MALAYA JOURNAL OF MATEMATIK
Malaya J. Mat. **13(01)**(2025), 1–7.
http://doi.org/10.26637/mjm1301/001

Skew codes over the split quaternions

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*R*eceived 17 March 2024; *A*ccepted 04 November 2024

Abstract. In this paper, the structures of linear codes over the split quaternions with coefficient from $\mathbb{Z}_3$, $\mathcal{H}_{s,3} = \mathbb{Z}_3 + i\mathbb{Z}_3 + j\mathbb{Z}_3 + k\mathbb{Z}_3$ are determined with $i^2 = -1, j^2 = k^2 = 1, ij = k = -ji, jk = -i = -kj, ki = j = -ik, ijk = 1$. It is shown that the split quaternions over $\mathbb{Z}_3$ decompose into two parts from $\mathbb{Z}_3 + i\mathbb{Z}_3$ with idempotent coefficients. The structures of the skew cydic and skew constacyclic codes over $\mathcal{H}_{s,3}$ are obtained.

AMS Subject Classifications: 11R52, 94B05.

Keywords: Skew constacyclic code, split quaternions.

Contents

1. Introduction

W. R. Hamilton invented the quaternions in 1843, [1]. It is generalization of complex numbers. These quaternions turned out to be the only associative division algebra over $\mathbb{R}$ with dimension > 2. Later quaternion algebras over arbitrary fields were introduced. A quaternion algebra over arbitrary field F means a unital, associative, four dimensional algebra over F with a basis $\{1, i, j, k\}$ and $a, b \in F^*$. The product was given by $i^2 = a, j^2 = b$ and $ij = -ji = k$. Firstly, quaternions over $\mathbb{Z}_p$ were introduced by Kandasamy, [2]. In [3], the structures of linear and cyclic codes over H_3 were given. It was shown that H_3 decomposed into two parts. The cyclic codes over H_3 were investigated.

If $a = -1, b = 1, F = \mathbb{Z}_p$, where p is a prime, then it is called the split quaternions over $\mathbb{Z}_p$. In [4] S. M. Kong et al. gave some key differences in the algebras of quaternions and split quaternions over $\mathbb{Z}_p$, for some prime p.

In the light of all this information, in this study, we investigated skew codes obtained from split quaternions.

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2. Preliminaries

Let $\mathcal{H}_{s,3} = \{a + bi + cj + dk : a, b, c, d \in \mathbb{Z}_3\}$ be a non-commutative ring, where $\mathbb{Z}_3 = \{0, 1, 2\}$ be a finite field, $i^2 = -1, j^2 = k^2 = 1$ and $ij = -ji = k$ with the sum and left (right) product operations;

$$
\begin{aligned}
&+ : H_{s,3} \times \mathrm{H}_{s,3} \longrightarrow H_{s,3} \\
&(a_1 + ib_1 + jc_1 + kd_1, a_2 + ib_2 + jc_2 + kd_2) \mapsto a_1 + a_2 + i(b_1 + b_2) + \\
&j(a_1 + c_2) + k(d_1 + d_2)
\end{aligned}
$$

$$
\begin{aligned}
&*_L : H_{s,3} \times \mathrm{H}_{s,3} \longrightarrow \mathrm{H}_{s,3} \\
&(a_1 + ib_1 + jc_1 + kd_1, a_2 + ib_2 + jc_2 + kd_2) \mapsto (a_1a_2 - b_1b_2 + c_1c_2 + d_1d_2) + \\
&i(a_1b_2 + b_1a_2 + d_1c_2 + 2c_1d_2) + j(a_1c_2 + a_2c_1 + d_1b_2 + 2d_2b_1) + \\
&k(a_1d_2 + d_1a_2 + b_1c_2 + 2c_1b_2)
\end{aligned}
$$

$$
\begin{aligned}
&*_R : H_{s,3} \times \mathrm{H}_{s,3} \longrightarrow \mathrm{H}_{s,3} \\
&(a_1 + ib_1 + jc_1 + kd_1, a_2 + ib_2 + jc_2 + kd_2) \mapsto (a_1a_2 - b_1b_2 + c_1c_2 + d_1b_2) + \\
&i(a_1b_2 + b_1a_2 + 2c_2d_1 + d_2c_1) + j(a_1c_2 + a_2c_1 + 2b_2d_1 + b_1d_2) + \\
&k(a_1d_2 + d_1a_2 + 2b_1c_2 + c_1b_2)
\end{aligned}
$$

