Malaya Journal of Matematik, Vol. 8, No. 4, 2277-2281, 2020

https://doi.org/10.26637/MJM0804/0163

Some results on harmonic index of root square mean graphs

S.S. Sandhya[1]* and H. Aswathy[2]

Abstract

The Harmonic index $H(G)$ of a graph G is defined as the sum of weights $\frac{2}{d(u)+d(v)}$ of all edges uv of G , where $d(u)$ denotes the degree of a vertex u in G. In this paper, we introduce harmonic index of some root square mean graphs.

Keywords

Root square mean graphs, Harmonic index, Crown, Dragon, Kite, $K_{n,n}, K_{1,n}$.

AMS Subject Classification

05C12, 92E10.

[1,2] *Department of Mathematics, Sree Ayyappa College for Women, Chunkankadai, Kanyakumari District, Tamil Nadu, India. Affiliated to Manonmaniam Sundaranar University, Abishekapatti-Tirunelveli-627012.*

***Corresponding author**: [1] sssandhya2009@gmail.com; [2]aswathyrh@gmail.com

Article History: Received **10** July **2020**; Accepted **22** November **2020**

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1. Introduction

Throughout this paper, all graphs are finite , simple ,undirected and connected. Zhong introduce the Harmonic index for graphs. Harmonic index is one of the most important indices in chemical and mathematical fields. It is a variant of the Randic index which is the most successful molecular descriptor in structure property and structure activity relationship studies. For a graph G, the harmonic index is defined as $H(G)=\sum_{u,v\in E(G)}\frac{2}{d(u)+dv)}$, where $d(u)$ is the degree of the vertex u in G. In this paper we consider Harmonic index of some standard graphs which admit Root square mean graphs. The Harmonic index of a graph G is denoted by $H(G)$.

Definition 1.1. *A graph $G=(V,E)$ with p vertices and q edges is said to be a Root Square Mean graph if it is possible to label the vertices $x\in V$ with distinct labels $f(x)$ from $1,2\ldots q+1$ in such a way that when each edge $e=uv$ is labeled with $f(uv)=\left\lceil\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rceil$ or $\left\lfloor\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rfloor$, then the edge labels are distinct. In this case, f is called a Root Square Mean labeling of G.*

Remark 1.2. *If G is a Root Square mean graph, then the vertices get labels from $1,2,\ldots,q+1$ and the edges get labels from $1,2,\ldots,q$.*

Remark 1.3. *If $p>q+1$ then the graph is not a Root Square Mean Graph, since we dont have sufficient labels from $1,2,\ldots,q+1$ for the vertices of G.*

2. Main Results

Theorem 2.1. *The Harmonic index of Crown graph $C_n\odot K_1$ is $\frac{1}{5n}$*

Proof. Consider $G=C_n\odot K_1$ be a Root square mean graphs.

$H(G)=\sum_{u,v\in E(G)}\frac{2}{d(u)+d(v)}$

$$=\frac{2}{\begin{pmatrix}[d(u_1)+d(u_2)]+[d(u_2)+d(u_3)]+\cdots+[d(u_{n-1})+\\ +d(u_n)]+[d(u_n)+d(u_1)]+[d(u_1)+d(v_1)]+\\ [d(u_2)+d(v_2)]+\cdots+[d(u_n)+d(v_n)]\end{pmatrix}}$$

$$=\frac{2}{[3+3]+[3+3]+\cdots+[3+3]+[3+1]+\cdots+[3+1]}$$

$$=\frac{2}{[6+6+\cdots+6]+[4+4+\cdots+4]}$$

$$=\frac{2}{[6\times n]+[4\times n]}$$

$$=\frac{2}{10n}$$

$$=\frac{1}{5n}$$

□

Example 2.2. *Harmonic index of $C_3 \odot K_1$ is given below.*

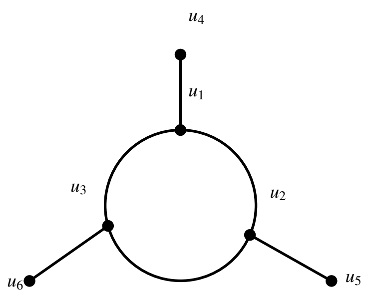

Figure : 2.1

$$H(C_3 \odot K_1) = \frac{2}{[3+3]+[3+3]+[3+3]+[3+1]+[3+1]+[3+1]}$$
$$= \frac{2}{[6\times 3]+[4\times 3]} = \frac{1}{15}$$
$$= \frac{2}{[6\times n]+[4\times n]} = \frac{1}{5n}$$

