,n-1})=n$.*

Proposition 2.5. *[4] There exists a graph G for which $\gamma_r(G)-\gamma(G)$ can be made arbitrarily large.*

The following results are follows from the definition of IRD.

Proposition 2.6. *If $n\neq 2$ is a positive integer, then $\gamma_{r,0}(K_n)=1$.*

Proposition 2.7. *If $n > 2$ is an integer, then $K_{1,n-1}$ does not admit IRDS.*

Proposition 2.8. *There exists a graph G for which $\gamma_{r,0}(G) - \gamma(G)$ can be made arbitrarily large.*

Proof. Let G be a graph such that the cycle $C_3(V(C_3) = \{v_1, v_2, v_3\}$ such that many pendents are joined with one vertex(say v_1)).
Note that $\{v_1\}$ is a dominating set and so $\gamma(G) = 1$.
Let D be a minimum IRDS and x be isolated in $< D >$.
Suppose $x = v_1$ then no other vertex will be in D, D will not be retrained(since the pendent vertices have no adjacent vertex outside D).
Suppose $x = v_2$, then $v_3, v_1 \notin D$ and so $D = \{v_2\} \cup \{all\ pendent\ vertices\}$. Thus $\gamma_{r,0}(G) = |V(G)| - 2$.
Suppose $x = v_3$, then $v_2, v_1 \notin D$ and so $D = \{v_3\} \cup \{all\ pendent\ vertices\}$. Thus $\gamma_{r,0}(G) = |V(G)| - 2$.
Suppose x is a pendent vertex adjacent to v_1. In this case, $v_1 \notin D$ and so all the pendent vertices will be in D. Since v_1 is adjacent to at least one vertex outside of D, either v_2 or v_3 will not be in D, without loss of generality, assume that $v_2 \notin D$. In this case, $V(G) - \{v_1, v_2\}$ is a minimum isolate restrained dominating set with $|V(G)| - 2$ elements. □

Proposition 2.9. *[4] If n_1 and n_2 are integers such that $min\{n_1, n_2\} \geq 2$, then $\gamma_r(K_{n_1,n_2}) = 2$.*

Lemma 2.10. *Let $G = K_{m,n} = (M,N)$ be a complete bipartite graph. Then G does not admit IRDS.*

Proof. Suppose there exists a minimum IRDS of G, say D. Let x be an isolated vertex in $< D >$. With out loss of generality, assume that $x \in M$. Since $x \in D$ is isolated in $< D >$, D must not have any vertex of N. Thus to dominate all the vertices of M, D must include all the vertices of M. Thus $M = D$. In this case, any vertex of N will not have a neighbor in $V - D$, a contradiction. □

Lemma 2.11. *Let $k \geq 3$ be an integer and $G = K_{m_1,m_2,\ldots,m_k} = (M_1, M_2, \ldots, M_k)$ be a complete $k-$partite graph. Then G admits an IRDS with IRDN $m = min\{mi\}$.*

Proof. With out loss of generality, let $m_1 = min\{mi\}$. Since $k \geq 3$, M_1 is a IRDS of G and so $\gamma_{r,0}(G) \leq m_1$.
Let D be a minimum IRDS of G and x be an isolated vertex in $< D >$. Then $x \in M_i$ for some i with $1 \leq i \leq k$. Since $x \in D$ is isolated in $< D >$, D must not have any vertex of $M_j(j \neq i)$. Thus to dominate all the vertices of M_i, D must include all the vertices of M_i. Thus $m_1 \leq |M_i| \leq |D|$ and so $m_1 \leq \gamma_{r,0}(G)$. □

Theorem 2.12. *Let $n \geq 2$ be an integer and let G be a disconnected graph with n components $G_1, G_2, \ldots, G_n$ such that the first r components $G_1, G_2, \ldots, G_r$ admit IRDS. Then $\gamma_{r,0}(G) = \min_{1 \leq i \leq r} \{t_i\}$, where $t_i = \gamma_{r,0}(G_i) + \sum_{j=1, j \neq i}^{n} \gamma_r(G_j)$ for $1 \leq i \leq r$.*