In this paper, the left product $*_L$ will be used as an operation for this ring. In $\mathcal{H}_{s,3}$ with the operations;

- There are 48 units. The number of units λ which satisfy $\lambda^2 = 1$ is 14. They are $1, 2, j, k, 2j, 2k, i + j + k, 2i + 2j + 2k, 2i + j + k, i + 2j + 2k, 2i + 2j + k, i + j + 2k, 2i + j + 2k$ and $i + 2j + k$. The number of units $\lambda = a + bi + cj + dk$ which satisfy $\lambda\lambda' = 1$ is 34 , where $\lambda' = a' + b'i + jc' + kd'$ is a unit.

$abcd$	$a\prime b\prime c\prime d\prime$
0012	0021
0011	0022
0100	0200
1100	2100
1101	1202
1110	1220
1120	1210
2200	1200
2202	2101
2210	2120
2220	2110
1011	2011
1012	2012
1021	2021
1022	2022
1201	1102
2201	2102

- There are 14 idempotent elements which are $0, 1, 2 + j, 2 + 2j, 2 + k, 2 + 2k, 2 + i + j + k, 2 + 2i + 2j + 2k, 2 + i + 2j + 2k, 2 + 2i + j + k, 2 + 2i + j + 2k, 2 + i + 2j + k, 2 + 2i + 2j + k, 2 + i + j + 2k,$

- There are 33 zero divisor. They were given in [4].

- There are 6 idempotent pairs which are $\{2+j, 2+2j\}$, $\{2+k, 2+2k\}$, $\{2+i+j+k, 2+2i+2j+2k\}$, $\{2+i+2j+2k, 2+2i+j+k\}$, $\{2+2i+j+2k, 2+i+2j+k\}$, $\{2+2i+2j+k, 2+i+j+2k\}$. All pairs are orthogonal and their sum is equal to 1.

Remark 2.1. *Without loss of generality, we choose the pair $\{2+j, 2+2j\}$. For any of the idempotent pair, similar things is made.*

Proposition 2.2. *Every element $h \in \mathcal{H}_{s,3}$ is uniquely written as*

$$h = h_1 *_L z_1 + h_2 *_L z_2$$

where $z_1, z_2 \in \mathbb{Z}_3 + i\mathbb{Z}_3 \cong \mathbb{Z}_3[x]\left\langle x^2+1\right\rangle \cong \mathbb{F}_9$, according to spesific central orthogonal idempotent pair $\{h_1 = 2+j, h_2 = 2+2j\}$.

Proof. Let $h = a_1 + b_1 i + c_1 j + d_1 k \in \mathcal{H}_{s,3}$ be an arbitrary element. Assume that $h = h_1 *_L z_1 + h_2 *_L z_2$, where $z_1 = s_1 + t_1 i$, $z_2 = s_2 + t_2 i \in \mathbb{Z}_3 + i\mathbb{Z}_3$. We know that $h_1 *_L h_1 = h_1$ and $h_1 *_L h_2 = 0$. So we have

$$h_1 *_L h = h_1 *_L z_1$$
$$(2+j) *_L (a_1 + b_1 i + c_1 j + d_1 k) = (2+j) *_L (s_1 + t_1 i).$$

Therefore $s_1 = a_1 + 2c_1, t_1 = b_1 + d_1$. We know that $h_2 *_L h_2 = h_2$ and $h_2 *_L h_1 = 0$. So we get

$$h_2 *_L h = h_2 *_L z_2$$
$$(2+2j) *_L (a_1 + b_1 i + c_1 j + d_1 h) = (2+2j) *_L (s_2 + t_2 i)$$

Therefore $s_2 = a_1 + c_1, t_2 = b_1 + 2d_1$. Hence.

$$h = h_1 *_L ((a_1 + 2c_1) + i(b_1 + d_1)) + h_2 *_L ((a_1 + c_1) + i(b_1 + 2d_1)) \in \mathcal{H}_{s,3}$$

where $a_1, b_1, c_1, d_1 \in \mathbb{Z}_3$. It is easily seen that the uniqueness. ■