Theorem 2.3. *Then Harmonic index of $C_n@P_m$ graph is*

$$H(C_n@P_m)=\begin{cases} \frac{1}{4n-1} \; if \; n=m \;\; \& \;\; n>2 \\ \frac{1}{4n+1} \; if \; m>n \;\; \& \;\; m=n+1 \\ \frac{1}{4n-3} \; if \; m<n \;\; \& \;\; n=m+1 \end{cases}$$

Proof. Consider $G = C_n@P_m$ be a Root square mean graph. Let $u_1u_2\dots u_nu_1$ be the Cycle of length n and $v_1v_2\dots v_m$ be the Path of length m. Here $u_n = v_1$

Case (i) if $m = n, \;\; n > 2$

$$H(G) = \sum_{u,v\in E(G)} \frac{2}{d(u)+dv)}$$
$$= \frac{2}{\begin{pmatrix} [d(u_1)+d(u_2)]+[d(u_2)+d(u_3)]+\cdots+[d(u_n)+ \\ +d(u_1)]++[d(v_1)+d(v_2)]+\cdots+[d(v_{m-1})+d(v_m)]\end{pmatrix}}$$
$$= \frac{2}{\begin{pmatrix} [2+2]+\cdots+[2+2]+[3+2]+\cdots+ \\ +[3+2]+[2+2]+\cdots+[2+1]\end{pmatrix}}$$
$$= \frac{2}{[4+4+\cdots+4]+[5+5+5]+[3]}$$
$$= \frac{2}{[4\times(2n-5)]+15+3}$$
$$= \frac{2}{8n-20+18}$$
$$= \frac{2}{8n-2}$$
$$= \frac{1}{4n-1}$$

Example 2.4. *Harmonic index of $C_4@P_4$ is given below.*

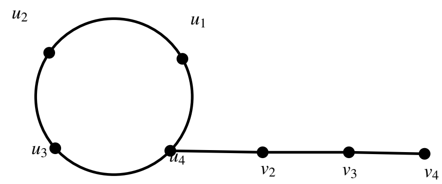

Figure : 2.2

$$H(C_4@P_4) = \frac{2}{[2+2]+[2+2]+[3+2]+[3+2]+[3+2]+[2+2]+[2+1]}$$
$$= \frac{2}{4+4+5+5+5+4+3}$$
$$= \frac{2}{[4\times 3]+[18]}$$
$$= \frac{2}{30} = \frac{1}{15}$$
$$= \frac{1}{4n-1}$$

Case (ii) if $m > n, \;\; \& \;\; m = n+1$

$$H(G) = \frac{2}{\begin{pmatrix} [d(u_1)+d(u_2)]+\cdots+[d(u_{n-1})+d(u_n)]+ \\ [d(u_n)+d(u_1)]+[d(v_1)+d(v_2)]+[d(v_2)+d(v_3)] \\ +\cdots+[d(v_{m-1})+d(v_m)]\end{pmatrix}}$$
$$= \frac{2}{[2+2]+[2+1]+\cdots+[2+2]+[2+3]+[2+3]+[2+3]+[2+1]}$$
$$= \frac{2}{4+4+\cdots+4+5+5+5+3}$$
$$= \frac{2}{[4(2n-4)]+18}$$
$$= \frac{2}{8n-16+18}$$
$$= \frac{2}{8n+2}$$
$$= \frac{1}{4n+1}$$

Example 2.5. *Harmonic index of $C_3@P_4$ is given below.*

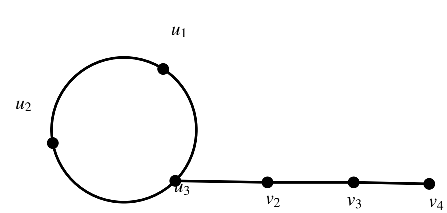

Figure : 2.3

$$H(C_3@P_4) = \frac{2}{\begin{pmatrix}[2+2]+[2+3]+[2+3]+\\+[2+3]+[2+2]+[2+1]\end{pmatrix}}$$
$$= \frac{2}{4+5+5+5+4+3}$$
$$= \frac{2}{[4\times 2]+[18]}$$
$$= \frac{2}{26} = \frac{1}{13}$$
$$= \frac{1}{4n+1}$$

