Malaya Journal of Matematik, Vol. 8, No. 3, 1237-1239, 2020
https://doi.org/10.26637/MJM0803/0089

r_o-operator in topological spaces

N. Anuradha[1]

Abstract
In this paper a new operator called r_o-operator in topological spaces is introduced for which regular open set is a fixed point. Properties of the operator is also studied.

Keywords
r_o-operator, Closure, Interior.

AMS Subject Classification
54C10.

[1] *Department of Mathematics, Government Brennen College, Dharmadam, Thalassery-670106, Kerala, India.*
Article History: Received **12** April **2020**; Accepted **16** July **2020**

Contents

1. Introduction

In 1937, M.H.Stone [4] introduced regular open set. R.C.Jain [1] in 1980, worked on role of regularly open sets. In this paper an attempt is done to find an operator for which regular open set is a fixed point. In section 2 , preliminary ideas are given. In section 3, r_o-operator is defined. Section 4 discusses about properties of r_o-operator and its fixed points.

2. Preliminaries

Non empty set X with topology τ is denoted as (X, τ). (X, τ) is abbreviated as X . For a set A, its closure is denoted as Cl(A) and interior is denoted as Int(A).

2.1 Definition[4]

A subset A of X is said to be

(i.) regular open, if $A = Int(Cl(A))$.

(ii.) regular closed, if $A = Cl(Int(A))$.

(iii.) clopen, if A is both open and closed.

2.2 Properties of regular open sets[4]

(i.) If a set is clopen, then it is regular open and if a set is regular open then it is open.

(ii.) Finite union of regular open sets is not always regular open.

(iii.) Finite intersection of regular open sets is regular open.

3. r_o-operator

Definition 3.1. *Consider the topological space X. The operator $r_o : P(X) \to P(X)$ defined by $r_o(A) = Int(Cl(A))$ is known as r_o-operator.*

Example 3.2. *Let $X = \{a,b,c\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$.*
Then $r_o(\{a\}) = \{a\}, r_o(\{b\}) = \{b\}, r_o(\{a,b\}) = X$, $r_o(\{c\}) = \phi$

Example 3.3. *Consider (R, τ), where R is the set of real numbers and τ is*
the usual topology. Then,

1. *$r_o(\{(a,b)\}) = (a,b)$ for any open interval (a,b) in R.*
2. *$r_o(\{[a,b]\}) = (a,b)$ for any closed interval [a,b] in R.*
3. *$r_o(\{[a,b)\}) = (a,b) = r_o(\{(a,b]\})$ for any half open intervals in R*

4. Properties of r_o-operator

Theorem 4.1. 1. *If $A \subseteq X$, then $Int(A) \subseteq r_o(A)$.*

2. *If A is an open set, then r_o is an expansive operator. That is $A \subseteq r_o(A)$ for any open set A.*

3. *If $A \subseteq X$, then $r_o(A) \subseteq Cl(A)$.*

4. *If A is a closed set, then r_o is a shrinking operator. That is $r_o(A) \subseteq A$, for any closed set A.*

5. *If A is a clopen set, then r_o is an invariant operator. That is $r_o(A) = A$ for any clopen set A of X.*

6. *The operator r_o is Idempotent. That is $r_o(r_oA) = r_o(A)$*

Proof. 1. $A \subseteq Cl(A)$
$\implies Int(A) \subseteq Int(Cl(A))$
$\implies Int(A) \subseteq r_o(A)$

2. $Int(A) \subseteq r_o(A)$ .... (by (1)).
A open $\implies Int(A) = A$.
Hence, $A \subseteq r_o(A)$.

3. $Int(Cl(A)) \subseteq Cl(A)$, by definition of Interior.
$\implies r_o(A) \subseteq Cl(A)$.

4. $r_o(A) \subseteq Cl(A)$ ......(by (3)).
A closed $\implies Cl(A) = A$
Hence, $r_o(A) \subset A$.