Example 2.3. $1 + i + j + k \in \mathcal{H}_{s,3}$ *is uniquely written as*

$$1 + i + j + k = (2+j) *_L (2i) + (2+2j) *_L (2)$$

for the idempotent pair $\{h_1 = 2+j, h_2 = 2+2j\}$. For the other idempotent pair $\{h_1 = 2+i+2j+2k, h_2 = 2+2i+j+k\}$, $1+i+j+k \in \mathcal{H}_{s,3}$ is uniquely written as;

$$1 + i + j + k = (2+i+2j+2k) *_L (2+2i) + (2+2i+j+k) *_L (2i).$$

Proposition 2.4. *$\mathcal{H}_{s,3}$ is a $\mathbb{Z}_3 + i\mathbb{Z}_3$-module and $\mathcal{H}_{s,3} = (\mathbb{Z}_3 + i\mathbb{Z}_3) \oplus (\mathbb{Z}_3 + i\mathbb{Z}_3)$ by using these central orthogonal idempotents.*

Remark 2.5. *Since $\mathbb{Z}_3[x]/\left\langle x^2+1\right\rangle \cong \mathbb{F}_9$, the split quaternions $\mathcal{H}_{s,3}$ just only can be decomposed into $(\mathbb{Z}_3 + i\mathbb{Z}_3) \oplus (\mathbb{Z}_3 + i\mathbb{Z}_3)$.*

Definition 2.6. *A linear code C of length n over $\mathcal{H}_{s,3}$ is a left (right) $\mathcal{H}_{s,3}$-submodule of $(\mathcal{H}_{s,3})^n$.*

Define a Gray map as follows;

$$\Omega : \mathcal{H}_{s,3} \longrightarrow (\mathbb{Z}_3 + i\mathbb{Z}_3) \oplus (\mathbb{Z}_3 + i\mathbb{Z}_3)$$
$$s = h_1 *_L z_1 + h_2 *_L z_2 \longmapsto (z_1, z_2)$$

We can extended the map from $(\mathcal{H}_{s,3})^n$ to $(\mathbb{Z}_3 + i\mathbb{Z}_3)^n \oplus ((\mathbb{Z}_3 + i\mathbb{Z}_3)^n$.

The Gray weight of an element $s \in \mathcal{H}_{s,3}$ is defined as

$$w_G(s) = w_H(\Omega(s))$$

where w_H denotes the Hamming weight. The Gray distance of $c = (c_0, \ldots, c_{n-1}) \in (\mathcal{H}_{s,3})^n$ is defined by

Example 2.7. *Since*

$$w_G(1+i+j+k,2i) = w_H(\Omega(1+i+j+k,2i)),$$

we have $w_G(1+i+j+k,2i)=4$*, by using* $\Omega(1+i+j+k)=(2i,2)$ *and* $\Omega(2i)=(2i,2i)$.

Proposition 2.8. *The Gray map is a linear map and isometry from* $((\mathcal{H}_{s,3})^n$*, Gray distance) to* $((\mathbb{Z}_3+i\mathbb{Z}_3)^{2n}$*, Hamming distance).*

3. Skew cyclic codes over $\mathcal{H}_{s,3}$

By defining an automorphism θ over $\mathcal{H}_{s,3}$ as follows, we can define the left (right) skew cyclic codes over $\mathcal{H}_{s,3}$,

$$\begin{aligned} \theta : \mathcal{H}_{s,3} &\longrightarrow \mathcal{H}_{s,3} \\ a+bi+cj+dk &\longmapsto a+2bi+cj+2dk \end{aligned}$$

The order of θ is 2.

Definition 3.1. *A subset C of $(\mathcal{H}_{s,3})^n$ is said to be left (right) skew cyclic code of length n if two conditions are satisfied;*

(i) C is a left (right) $\mathcal{H}_{s,3}$-submodule of $(\mathcal{H}_{s,3})^n$

(ii) If $c=(c_1,\ldots,c_{n-1})\in C$, then $\rho(c)=(\theta(c_{n-1}),\theta(c),\ldots,\theta(c_{n-2}))\in C$.

The ring $\mathcal{H}_{s,3}[x,\theta]=\left\{s_0+s_1x+\cdots+s_{n-1}x^{n-1} : s_t\in\mathcal{H}_{s,3}, n\in\mathbb{N}, t=0,1,\ldots n-1\right\}$ are called skew polynomial ring with the usual polynomial addition and the multiplication as follows;

$$(ax^m)*_L(bx^s)=a*_L\theta^m(b)x^{m+s}$$

The ring is a non-commutative ring. In the polynomial representation a skew cyclic code of length n over $\mathcal{H}_{s,3}$ is defined as left ideal of quotient ring $\Re=\mathcal{H}_{s,3}[x,\theta]/\langle x^n-1\rangle$, if the order of θ divides n, that is n is even. If the order of θ does not divide n, a left (right) skew cyclic code of length n over $\mathcal{H}_{s,3}$ is defined as a left (right)-$\mathcal{H}_{s,3}[x,\theta]$-submodule of $\Re$.