Case (iii) if $m < n$, & $n = m+1$

$$H(G) = \frac{2}{\begin{pmatrix}[d(u_1)+d(u_2)]+\cdots+[d(u_{n-1})+d(u_n)]+\\ [d(u_n)+d(u_1)]+[d(v_1)+d(v_2)]+[d(v_2)+\\ +d(v_3)]+\cdots+[d(v_{m-1})+d(v_m)]\end{pmatrix}}$$
$$= \frac{2}{\begin{pmatrix}[2+2]+[2+2]+\cdots+[2+2]+[2+3]+\\ +[2+3]+[2+3]+[2+1]\end{pmatrix}}$$
$$= \frac{2}{4+4+\cdots+4+5+5+5+3}$$
$$= \frac{2}{[4(2n-6)]+18}$$
$$= \frac{2}{8n-24+18}$$
$$= \frac{2}{8n-6}$$
$$= \frac{1}{4n-3}$$

Example 2.6. *Harmonic index of $C_4@P_3$ is given below.*

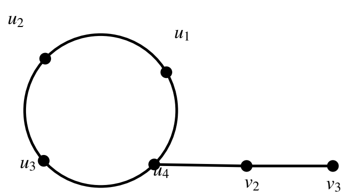

Figure : 2.4

$$H(C_3@P_4) = \frac{2}{\begin{pmatrix}[2+2]+[3+2]+[3+2]+\\+[3+2]+[2+2]+[1+2]\end{pmatrix}}$$
$$= \frac{2}{4+5+5+5+4+3}$$
$$= \frac{2}{[4\times 2]+[18]}$$
$$= \frac{2}{26} = \frac{1}{13}$$
$$= \frac{1}{4n-3}$$

∎

Theorem 2.7. *Then Harmonic index of Kite graph $K(m,n)$ is*

$$H(Km,n) = \begin{cases}\frac{1}{4n+1} \; if\ n=m \ \& \ n>2\\ \frac{1}{4n+3} \; if\ m>n \ \& \ m=n+1\\ \frac{1}{4n-1} \; if\ m<n \ \& \ n=m+1\end{cases}$$

Proof. Case (i) if $m = n$, $n > 2$

$$H(G) = \frac{2}{\begin{pmatrix}[d(u_1)+d(u_2)]+\cdots+[d(u_{n-1})+d(u_n)]+\\ [d(u_1)+d(u_3)]+[d(u_3)+d(u_4)]+[d(u_3)+d(u_5)]\\ +[d(u_4)+d(u_5)]\end{pmatrix}}$$
$$= \frac{2}{\begin{pmatrix}[2+2]+[2+2]+\cdots+[2+2]+\\ [2+3]+[2+3]+[2+3]+[2+1]\end{pmatrix}}$$
$$= \frac{2}{[4+4+\cdots+4]+[5+5+5]+[3]}$$
$$= \frac{2}{8n-16+18}$$
$$= \frac{2}{8n+2}$$
$$= \frac{1}{4n+1}$$

Example 2.8. *Harmonic index of $K(4,4)$ is given below.*

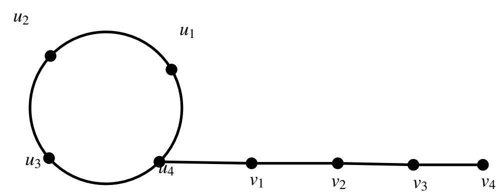

Figure : 2.5

$H(G)$
$$= \frac{2}{\begin{pmatrix}[2+2]+[2+2]+[2+3]+[2+3]+\\ +[2+3]+[2+2]+[2+2]+[2+1]\end{pmatrix}}$$
$$= \frac{2}{4+4+4+4+5+5+5+3}$$
$$= \frac{2}{[4\times 4]+18}$$
$$= \frac{2}{34} = \frac{1}{17}$$
$$= \frac{1}{4n+1}$$

Case (ii) if $m > n$, & $m = n+1$

$$H(G) = \frac{2}{\begin{pmatrix}[d(u_1)+d(u_2)]+\ldots[d(u_{n-1})+d(u_n)]+\\ [d(u_n)+d(u_1)]+[d(v_1)+d(v_2)]+\\ [d(v_2)+d(v_3)]+\cdots+[d(v_{m-1})+d(v_m)]\end{pmatrix}}$$
$$= \frac{2}{[2+2]+[2+2]+\cdots+[2+2]+[2+3]+[2+3]+[2+3]+[2+1]}$$
$$= \frac{2}{4+4+\cdots+4+5+5+5+3}$$

