



Combined impact of variable internal heat source and variable viscosity on the onset of convective motion in a porous layer

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Abstract

The qualitative effect of variable internal heat source and temperature dependence of fluid viscosity on the onset of convection in a horizontal fluid saturated porous layer is investigated using linear stability analysis. The temperature-dependence of viscosity is considered to be exponential. A parametric study is performed out by varying the following parameters: viscosity parameter (B) and internal heat source parameter (N_s). We addressed four cases of variance in the internal heat source : (i) $N(z) = z$, (ii) $N(z) = z^2$, (iii) $N(z) = z^3$ and (iv) $N(z) = e^z$. Results indicate that for both parameters of the factor viscosity and heat source variance are to delay the beginning of convective moment. It seen that the system is to be more unstable for case (iii), while more stable for case (iv).

Keywords

Variable viscosity: Variable internal heat source: Stability: Rayleigh Benard convection.

AMS Subject Classification

35Q30.

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1. Introduction

The study of buoyancy-driven flows in porous media is vital, as it has many applications in fields such as insulation of buildings, oil recovery in the petroleum industry, geothermal reservoirs, and chemical reactor engineering. In various physical models for porous structure, several researchers analyzed the instability including Nield [1–4], Vafi [5], Wang and Tan [6], Celli et al. [7], Pop and Ingham [8], Gasser and Kazimi [9], Banu and Rees [10], Mahajan et al. [11].

The temperature dependence of the liquid properties can change the flow behavior in flows with heat transfer: in particular its stability characteristics is well established. The viscosity shows a rather pronounced temperature variation for most of the practical liquids, since viscosity is more temperature resistant than heat and thermal conductivity. Rossby [12] measured the thermal conductivity and water viscosity values between 20 & 25^oC and found that the kinematic viscosity parameter varies approximately around 10% between 20 to 25^oC whereas the water thermal conductivity varies just 1.5%. Torrance and Turcotte [13] observed that fluid viscosity decreases as temperature rises, while gases exhibit a reverse pattern. Several researchers have been studying the effect of viscosity varying temperature in Rayleigh – Benard convection problems in recent years (Booker [14], Solomatov and Barr [15], Barletta and Nield [16]).

There are several studies that have appeared in the literature on how the onset of Rayleigh-Bénard convection is influenced by a periodical boundary temperature. Davis [17] has reviewed several of the results related to those issues. At the other hand, limited attention has been paid to the studies relating to the

influence of thermal modulation on the onset of convection in a fluid saturated porous medium. Rudraiah and Malashetty [18] have studied the influence of time dependent wall temperature on the onset of convection in a porous medium. Study of the effect on convection in a fluid- and fluid-saturated porous layer of complex body forces in a has been of great interest [19–22]. The effect of internal heating on convection studied by Joseph and Shir [23] and Joseph [24] exploited nonlinear energy methods to find the critical Rayleigh number for an internal heat source for a fluid saturated porous sheet. The aim of this research is to study the effect of temperature dependent viscosity and varying internal heat source on the convective motion in a porous medium.

2. Conceptual Model

Figure 1 illustrates the physical configuration of the present study. The physical model under consideration is a horizontal isotropic porous bed bounded between planes at $z = 0$ & $z = L$. Further we assume that the heat source Q depends on the vertical coordinate z . We assume that the viscosity μ has a exponential temperature dependence of the form

$$\mu = \mu_0 \exp[-A(T - T_0)],$$

for a constant $A > 0$ and T , μ_0 and T_0 are temperature and reference viscosity and values of temperature, respectively.

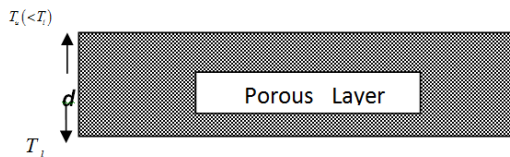


Figure 1. Physical configuration

3. Mathematical Formulation

The porous layer governing equations are

$$\nabla \cdot \vec{V} = 0 \tag{3.1}$$

$$0 = -\nabla p - \frac{\mu(T)}{K} \vec{V} + \rho_0 [1 - \beta(T - T_0)] \vec{g} \tag{3.2}$$

$$A \frac{\partial T}{\partial t} + (\vec{V} \cdot \nabla) T = \kappa \nabla^2 T + Q(z) \tag{3.3}$$