Proof. With out loss of generality, let $t_1 = \min_{1 \leq i \leq r} \{t_i\}$.
Let S be a $\gamma_{r,0}$- set of G_1 and D_i be a γ_r- set of G_i for each i with $2 \leq i \leq n$. Then $S \cup (\bigcup_{i=2}^{n} D_i)$ is an IRDS of G with cardinality $\gamma_{r,0}(G_1) + \sum_{i=2}^{n} \gamma_r(G_i)$ and so $\gamma_{r,0}(G) \leq \gamma_{r,0}(G_1) + \sum_{i=2}^{n} \gamma_r(G_i) = t_1$.
Let S be a minimal IRDS of G. Then S must intersect $V(G_i)$ for each $1 \leq i \leq n$. Further, there exists an integer j such that $S \cap V(G_j)$ is a minimum IRDS of G_j and $1 \leq j \leq r$. Also for each $1 \leq i \leq n, i \neq j$, the set $S \cap V(G_i)$ is a minimal restrained dominating set of G_i.
Therefore $|S| \geq \gamma_{r,0}(G_j) + \sum_{i=1, i \neq j}^{n} \gamma_r(G_i) \geq t_1$ and hence $\gamma_{r,0}(G) = \min_{1 \leq i \leq r} \{t_i\}$. □

Proposition 2.13. *[4] If G is a graph; then $\gamma_r(G) = 1$ if and only if $G \equiv K_1 + H$ where H is a graph with no isolated vertices.*

Lemma 2.14. *If G is a graph of order greater than 2; then $\gamma_{r,0}(G) = 1$ if and only if G has a full vertex and $delta(G) \geq 2$.*

Proof. Suppose $\gamma_{r,0}(G) = 1$. Then there exists a vertex $x \in V(G)$ such that $D = \{x\}$ is a minimum IRDS. Clearly x is a full vertex. Let $v \in V(G) - D$. Then v is adjacent with x as well as v is adjacent with another of $V(G) - \{x\}$. Thus $deg(v) \geq 2$.
Conversely suppose G has a full vertex, say x and $\delta(G) \geq 2$. In this case, $\{x\}$ is minimum IRDM(Since $\delta(G) \geq 2$, every vertex $u \neq x$ is adjacent to a vertex in $V(G) - \{x\}$). □

Theorem 2.15. *Let G be a connected graph of order $n = 4$. Then $\gamma_r(G) = 2$ if and only if $G \notin \{K_4, K_{1,3}\}$.*

Theorem 2.16. *Let G be a connected graph of order $n = 4$. Then $\gamma_{r,0}(G) = 2$ if and only if $G \notin \{K_4, K_{1,3}, C_4\}$.*

Proof. Suppose that $\gamma_{r,0}(G) = 2$. Suppose $\gamma_r(G) = 1$, then G has a full vertex and so $\gamma_{r,0}(G) = 1$, a contradiction. Thus $\gamma_r(G) = 2$ and hence by Theorem 2.15, $G \neq K_4$ and $G \neq K_{1,3}$.
Note that the graph C_4 does not admit IRDS.(For, let D be a minimum IRDS of C_4. Since $\gamma_{r,0}(C_4) = 2$, D must be a set of two adjacent vertices or two non adjacent vertices. If D contain two adjacent vertices, the $< D >$ has no isolated vertex, a contradiction. If D contain two non adjacent vertices, then no vertex which lies out of D has a neighbor in D, a contradiction.) Thus $G \neq C_4$.
Conversely, suppose $G \notin \{K_4, K_{1,3}, C_4\}$.
If G is a path on four vertices, then the set of two pendent vertices is a minimum IRDS of G.
If $G \cong K_4 - e$, where e is any edge of K_4, then the set of two vertices with degree 2 is a minimum IRDS of G.
Otherwise, G is isomorphic to a graph such that the cycle $C_3(V(C_3) = \{v_1, v_2, v_3\})$ is attached with one pendent v_4 with one vertex, say v_1. In this case, the vertex v_4 together with a vertex of degree two forms a minimum IRDS of G. □

Theorem 2.17. *[4] Let G be a connected graph of order n. Then $\gamma_r(G) = n$ if and only if G is a star.*

Theorem 2.18. *Let G be a connected graph of order n. Then $\gamma_{r,0}(G) = n$ if and only if $G \cong K_1$.*