5. A clopen $\implies Int(Cl(A)) = A$
$\implies r_o(A) = A$.

6. $r_o(r_o(A)) = Int(Cl(Int(Cl(A))))$
$\implies r_o(r_o(A)) = Int(Cl(Cl(A)))$
$\implies r_o(r_o(A)) = Int(Cl(A))$
$\implies r_o(r_o(A)) = r_o(A)$

□

Theorem 4.2. 1. *If $A \subseteq X$, then $r_o(Cl(A)) = r_o(A)$.*

2. *If $A \subseteq B$, then $r_o(A) \subseteq r_o(B)$, where $A, B \subset X$.*

3. *$r_o(A \cap B) \subseteq r_o(A) \cap r_o(B)$*

4. *$r_o(A \cup B) \supseteq r_o(A) \cup r_o(B)$.*

Proof. 1. $r_o(Cl(A)) = Int(Cl(Cl(A)))$
$\implies r_o(Cl(A)) = Int(Cl(A))$
$\implies r_o(Cl(A)) = r_o(A)$

2. $A \subseteq B \implies Cl(A) \subseteq Cl(B)$
$\implies Int(Cl(A)) \subseteq Int(Cl(B))$
$\implies r_o(A) \subseteq r_o(B)$

3. $A \cap B \subseteq A$ and $A \cap B \subseteq B$.
Then (2) $\implies r_o(A \cap B) \subseteq r_o(A)$ and $r_o(A \cap B) \subseteq r_o(B)$
$\implies r_o(A \cap B) \subseteq r_o(A) \cap r_o(B)$

4. $A \subseteq A \cup B$ and $B \subseteq A \cup B$.
Then (2) $\implies r_o(A \cup B) \supseteq r_o(A)$ and $r_o(A \cup B) \supseteq r_o(B)$
$\implies r_o(A \cup B) \supseteq r_o(A) \cup r_o(B)$

□

Theorem 4.3. 1. *Regular open sets are fixed points of r_o-operator. That is, $r_o(A) = A$.*

2. *Clopen sets are fixed points of r_o-operator. That is, $r_o(A) = A$.*

3. *ϕ and X are fixed points r_o-operator. That is, $r_o(\phi) = \phi, r_o(X) = X$.*

Proof. 1. If A is a regular open set $Int(Cl(A)) = A$. $\implies r_o(A) = A$.

2. Every clopen set is regular open. Hence $r_o(A) = A$.

3. Trivial.

□

Theorem 4.4. 1. *For two non empty regular open sets A and B, $r_o(A \cup B) \neq A \cup B$.*

2. *For two non empty regular open sets A and B, $r_o(A \cap B) = A \cap B$.*

Proof. 1. Union of regular open sets is not always regular open. So $r_o(A \cup B) \neq A \cup B$.

Example 4.5. *Let $X = \{a,b,c\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$.*

Let $A = \{a\}$, $B = \{b\}$. Then A and B are regular open sets. But $(A \cup B)$ is not regular open.

$r_o(A) = \{a\}, r_o(B) = \{b\}, r_o(A \cup B) = X$.

That is $r_o(A \cup B) \neq A \cup B$.

2. In the case of two regular open sets A and $B, A \cap B$ is regular open and hence $r_o(A \cap B) = A \cap B$.

□

References

[1] R.C. Jain, *The Role of Regularly Open Sets in General Topology*, Ph. D Thesis, Meerut University, Institute of advanced studies, Meerut-India, 1980.
[2] James R Munkres, *Topology*, Second Edition, PHI Learning Private Limited, 2012.
[3] G.F. Simmons, *Introduction to Topology and Modern Analysis*, Tata Mc Graw Hill Education Private Limited, Fifteenth Reprint, 2011.

[4] M.H. Stone, Application of the theory of Boolean Algebras to General Topology, *Trans. Amer. Math. Soc.,* 41(1937), 375–481.

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