In these equations, \vec{V} denotes the velocity vector, p is the pressure, κ is the thermal diffusivity, A is the ratio of heat capacities, ρ_0 is the reference fluid density and T is the temperature. The basic steady state solution is of the form

$$(u, v, w, p, T) = (0, 0, 0, p_b(z), T_b(z))$$

Then equation (3.3) can be written for basic temperature T_b as:

$$\frac{d^2 T_b}{dz^2} - \frac{1}{\kappa} Q(z) = 0$$

Integrating the above equation twice, we get

$$T_b(z) = \frac{-1}{\kappa} \int_0^z \int_0^\xi Q(\lambda) d\lambda d\xi$$

Applying the boundary conditions

$$T_b = T_L \text{ at } z = 0 \text{ \& } T_b = T_u \text{ at } z = d,$$

we obtain

$$T_b(z) = \frac{-1}{\kappa} \int_0^z \int_0^\xi Q(\lambda) d\lambda d\xi - Cz + T_l,$$

where the constant C is given by

$$C = \frac{1}{d}(T_l - T_u) - \frac{1}{\kappa d} \int_0^d \int_0^\xi Q(\lambda) d\lambda d\xi.$$

Basic state is slightly perturbed using the relation given by

$$\vec{V} = \vec{V}', \quad p = p_b(z) + p', \quad T = T_b(z) + \theta \tag{3.4}$$

Substituting equations (3.4) into equations (3.1)-(3.3). linearizing, by eliminating the term ∇p in the momentum equation and retaining the vertical component, we have:

$$f(z) \nabla^2 w + f'(z) \frac{\partial w}{\partial z} = R \nabla_{hm}^2 T \tag{3.5}$$

$$\left(A \frac{\partial}{\partial t} - \nabla^2 \right) T_m = w N(z), \tag{3.6}$$

where

$$f(z) = \exp \left[B \left(z - \frac{1}{2} \right) \right], \quad B = \left(\frac{v_{\max}}{v_{\min}} \right)$$

We assume the solution are of the form

$$(w, T) = [W(z), \ominus(z)] e^{i(lx + my)}. \tag{3.7}$$

Substituting equation (3.7) into equations (3.5)-(3.6), we obtain the following ordinary differential equations

$$f(z) (D^2 - a^2) w + f'(z) D w = -R a^2 \ominus \tag{3.8}$$

$$(D^2 - a^2) \ominus = -W L(z) \tag{3.9}$$

where \ominus is the amplitude of perturbed temperature, W is the amplitude of perturbed vertical velocity and $R = \alpha g_0 (T_l - T_u) d^3 / \nu \kappa$ is the Rayleigh number and $L(z) = 1 + N_s N(z)$. The boundary conditions take the form

$$W = \ominus = 0 \text{ at } z = 0, 1. \tag{3.10}$$

4. Technique of Solution

Equations (3.8) and (3.9) along with the boundary conditions given by equation (3.10) constitute an eigen value



problem with as the eigen value. Accordingly W and \ominus are written as

$$W = \sum_{i=1}^n A_i W_i, \quad \ominus = \sum_{i=1}^n B_i \ominus_i$$

$$W_i = \ominus_i = \sin(iz) \quad (4.1)$$

where A_i & B_i are constants to be determined. Substituting equation (4.1) into equations (3.8)-(3.9) and using trial functions, we obtain a system of linear homogeneous algebraic equations in A_i & B_i . A nontrivial solution to the system requires the characteristic determinant of the coefficient matrix must vanish and this leads to a relation involving the physical parameters R , Ns , B and a in the form

$$f(R, Ns, B, a) = 0.$$

The critical value of R^c is determined numerically with respect to a for different values of Pe , B & Ns .

5. Results and Discussion

Using the Galerkin process, the joint effect of variable viscosity and variable internal heat source with four different cases of linear & non-linear variation of heat source cases: (i) $N(z) = z$, (ii) $N(z) = z^2$, (iii) $N(z) = z^5$ and (iv) $N(z) = e^z$ is investigated. The governing parameters considered in the present study are: internal heat source parameter Ns and viscosity parameter (B). The stability for the system is obtained in terms of R^c and corresponding a_c for various values of Ns and B with Pe . To validate the numerical procedure used in the present study, the R^c and the corresponding a_c obtained under the limiting case of $B = 0$ (constant case of viscosity) and $Ns = 0$ (absence of heat source). That is as $Ns = 0$ or $B = 0$, the result $R^c \rightarrow 39.479$ and $a_c = 3.14$ which is the known exact value Rionero and Straughan [15]. **Figures 2–5** illustrate