Proof. Let G be a connected graph of order n and suppose $\gamma_{r,0}(G) = n$.
On the contrary assume that $n \geq 2$. Let D be a $\gamma_{r,0}$-set of G. Then $< D >$ must contain an isolated vertex, say x. Since G is connected, $|N(x)| \geq 1$ and $N(x) \cap D = \phi$. Thus $|D| \leq n-1$, a contradiction. Thus $n = 1$ and so $G \cong K_1$.
Conversely, suppose $G \equiv K_1$. Then $\gamma_{r,0}(G) = 1$. □

Theorem 2.19. *[4] Let G be a connected graph of order n containing a cycle. Then $\gamma_r(G) = n-2$ if and only if G is C_4 or C_5 or G can be obtained from C_3 by attaching zero or more leaves to at most two of the vertices of the cycle.*

Theorem 2.20. *Let G be a graph such that G can be obtained from $C_n(V(C_n) = \{u_1, u_2, \ldots u_n\})$ by attaching zero or more leaves to some or all the vertices of the cycle C_n. Then $\gamma_{r,0}(G) = n-2$ if and only if G can be obtained from C_3 by attaching zero or more leaves to at most two vertices of C_3.*

Proof. Suppose $\gamma_{r,0}(G) = n-2$. Let D be a IRDS of G such that $|D| = n-2$ and x is isolated in $< D >$.
Case 1: Suppose $n \geq 5$.
Sub case 1.1: Suppose $x = u_i$ for some i with $1 \leq i \leq n$.
With out loss of generality, let us assume $x = u_1$. Then $N(u_1) = \{u_2, u_n\}$ and $u_2, u_n \notin D$. In this case the set $V(G) - \{u_2, u_3, u_n\}$ is an IRDS with less than $n-2$ elements, a contradiction.
Sub case 1.2: Suppose x is adjacent to u_i for some i with $1 \leq i \leq n$.
With out loss of generality, let us assume x is adjacent to u_1. In this case, $u_1 \notin D$ and so all the pendent vertices adjacent to u_1 are in D. Since D is restrained, u_1 must be adjacent with some other vertex which is not in D and it must be either u_2 or u_n and without loss of generality, let it be u_2. In this case the set $V(G) - \{u_1, u_2, u_n\}$ is an IRDS with less than $n-2$ elements, a contradiction.
Case 2: Suppose $n = 4$.
Sub case 2.1: Suppose $x = u_i$ for some i with $1 \leq i \leq 4$.
With out loss of generality, let us assume $x = u_1$. Then $N(u_1) = \{u_2, u_4\}$ and $u_2, u_4 \notin D$. Thus $D = V(G) - \{u_2, u_4\}$. In this case both the vertices u_2 and u_4 do not have neighbors in $V - D$, a contradiction to the definition of restrained dominating set.
Sub case 2.2: Suppose x is adjacent to u_i for some i with $1 \leq i \leq n$.
With out loss of generality, let us assume x is adjacent to u_1. In this case, $u_1 \notin D$ and so all the pendent vertices adjacent to u_1 are in D. Since D is restrained, u_1 must be adjacent with some other vertex which is not in D and it must be either u_2 or u_4 and without loss of generality, let it be u_2.
Sub case 2.2.1: Suppose the vertex u_3 is adjacent with some pendent vertices, then $V(G) - \{u_1, u_2, u_4\}$ is an IRDS with less than $n-2$ elements, a contradiction.
Sub case 2.2.2: Suppose the vertex u_4 is adjacent with some pendent vertices, then $V(G) - \{u_1, u_2, u_3\}$ is an IRDS with less than $n-2$ elements, a contradiction.
Sub case 2.2.3: Suppose both the vertices u_3 and u_4 are not adjacent pendent vertices, then $V(G) - \{u_1, u_2, u_4\}$ is an IRDS with less than $n-2$ elements, a contradiction.
Case 3: Suppose $n = 3$ and suppose all the three vertices u_1, u_2 and u_3 are adjacent to some pendant vertices.
Sub case 3.1: Suppose $x = u_i$ for some i with $1 \leq i \leq 3$.
With out loss of generality, let us assume $x = u_1$. In this case, any pendent vertex which is adjacent to $u_1 (= x)$ will not have a neighbor outside D, a contradiction.
Sub case 3.2: Suppose x is adjacent to u_i for some i with $1 \leq i \leq 3$.
In this case, all the pendent vertices forms a IRDS with less than $n-2$ elements, a contradiction.
From the above cases, it is concluded that G can be obtained from C_3 by attaching zero or more leaves to at most two vertices of C_3.
Conversely, suppose G can be obtained from C_3 by attaching zero or more leaves to at most two vertices of C_3.
Assume that u_1 is the vertex such that u_1 is not adjacent with pendent vertex. Let N be the set all pendent vertices which are adjacent to u_2 and u_3. In this case, the set $N \cup \{u_1\}$ is an IRDS of G with $n-2$ elements. Thus $\gamma_{r,0}(G) \leq n-2$. From Theorem 2.19 and Theorem 2.2, we have $n-2 = \gamma_r(G) \leq \gamma_{r,0}(G) = n-2$. □