By defining an automorphism ψ over $\mathbb{Z}_3+i\mathbb{Z}_3$, we can also define the skew cyclic codes over $\mathbb{Z}_3+i\mathbb{Z}_3$.

$$\begin{aligned} \Psi : \mathbb{Z}_3+i\mathbb{Z}_3 &\longrightarrow \mathbb{Z}_3+i\mathbb{Z}_3 \\ a+ib &\longmapsto a+2bi \end{aligned}$$

Theorem 3.2. *Let $C=h_1*_L C_1\oplus h_2*_L C_2$ be a linear code of length n over $\mathcal{H}_{s,3}$, where C_t $(t=1,2)$ are codes over $\mathbb{Z}_3+i\mathbb{Z}_3$. Then C is a skew cyclic code with respect to the automorphism θ if and only if C_t are skew cyclic codes over $\mathbb{Z}_3+i\mathbb{Z}_3$ with respect to the automorphism ψ, where $t=1,2$.*

Proof. For any $\mathbf{r}=(r_0,\ldots,r_{n-1})\in C$, let $r_y=h_1*_L a_y\oplus h_2*_L b_y\in\mathcal{H}_{s,3}$, for $y=0,1,\ldots,n-1$, where $\mathbf{a}=(a_1,\ldots,a_{n-1})\in C_1,\mathbf{b}=(b_0,\ldots,b_{n-1})\in C_2$. If $C_t(t=1,2)$ are skew cyclic codes over $\mathbb{Z}_3+i\mathbb{Z}_3$, then

$$\rho(\mathbf{r})=\rho(h_1*_L\mathbf{a}\oplus h_2*_L\mathbf{b})=h_1*_L\rho(\mathbf{a})\oplus h_2*_L\rho(\mathbf{b})\in C.$$

Since $\rho(\mathbf{a})=(\psi(a_{n-1}),\psi(a_0),\ldots,\psi(a_{n-2}))\in C_1$, for $(a_0,\ldots,a_{n-2})\in C_1$ and $\rho(\mathbf{b})=(\psi(b_{n-1}),\psi(b_0),\ldots,\psi(b_{n-2}))\in C_2$ for $(b_0,\ldots,b_{n-2})\in C_2$. This shows that C is a skew cyclic code over $\mathcal{H}_{s,3}$.

On the other hand, if C is a skew cyclic code over $\mathcal{H}_{s,3}$, we have $\rho(r)=(\theta(r_{n-2}),\theta(r_0),\ldots,\theta(r_{n-2}))=h_1*_L\rho(\mathbf{a})\oplus h_2*_L\rho(\mathbf{b})\in C$ which implies $\rho(\mathbf{a})\in C_1,\rho(\mathbf{b})\in C_2$. Hence C_t $(t=1,2)$ are skew cyclic codes over $\mathbb{Z}_3+i\mathbb{Z}_3$. ■

By taking $\theta'(a+bi+cj+dk)=a+2bi+2cj+dk$ and by choosing central orthogonal idempotent pair $h'_1=2+k, h'_2=2+2k$, then we have;

Theorem 3.3. *Let $D = h_1' *_L D_1 \oplus h_2' *_L D_2$ be a linear code of length n over $\mathcal{H}_{s,3}$, where D_t $(t = 1, 2)$ are codes over $\mathbb{Z}_3 + i\mathbb{Z}_3$. Then D is a skew cyclic code with respect to the automorphism θ' if and only if D_t are skew cyclic codes over $\mathbb{Z}_3 + i\mathbb{Z}_3$ with respect to the automorphism ψ, where $t = 1, 2$.*

Proof. It is made as in the proof of the Theorem 10. ■

4. Skew θ-λ-constacyclic codes over $\mathcal{H}_{s,3}$

Definition 4.1. *A subset C of $(\mathcal{H}_{s,3})^n$ is said to be left (right) skew θ-λ-constacyclic code of length n if two conditions are satisfied;*

(i) C is a left (right) $\mathcal{H}_{s,3}$-submodule of $(\mathcal{H}_{s,3})^n$

(ii) If $c = (c_0, \ldots, c_{n-1}) \in C$, then $\sigma_\lambda(c) = (\lambda *_L \theta(c_{n-1}), \theta(c_0), \ldots, \theta(c_{n-2})) \in C$, where $\theta \in Aut(\mathcal{H}_{s,3})$ and λ is a unit in $\mathcal{H}_{s,3}$.