$= \frac{2}{[4(2n-3)]+18}$
$= \frac{2}{8n-12+18}$
$= \frac{2}{8n+6}$
$= \frac{1}{4n+3}$

Example 2.9. *Harmonic index of $K(3,2)$ is given below.*

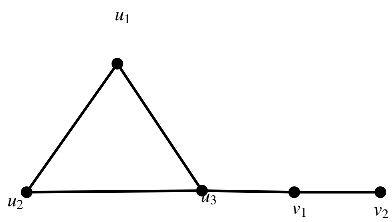

Figure : 2.6

$$H(G) = \frac{2}{[2+2]+[2+3]+[2+3]+[2+3]+[2+1]}$$
$$= \frac{2}{4+5+5+5+3}$$
$$= \frac{2}{[4\times 1]+[18]}$$
$$= \frac{2}{22} = \frac{1}{11}$$
$$= \frac{1}{4n+3}$$

Case (iii) if $m < n$, & $n = m+1$

$$H(G) = \frac{2}{\left(\begin{array}{c}[d(u_1)+d(u_2)]+[d(u_{n-1})+d(u_n)]\\ +[d(u_n)+d(u_1)]+[d(v_1)+d(v_2)]+\\ [d(v_2)+d(v_3)]+\cdots+[d(v_{m-1})+d(v_m)]\end{array}\right)}$$
$$= \frac{2}{\left(\begin{array}{c}[2+2]+[2+2]+\cdots+[2+2]+\\ +[2+3]+[2+3]+[2+3][2+1]\end{array}\right)}$$
$$= \frac{2}{4+4+\cdots+4+5+5+5+3}$$
$$= \frac{2}{[4(2n-5)]+18}$$
$$= \frac{2}{8n-20+18}$$
$$= \frac{2}{8n-2}$$
$$= \frac{1}{4n-1}$$

Example 2.10. *Harmonic index of $K(3,4)$ is given below.*

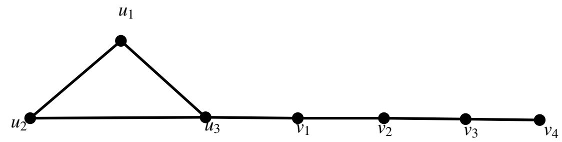

Figure : 2.7

$$H(G) = \frac{2}{\left(\begin{array}{c}[2+2]+[3+2]+[3+2]+\\ +[3+2]+[2+2]+[2+2]+[1+2]\end{array}\right)}$$
$$= \frac{2}{4+5+5+5+4+3}$$
$$= \frac{2}{[4\times 3]+18}$$
$$= \frac{2}{30} = \frac{1}{15}$$
$$= \frac{1}{4n-1}$$

□

Theorem 2.11. *Then Harmonic index of $K_{n,n}$ is*

$$H(K_{n,n}) = \begin{cases} 1 \; if \; n=1 \\ \frac{1}{8} \; if \; n=2 \end{cases}$$

Proof. Let $G = K_{n,n}$ be a Root square mean graph.
Case (i) Consider $K_{1,1}$

$$H(G) = \frac{2}{[1+1]} = \frac{2}{2} = 1$$

Case (ii) Consider $K_{2,2}$

$$H(G) = \frac{2}{[2+2]+[2+2]+[2+2]+[2+2]}$$
$$= \frac{2}{[4+4+4+4]} = \frac{2}{16} = \frac{1}{8}$$ □

Theorem 2.12. *Let $K_{1,n}$ be a Root square mean graph if $n \le 6$ then the harmonic index of $K_{1,n}$ is $\frac{2}{n(n+1)}$.*

Proof. Let $G = K_{1,n}$ be a Root square mean graph.

$$H(G) = \frac{2}{([d(u)+d(v_1)]+[d(u)+d(v_2)]+\cdots+[d(u_n)+d(v_n)])}$$
$$= \frac{2}{(n+1)+(n+1)+\cdots+(n+1)} = \frac{2}{n(n+1)}$$ □

Example 2.13. *Harmonic index of $K_{1,3}$ is given below.*

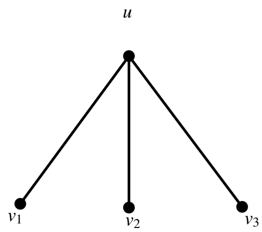

Figure.2.8

$$H(G) = \frac{2}{[1+3]+[1+3]+[1+3]}$$
$$= \frac{2}{4+4+4}$$
$$= \frac{2}{12}$$
$$= \frac{2}{n(n+1)}$$

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ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
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