Lemma 2.21. *[4] If $n \geq 1$ is an integer, then $\gamma_r(P_n) = n - 2\lfloor \frac{n-1}{3} \rfloor$.*

Lemma 2.22. *[4] If $n \geq 3$, then $\gamma_r(C_n) = n - 2\lfloor \frac{n}{3} \rfloor$.*

Here, we obtained the IRDN of paths and cycles.

Lemma 2.23. *Let P_n be a path of n vertices for $n \geq 3$ and $k \geq 1$ be an integer. Then $\gamma_{r,0}(P_n) = k+1$ if $n = 3k+1$,*
$\gamma_{r,0}(P_n) = k+2$ if $n = 3k+2$,
$\gamma_{r,0}(P_n) = k+2$ if $n = 3k$.

Proof. (a). Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$.
Case 1: Suppose $n = 3k+1$. Note that $D = \{v_{3i+1} : 0 \leq i \leq k\}$ is a minimum IRDS with $k+1$ elements. Thus $\gamma_{r,0}(P_n) \leq k+1$.
Let D be any minimum IRDS of P_n. Then D must be a dominating set. Note that every vertex of D can dominate a maximum of 3 vertices. Thus to dominate $3k$ vertices, D must have k vertices. Hence, to dominate the remaining one vertex of P_n, D must have one more vertex and so $|D| \geq k+1$. Thus $\gamma_{r,0}(P_n) \geq k+1$.
Case 2: Suppose $n = 3k+2$. Note that $D = \{v_{3i+1} : 0 \leq i \leq k\} \cup \{v_{3k+2}\}$ is a minimum IRDS with $k+2$ elements. Thus $\gamma_{r,0}(P_n) \leq k+2$.
Let D be any minimum IRDS of P_n.

Sub case 2.1: Suppose $v_1, v_2 \in D$, then to dominate the remaining $n-3 = 3(k-1)+2$ vertices namely, $v_4, v_5, \dots, v_n$, D must include another k vertices. Thus $|D| \geq k+2$.
Sub case 2.2: Suppose $v_2 \in D$ and $v_1 \notin D$, then there does not exist neighbor for v_1 in $V-D$ and so D is not a restrained dominating set, a contradiction.
Sub case 2.3: Suppose $v_1 \in D$ and $v_2 \notin D$. Then there exists a neighbor for v_2 in $V-D$ and it must be v_3. Thus $v_3 \notin D$ and so $v_4 \in D$. Suppose $v_5 \in D$, then as in the proof of Sub case1, we can prove that $|D| \geq k+2$. If $v_5 \notin D$, then v_6 should not be in D and so $v_7 \in D$. Proceeding like this, we have $v_1, v_4, v_7, \dots, v_{3k+1}$ are in D. In this case, the vertex v_{3k+2} does not have a private neighbor in $V-D$ and so $v_{3k+2} \in D$. Thus $|D| \geq k+2$ and so $\gamma_{r,0}(P_n) \geq k+2$.
Case 3: Suppose $n = 3k$. Note that $D = \{v_{3i+1} : 0 \leq i \leq k-1\} \cup \{v_{3k-1}, v_{3k}\}$ is a minimum IRDS with $k+2$ elements. Thus $\gamma_{r,0}(P_n) \leq k+2$.
Let D be any minimum IRDS of P_n.
Sub case 3.1: Suppose $v_1, v_2 \in D$, then as in the proof of Sub case 3 of Case 2, we can prove that $v_2, v_5, v_8, \dots, v_{3k+2}$. In this case v_{3k} does not have a private neighbor in $V-D$ and so $v_{3k} \in D$. Thus $|D| \geq 1+k+1 = k+2$.
Sub case 3.2: Suppose $v_2 \in D$ and $v_1 \notin D$, then there does not exist neighbor for v_1 in $V-D$ and so D is not a restrained dominating set, a contradiction.
Sub case 3.3: Suppose $v_1 \in D$ and $v_2 \notin D$. As in the proof of Sub case 3 of Case 2, we can prove that, $v_1, v_4, v_7, \dots, v_{3k-2}$ are in D. In this case, both the vertices v_{3k-1} and v_{3k} must be in D(otherwise the vertex not belongs to D does not have a neighbor in $V-D$). Thus $|D| \geq k+2$ and so $\gamma_{r,0}(P_n) \geq k+2$. □