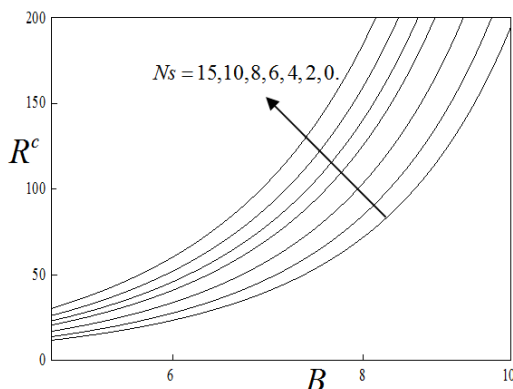


Figure 2. Showing the variation of R^c with respect to B for various values of $Ns = 0, 2, 4, 6, 10, 15, 20$ for case (iii) $N(z) = z^5$

the variation of the R^c and the a_c as a function of B different values of Ns for different linear and non-linear gravity variation: cases: (i) $N(z) = z$, (ii) $N(z) = z^2$, (iii) $N(z) = z^5$ and (iv)

$N(z) = e^z$ respectively. From these figures, it is established that the R^c increases with increasing viscosity parameter B for all different cases of internal heat variation and R^c decreases with increasing internal heat source parameter Ns . Therefore, the stability of the system is maintained by both parameters. An increase in viscosity parameter B increases the temperature between channel walls throughout the flow region. Hence viscosity parameter has stabilizing effect on the system. The effect of varying heat source is shows to be destabilizing in all cases. Due to more production of heat and this helps for the early convection. Furthermore, it is noticed that the system more unstable for case (iii), while for case (iv) it is found to be more stable.

6. Conclusions

Numerical analysis of convective instability in a porous layer with combined effect of variable heat source and temperature dependence of fluid viscosity. The study was conducted in four different cases of variance heat source: (i) $N(z) = z$, (ii) $N(z) = z^2$, (iii) $N(z) = z^5$ and (iv) $N(z) = e^z$. Results show that the effects of increasing the parameter of the viscosity parameter and the internal heat source delay the start of convection. It is noted that for case (iv) the system is more stable, while for case (iii) the system is more unstable, The above findings indicate that, by selecting acceptable values for these parameters, the onset of convection can be advanced or delayed.

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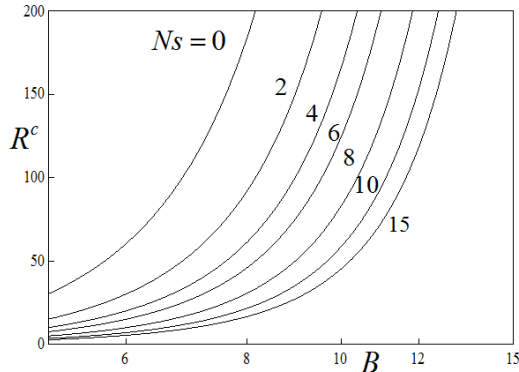


Figure 3. Showing the variation of R^c with respect to B for various values of Ns for case (i) $N(z) = z$

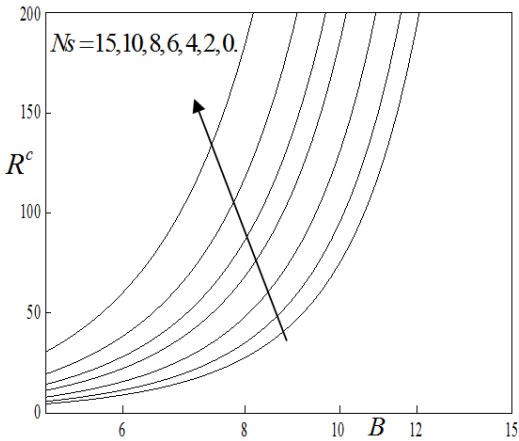


Figure 4. Showing the variation of R^c with respect to B for various values of Ns for case (ii) $N(z) = z^2$

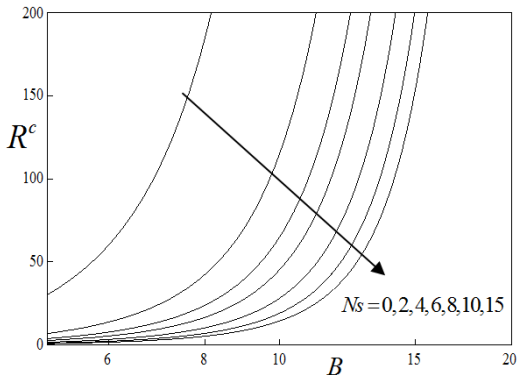


Figure 5. Showing the variation of R^c with respect to B for various values of Ns for case (iv) $N(z) = e^z$

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