In this section, we will take θ as in the section 3 and $\{h_1 = 2 + j, h_2 = 2 + 2j\}$.

Let $\lambda = \lambda_1 + \lambda_2 i + \lambda_3 j + \lambda_4 k \in \mathcal{H}_{s,3}$ such that where $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{Z}_3$. From the proposition 2, we know that

$$\lambda = h_1 *_L (\lambda_1 + 2\lambda_3 + i(\lambda_2 + \lambda_4)) + h_2 *_L (\lambda_1 + \lambda_3 + i(\lambda_2 + 2\lambda_4)) \qquad (*)$$

From table 1, it is easily seen that if an element $\lambda = h_1 *_L u_1 + h_2 *_L u_2$ is a unit in $\mathcal{H}_{s,3}$, then u_s are units in $\mathbb{Z}_3 + i\mathbb{Z}_3$, where $s = 1, 2$.

Proposition 4.2. *An element $\lambda = h_1 *_L u_1 + h_2 *_L u_2 = \lambda_1 + \lambda_2 i + \lambda_3 j + \lambda_4 k \in \mathcal{H}_{s,3}$ is fixed by θ if and only if u_s are fixed by ψ for $s = 1, 2$ and u_s's are as in $(*)$.*

Proof. Suppose that λ is fixed by θ. Then

$$\begin{aligned} \lambda_1 + \lambda_2 i + \lambda_3 j + \lambda_4 k &= \theta(\lambda_1 + \lambda_2 i + \lambda_3 j + \lambda_4 k) \\ &= \lambda_1 + 2\lambda_2 i + \lambda_3 j + 2\lambda_4 k \\ &= \psi(\lambda_1) + 2\psi(\lambda_2) i + \psi(\lambda_3) j + 2\psi(\lambda_4) k \end{aligned}$$

So $\psi(\lambda_t) = \lambda_t, t = 1, 3$ and $\psi(\lambda_r) = 2\lambda_r, r = 2, 4$.

Thus

$$\begin{aligned} \psi(\lambda_1 + 2\lambda_3 + i(\lambda_2 + \lambda_3)) &= \psi(\lambda_1) + 2\psi(\lambda_3) + \psi(i)[\psi(\lambda_2) + \psi(\lambda_3)] \\ &= \lambda_1 + 2\lambda_3 + 2i(2\lambda_2 + 2\lambda_4) \\ &= \lambda_1 + 2\lambda_3 + 2i(\lambda_2 + \lambda_4) \end{aligned}$$

Similarly, it can be shown that $\psi(u_2) = u_2$.

Conversely, suppose u_s's are fixed by ψ, where $s = 1, 2$. Thus $\psi(u_s) = u_s$, where $s = 1, 2$. From

$$\lambda = h_1 *_L u_1 + h_2 *_L u_2$$

we have

$$\begin{aligned} \theta(\lambda) = \theta(h_1 *_L u_1 + h_2 *_L u_2) &= h_1 *_L \psi(u_1) + h_2 *_L \psi(u_2) \\ &= h_1 *_L u_1 + h_2 *_L u_2 \end{aligned}$$

Hence α is fixed by θ. ■

Now, we will give the structures of skew θ-λ-constacyclic codes over $\mathcal{H}_{s,3}$, where a unit λ satisfies $\lambda *_L h_s = h_s *_L \lambda \quad (s = 1, 2)$ and $\{h_1 = 2 + j, h_2 = 2 + 2j\}$ is a central orthogonal idempotent pair.