Lemma 2.24. *Let $k \geq 1$ be an integer and C_n be a cycle($n \geq 3$). Then*
$\gamma_{r,0}(C_n) = k$ *if* $n = 3k$,
$\gamma_{r,0}(C_n) = k+1$ *if* $n = 3k+1$,
$\gamma_{r,0}(C_n) = k+2$ *if* $n = 3k+2$.

Proof. (a). Let $V(C_n) = \{v_1, v_2, \dots, v_n\}$.
Case 1: Suppose $n = 3k$. By Lemma 2.22, $\gamma_r(C_n) = n - 2\lfloor \frac{n}{3} \rfloor = 3k - 2\lfloor \frac{3k}{3} \rfloor = 3k-2k = k$. Thus by Theorem 2.2, $k = \gamma_r(C_n) \leq \gamma_{r,0}(C_n)$.
Let D be any minimum IRDS of C_n. Then D must be a dominating set. Note that every vertex of D can dominate a maximum of 3 vertices. Thus to dominate $3k$ vertices, D must have k vertices and so $|D| \geq k$. Thus $k \geq \gamma_{r,0}(C_n)$.
Case 2: Suppose $n = 3k+1$. Note that the graph C_4 does not admit IRDS. Assume that $k \geq 2$. By Lemma 2.22, $\gamma_r(C_n) = n - 2\lfloor \frac{n}{3} \rfloor = 3k+1-2\lfloor \frac{3k+1}{3} \rfloor = 3k+1-2k = k+1$. Thus by Theorem 2.2, $k+1 = \gamma_r(C_n) \leq \gamma_{r,0}(C_n)$.
Note that $D = \{v_{3i+1} : 0 \leq i \leq k-1\} \cup \{v_{3k-1}\}$ is a minimum IRDS with $k+1$ elements. Thus $\gamma_{r,0}(C_n) \leq k+1$.
Case 3: Suppose $n = 3k+2$. Note that the graph C_5 does not admit IRDS. Assume that $k \geq 2$. By Lemma 2.22, $\gamma_r(C_n) = n - (2\lfloor \frac{n-1}{3} \rfloor) = 3k+2-(2\lfloor \frac{3k+2-1}{3} \rfloor) = 3k+2-2(k) = k+2$. Thus by Theorem 2.2, $k+2 = \gamma_r(C_n) \leq \gamma_{r,0}(C_n)$.
Note that $D = \{v_{3i+1} : 0 \leq i \leq k-1\} \cup \{v_{3k-1}, v_{3k}\}$ is a minimum IRDS with $k+2$ elements. Thus $\gamma_{r,0}(C_n) \leq k+2$. □

Lemma 2.25. *For any integer $k \geq 1$, there exists a graph G such that $\gamma(G) = \gamma_{r,0}(G) = \gamma_r(G) = k$.*

Proof. As Proved in Lemma 2.24, $\gamma_{r,0}(C_{3k}) = k$.
By Lemma 2.22, $\gamma_r(C_{3k}) = 3k - 2\lfloor \frac{3k}{3} \rfloor = 3k-2k = k$.
Let $V(C_n) = \{v_1, v_2, \dots, v_n\}$. Then $D = \{v_{3i+1} : 0 \leq i \leq k-1\}$ is a minimum dominating set with k elements. Thus $\gamma(C_{3k}) = k$.

□

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