Theorem 4.3. *Let $\lambda = h_1 * u_1 + h_2 * u_2 = 2, j$ or $2j$ be a uni in $\mathcal{H}_{s,3}$ and $C = h_1 *_L C_1 + h_2 *_L C_2$ be a linear code of length n over $\mathcal{H}_{s,3}$ such that ord $\theta \mid n$, where C_1, C_2 are cedes over $\mathbb{Z}_3 + i\mathbb{Z}_3$. If C is a skew θ-λ-constacyclic code of length n over $\mathcal{H}_{s,3}$, then C_s are ψ-u_s-constacyclic codes of length n over $\mathbb{Z}_3 + i\mathbb{Z}_3$, where $s = 1, 2$.*

Proof. Let $C = h_1 *_L C_1 + h_2 *_L C_2$ be a linear code over $\mathcal{H}_{s,3}$. Then for each $c = (c_0, \ldots, c_{n-1}) \in C$, where $c_y = h_1 *_L a_y + h_2 *_L b_y$ and $a_y, b_y \in \mathbb{Z}_3 + i\mathbb{Z}_3$ for $y = 0, 1, \ldots, n-1$, we have $\mathbf{a} = (a_0, \ldots, a_{n-1}) \in C_1, \mathbf{b} = (b_0, \ldots, b_{n-1}) \in C_2$. Assume that C is a skew θ-λ-constacyclic code of length n over $\mathcal{H}_{s,3}$. Then $\sigma_\lambda(c) = (\lambda *_L \theta(c_{n-1}), \theta(c_0), ..., \theta(c_{n-2})) \in C$ for each $c = (c_0, \ldots, c_{n-1}) \in C$. Then we get

$$\begin{aligned}
\lambda *_L \theta(c_{n-1}) =& \lambda *_L \theta(h_1 *_L a_{n-1} + h_2 *_L b_{n-1}) \\
&= \lambda *_L [h_1 *_L \psi(a_{n-1}) + h_2 *_L \psi(b_{n-1})] \\
&= \lambda *_L h_1 *_L \psi(a_{n-1}) + \lambda *_L h_2 *_L \psi(b_{n-1}) \\
&= h_1 *_L \lambda *_L \psi(a_{n-1}) + h_2 *_L \lambda *_L \psi(b_L) \\
&= h_1 *_L (h_1 *_L u_1 + h_2 *_L u_2) *_L \psi(a_{n-1}) + h_2 *_L (h_1 *_L u_1 + h_2 *_L u_2) *_L \psi(b_{n-1})) \\
&= h_1 *_L u_1 *_L \psi(a_{n-1}) + h_2 *_L u_2 *_L \psi(b_{n-1})
\end{aligned}$$

So

$$\begin{aligned}
\sigma_\lambda(c) =& (\lambda *_L \theta(c_{n-1}), \theta(c_0), \ldots, \theta(c_{n-2})) \\
=& h_1 *_L (u_1 *_L \psi(a_{n-1}), \psi(a_0), \ldots, \psi(a_{n-2})) + h_2 *_L (u_2 *_L \psi(b_{n-2}), \psi(b_0), \ldots, \psi(b_{n-2})) \in C
\end{aligned}$$

Hence

$$\sigma_{u_1}(\mathbf{a}) = (u_1 *_L \psi(a_{n-1}), \psi(a_0), \ldots, \psi(a_{n-2})) \in C_1$$

$$\sigma_{u_2}(\mathbf{b}) = (u_2 *_L \psi(b_{n-1}), \psi(b_0), \ldots, \psi(b_{n-2})) \in C_2$$

Therefore C_s's are skew ψ-u_s-constacyclic codes of length n over $\mathbb{Z}_3 + i\mathbb{Z}_3$, where $s = 1, 2$. ■

Theorem 4.4. *For u_1, u_2, let $\lambda = h_1 *_L u_1 + h_2 *_L u_2$ be a unit in $\mathcal{H}_{s,3}$. If C_s are skew ψ-u_s-constacyclic codes of length n over $\mathbb{Z}_3 + i\mathbb{Z}_3$, where $s = 1, 2$, then C is a skew θ-λ-constacyclic code over $\mathcal{H}_{s,3}$ of length n.*

Proof. Assume that C_s's are skew ψ-u_s-constacyclic codes of length n over $\mathbb{Z}_3 + i\mathbb{Z}_3$, where $s = 1, 2$. Then for each $\mathbf{a} \in C_1, \mathbf{b} \in C_2$, we have $\sigma_{u_1}(\mathbf{a}) \in C_1, \sigma_{u_2}(\mathbf{b}) \in C_2$. Note that

$$h_1 *_L \sigma_{u_1}(\mathbf{a}) + h_2 *_L \sigma_{u_2}(\mathbf{b}) = (\lambda *_L \theta(c_{n-1}), \theta(c_0), \cdots, \theta(c_{n-2})) = \sigma_\lambda(\mathbf{c}).$$

Therefore C is a skew θ–λ-constacyclic code of length n over $\mathcal{H}_{s,3}$. ■

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